CP violation in $t \bar{t}$ production and decay (at the LHC)

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CP Violation at LHC

- Would like some null-tests of CP violation at LHC (observables that vanish if CP is conserved):
  - "naive-T" odd triple products
  - ideally truly CP odd
- Start with a process that contains only SM particles in the final state
  - \( pp \rightarrow \bar{t} \ t \rightarrow \bar{b} \ W^- \ b \ W^+ \rightarrow \bar{b} \ l^- \ b \ l^+ (\nu\nu) \)
- Consider CP violation via new physics in both the production and decay vertices for top
  - two examples: Higgs and top (chromo)edm
**CP properties**

- pp initial state is **not** a CP eigenstate
  - get good CP properties from final state, for example $H$ decay to $t\bar{t}$, view LHC as Higgs factory (or $t\bar{t}$ factory)
  - this can be extended to $g\ g$ initial state by considering inclusive observables where we sum over gluon spin and color (which we do anyway)
  - also works for $q\bar{q}$ initial state
- observables may also involve jet momenta and thus represent sums over many processes
FIG. 5. $t\bar{t}$ production mechanisms at the LHC. The larger cross section indicated by the dashed line corresponds to gluon fusion. The dotted line indicates light $q\bar{q}$ annihilation. We also show the resonant Higgs production for two different values of the Higgs mass.
• gluon fusion dominant (but $q \bar{q}$ also studied)

• framework:
Spin correlations

- The underlying asymmetries are spin correlations
- The observables correspond to different ways of analyzing the spin

\[ \varepsilon(p_t, p_b^-, s_t, s^-) \]

CP violation in the production vertex

\[ \varepsilon(p_t, p_b, p_{l^+}, s_t) \]

CP violation in the decay vertex

\[ \varepsilon(p_t^-, p_b^-, p_{l^-}, s^-) \]
Parton CM asymmetry: \( bW \) “final states” or \( b l \ \nu \) “final states”

- Starting from the correlation:
- In parton CM:\[
\begin{align*}
\sqrt{s} p_t \cdot (p_b \times p_{\bar{b}}) & \xrightarrow{\text{CP}} \sqrt{s} (-p_i) \cdot (-p_{\bar{b}}) \times (-p_b)) \\
& = -\sqrt{s} p_t \cdot (p_b \times p_{\bar{b}})
\end{align*}
\]

- can be studied with the counting asymmetry:

\[
A_{CP} \equiv \frac{N_{\text{events}}(p_{\bar{b}} \cdot (p_b \times p_t) > 0) - N_{\text{events}}(p_{\bar{b}} \cdot (p_b \times p_t) < 0)}{N_{\text{events}}(p_{\bar{b}} \cdot (p_b \times p_t) > 0) + N_{\text{events}}(p_{\bar{b}} \cdot (p_b \times p_t) < 0)}
\]

- or the corresponding one with lepton momenta
‘Mixed’ helicity framework

start from:

\[
\mathcal{M} = -\frac{\bar{u}_b \Gamma_D (p_t + m_t) \Gamma_P (-\bar{p}_t + m_t) \Gamma_D v_b}{(p_t^2 - m_t^2)(p_t^2 - m_t^2)}.
\]

use the narrow width approximation for the top and rewrite numerators as top polarization sums.

square the amplitude and split into production and decay factors, first summing over b (and lepton) spin

\[
|\mathcal{M}|^2 = \left(\frac{\pi}{m_t \Gamma_t}\right)^2 \delta(p_t^2 - m_t^2) \delta(p_t^2 - m_t^2) \sum_{\lambda',\lambda,\sigma,\sigma'} \mathcal{T}_t(\lambda', \lambda) \mathcal{T}_t(\sigma, \sigma') \mathcal{T}_P(\lambda, \sigma, \sigma', \lambda')
\]

where we have defined the helicity factors

\[
\mathcal{T}_t(\lambda', \lambda) \equiv (\bar{u}_{t\lambda'} \gamma^0 \Gamma_D^\dagger \gamma^0 \bar{b} \Gamma_D u_{t\lambda})
\]

\[
\mathcal{T}_t(\sigma, \sigma') \equiv (\bar{v}_{\bar{t}\sigma} \Gamma_D \bar{p} \gamma^0 \Gamma_D^\dagger \gamma^0 v_{\bar{t}\sigma'})
\]

\[
\mathcal{T}_P(\lambda, \sigma, \sigma', \lambda') \equiv (\bar{u}_{t\lambda} \Gamma_P v_{t\sigma} \bar{v}_{\bar{t}\sigma'} \gamma^0 \Gamma_P^\dagger \gamma^0 u_{t\lambda'})
\]
• With these expressions we can find compact analytic results.
• we find results that factorize as follows:

\[
|M|^2_{CP} = C_1(s, t, u) (t - u) \epsilon(p_t, p_t, p_D, p_D) \\
+ C_2(s, t, u) \frac{s}{2} \epsilon(p_D, p_D, P, q) \\
+ C_3(s, t, u) (P \cdot p_D \epsilon(p_D, p_t, p_{\bar{t}}, q) + P \cdot p_{\bar{D}} \epsilon(p_D, p_t, p_{\bar{t}}, q))
\]

with

\[P \equiv p_1 + p_2 \quad q \equiv p_1 - p_2\]

for hadronic W decay \( p_D \rightarrow p_b, \quad p_{\bar{D}} \rightarrow p_{\bar{b}} \)

for leptonic W decay \( p_D \rightarrow p_{\ell^+}, \quad p_{\bar{D}} \rightarrow p_{\ell^-} \)
CP Violation in Neutral Higgs Sector

- Start with simple example: CP nature of a Higgs (heavy enough to decay into top pairs)
- coupling: \[-\frac{m_t}{\nu} H \bar{t}(A + i B \gamma_5) t.\]
- CP violation if both $A, B$ non-zero at the same time (multi-Higgs models)
- Look for $H \rightarrow \bar{t} t$ decays with subsequent top decay $t \rightarrow b W$
- $\Gamma \sim |A|^2 + |B|^2$ but $A_{CP} \sim AB$
Neutral Higgs production at LHC

\[ -[F_a(s)(-q_1 \cdot q_2 g_{\mu\nu} + q_{2\mu} q_{1\nu}) + F_b(s)\epsilon(\mu, \nu, q_1, q_2)]. \]

with

\[
F_a = \frac{g \alpha_s A}{4\pi M_W} \delta_{ab} \tau_t [1 + (1 - \tau_t)f(\tau_t)],
\]

\[
F_b = \frac{g \alpha_s B}{4\pi M_W} \delta_{ab} \tau_t f(\tau_t),
\]

where \( \tau_t = 4m_t^2/s \) and for \( \tau_t < 1 \)

\[
f(\tau) = -\frac{1}{4} \left[ \log \left( \frac{1 + \sqrt{1 - \tau}}{1 - \sqrt{1 - \tau}} \right) - i\pi \right]^2.
\]
Form Factors

- **Higgs case, bW final states:**
- **s-channel squared**

\[
C_1(s, t, u) = \frac{3}{32} K_{bb} \left( AB \left| F_a \right|^2 + \left| F_b \right|^2 \right) \frac{s^2}{\pi^2 \alpha_s^2} \frac{m_t^4}{(s - m_H^2)^2 + m_H^2 \Gamma_H^2} \frac{m_t^4}{v^2}
\]

\[
C_2(s, t, u) = C_3(s, t, u) = 0
\]

- **interference between s-channel and t and u channels**

\[
C_1(s, t, u) = -\frac{1}{2} K_{bb} \left( B \Re (F_a) + A \Re (F_b) \right) \frac{s^2}{\pi \alpha_s} \frac{m_t^4}{(s - m_H^2)(s^2 - (t - u)^2)} \frac{m_t^4}{v}
\]

\[
C_2(s, t, u) = C_3(s, t, u) = 0
\]

- **with:**

\[
K_{bb} \equiv (\pi^2 \alpha_s^2 g^4) \left( 2 - \frac{m_t^2}{M_W^2} \right)^2 \left( \frac{\pi}{m_t \Gamma_t} \right)^2 \delta(p_t^2 - m_t^2) \delta(p_t^2 - m_t^2).
\]
• Higgs case but b l ν final states:
  - $K_{bb} \rightarrow K_{ll}$ with

$$K_{\ell\ell} \equiv 16 (\pi^2 \alpha_s^2 g^8) (p_b \cdot p_\nu) (p_\bar{b} \cdot p_{\bar{\nu}}) \left( \frac{\pi}{m_t \Gamma_t} \right)^2 \left( \frac{\pi}{M_W \Gamma_W} \right)^2 \delta(p_t^2 - m_t^2) \delta(p_{\bar{t}}^2 - m_{\bar{t}}^2) \delta(p_{W^+}^2 - M_W^2) \delta(p_{W^-}^2 - M_W^2);$$

• And the correlations now involve the lepton momenta (instead of the b-jet)
Anomalous, CP violating, top-quark couplings

• For CP violation in the production process we may have a (chromo)edm of the top-quark:

\[ \mathcal{L}_{cdm} = -i g_s \frac{\tilde{d} t}{2} \sigma_{\mu \nu} \gamma_5 t G^{\mu \nu} \]

• It generates two types of vertices but the "seagulls" do not contribute to the CP asymmetry

\[ g t \tilde{t} \rightarrow -i g_s \frac{\lambda^a}{2} (\gamma_\mu + \tilde{d} \sigma_{\mu \nu} q^\nu \gamma_5) \]
\[ gg \tilde{t} \rightarrow i \pi \alpha_s [\lambda^b, \lambda^c] \tilde{d} \sigma_{\mu \nu} \gamma_5. \]
• Form factors for a top (chromo)edm
  
  - s-channel squared
  \[ C_1(s, t, u) = C_2(s, t, u) = C_3(s, t, u) = \frac{3}{2} \hat{d} K_{bb} m_t \frac{(t - u)}{s^2}. \]

  - t and u channels (squared + interf.)
  \[ C_1(s, t, u) = -\frac{1}{48} \hat{d} K_{bb} \frac{m_t}{s^2(t - m_t^2)^2(u - m_t^2)^2} \left[ 9(t - u)^5 - 2(5s - 36m_t^2)s(t - u)^3 
  + s^2(s^2 - 22sm_t^2 + 144m_t^4)(t - u) + \frac{14m_t^2s^4(s + 8m_t^2)}{(t - u)} \right] \]
  \[ C_2(s, t, u) = -\frac{1}{48} \hat{d} K_{bb} \frac{m_t}{s^2(t - m_t^2)^2(u - m_t^2)^2} \left[ 9(t - u)^5 - 2(5s - 9m_t^2)s(t - u)^3 
  + s^2(s^2 + 46sm_t^2)(t - u) \right] \]
  \[ C_3(s, t, u) = C_2(s, t, u). \]

  - interf, between s-channel and t,u channels
  \[ C_1(s, t, u) = -\frac{3}{4} \hat{d} K_{bb} \frac{m_t(t - u)}{s^2(t - m_t^2)(u - m_t^2)} \left( -4sm_t^2 + s^2 - (t - u)^2 \right) \]
  \[ C_2(s, t, u) = -\frac{3}{4} \hat{d} K_{bb} \frac{m_t(t - u)}{s^2(t - m_t^2)(u - m_t^2)} \left( -2sm_t^2 + s^2 - (t - u)^2 \right) \]
  \[ C_3(s, t, u) = C_2(s, t, u). \]
the form factors for leptonic final states can be obtained from these simply by replacing $K_{bb}$ with $K_{ll}$ where

$$K_{ll} \equiv 16 (\pi^2 \alpha_s^2 g^8) (p_b \cdot p_\nu) (p_\bar{b} \cdot p_\bar{\nu}) \left( \frac{\pi}{m_t \Gamma_t} \right)^2 \left( \frac{\pi}{M_W \Gamma_W} \right)^2 \times \delta(p_t^2 - m_t^2) \delta(p_i^2 - m_i^2) \delta(p_{W+}^2 - M_W^2) \delta(p_{W-}^2 - M_W^2);$$

now the narrow width approximation has been used for the $W$ as well.

similar expressions are easily obtained for the case of one $W$ decaying leptonically.
For CP violation in the decay vertex:

- Higgs exchange models are very suppressed by lepton and light quark masses.
- For anomalous $tbW$ couplings, there is only one that interferes with the SM amplitude in the limit of massless $b$ quark:

\[
\Gamma_{Wtb}^\mu = -\frac{g}{\sqrt{2}} V_{tb}^* \bar{u}(p_b) \left[ \gamma_\mu (f_1^L P_L + f_1^R P_R) - i \sigma^{\mu\nu} (p_t - p_b) \nu (f_2^L P_L + f_2^R P_R) u(p_t), \right. \\
\Gamma_{Wtb}^\bar{\mu} = -\frac{g}{\sqrt{2}} V_{tb} \bar{\nu}(p_{\bar{t}}) \left[ \gamma_\nu (\bar{f}_1^L P_L + \bar{f}_1^R P_R) - i \sigma^{\nu\mu} (p_{\bar{t}} - p_{\bar{b}}) \nu (\bar{f}_2^L P_L + \bar{f}_2^R P_R) \right] \nu(p_{\bar{b}}),
\]

- include absorptive phases this time and take:

\[
f_1^L = \bar{f}_1^L = 1, \quad f_2^R = f e^{i(\phi_f + \delta_f)}, \quad \bar{f}_2^L = f e^{i(-\phi_f + \delta_f)}.
\]

- no seagulls (with a gluon) are present
• Now the triple products naturally take the form

\[ |\mathcal{M}|_T^2 = f \sin(\delta_f + \phi_f) \epsilon(p_t, p_b, p_{\ell+}, Q_t) + f \sin(\delta_f - \phi_f) \epsilon(p_\bar{t}, p_\bar{b}, p_{\ell-}, Q_\bar{t}) \]

- Q is a four momentum that analyzes the top spin and is a linear combination of the other momenta in the diagram (those not related to top decay)
- the correlations are NOT CP odd, they can be faked by absorptive phases
- true CP odd observables can be constructed by comparing the top and anti-top decays
**top-spin analyzer** \( Q_t : \)

- **s-channel squared for example yields:**

\[
Q_t = -K_{\ell\ell} \frac{3m_t}{2s^2} \left\{ ((t-u)^2 - s^2)p_{\ell^-} + 2(s p_{\ell^-} \cdot (p_t + p_i) - (t-u)p_{\ell^-} \cdot q)p_i \right. \\
+ 2((t-u)p_{\ell^-} \cdot (p_t + p_i) - s p_{\ell^-} \cdot q)q \right\}
\]

- **t and u channels and their interference with the s-channel give more complicated but similar expressions**

- **need at least one leptonic W decay to get a non-zero result**
Some Numerics
Look at an isolated $H$ decay

- For $H \rightarrow \bar{t} \; t \rightarrow \bar{b} \; W^- \; b \; W^+$ we can use the invariant calculated before $\varepsilon(p_t,p_{\bar{t}},p_b,p_{\bar{b}})$
- Construct the counting asymmetry:

$$A_{CP} \equiv \frac{N_{\text{events}}(\vec{p}_{\bar{b}} \cdot (\vec{p}_{\bar{b}} \times \vec{p}_t) > 0) - N_{\text{events}}(\vec{p}_{\bar{b}} \cdot (\vec{p}_{\bar{b}} \times \vec{p}_t) < 0)}{N_{\text{events}}(\vec{p}_{\bar{b}} \cdot (\vec{p}_{\bar{b}} \times \vec{p}_t) > 0) + N_{\text{events}}(\vec{p}_{\bar{b}} \cdot (\vec{p}_{\bar{b}} \times \vec{p}_t) < 0)}$$

$$= \frac{\pi}{4} \sqrt{1 - \frac{4m_t^2}{M_H^2}} \frac{AB}{|A|^2 + |B|^2} \frac{(1 - \frac{2M_w^2}{m_t^2})^2}{(1 + \frac{2M_w^2}{m_t^2})^2} \cdot \left( \frac{1}{1 - \frac{2m_t^2}{M_H^2} - 2 \frac{|A|^2 - |B|^2}{|A|^2 + |B|^2} \frac{m_t^2}{M_H^2}} \right)$$
How large can it be?

• With some assumptions:
  - lightest neutral mass eigenstate dominate
  - different vevs have comparable sizes

• Weinberg finds inequalities that translate into

\[ |AB| \leq \frac{1}{\sqrt{2}} \]

• and shows models where the upper bound is reached (Phys.Rev. D42 (1990) 860)
In H decay (H rest frame)

\[ A_{CP} \]

\[ M_H(\text{GeV}) \]

\[ A B = 1/\sqrt{2} \]
LHC?

- There can be a large "intrinsic" asymmetry
- In calculating $A_{CP}$ we didn’t "use the LHC" so $X \rightarrow X_{CP}$ didn’t affect us
- treat the LHC as a “Higgs factory” and start with the $H$ as initial state
- to what extent can we do the same for $\bar{t}t$ initial states?
- Assume we can, but there are still other complications to consider
Non-resonant $t\bar{t}$ production

- At the LHC the signal gets diluted by non-resonant $t\bar{t}$ pair production that enters mostly the denominator.
- There was also a new contribution to the numerator from the interference between the diagrams:

![Diagrams](image)

FIG. 3. Three diagrams responsible for CP asymmetry in top-quark pair production.
More dilution

• The asymmetry also requires reconstruction of t and b directions in parton CM frame (H rest frame)

\[
A_{CP} = \frac{N_{\text{events}}(\vec{p}_t \cdot (\vec{p}_b \times \vec{p}_t) > 0) - N_{\text{events}}(\vec{p}_t \cdot (\vec{p}_b \times \vec{p}_t) < 0)}{N_{\text{events}}(\vec{p}_t \cdot (\vec{p}_b \times \vec{p}_t) > 0) + N_{\text{events}}(\vec{p}_t \cdot (\vec{p}_b \times \vec{p}_t) < 0)}
\]

(13)

• This is not CP-odd in the lab frame, however the invariant form gives rise to more complicated asymmetries for the lab frame - more dilution

• Assuming reconstruction of top and bottom momenta directions in the lab frame we could use:

\[
\hat{A}_{CP} = \frac{N_{\text{events}}((\vec{p}_t - \vec{p}_t) \cdot (\vec{p}_b \times \vec{p}_b) > 0) - N_{\text{events}}((\vec{p}_t - \vec{p}_t) \cdot (\vec{p}_b \times \vec{p}_b) < 0)}{N_{\text{events}}((\vec{p}_t - \vec{p}_t) \cdot (\vec{p}_b \times \vec{p}_b) > 0) + N_{\text{events}}((\vec{p}_t - \vec{p}_t) \cdot (\vec{p}_b \times \vec{p}_b) < 0)}
\]
recall the "intrinsic" asymmetry could be as large as 0.065
Comments

- The resulting asymmetry is purely $CP$ violating, even if the calculation includes absorptive phases in the Higgs production vertex.
- For LHC numbers we used CTEQ6M pdf’s at $\sqrt{S} = 14$ TeV.
- The asymmetry is smaller than in Higgs decay in this model since non-resonant production is larger and $CP$ conserving.
- $q \bar{q}$ annihilation very small contribution to asymmetry.
For the anomalous couplings

- The asymmetries are not always CP odd, in particular for CP violation in the decay vertex
- Size of CP even terms could be comparable
- Phenomenology for this case in progress but there exists much previous work:
## Some Numerics (previous work)

<table>
<thead>
<tr>
<th></th>
<th>$\tilde{d} \left[ \frac{1}{m_t} \right]$</th>
<th>$f \left[ \frac{1}{m_t} \right]$</th>
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<tr>
<td><strong>Theory Estimate</strong></td>
<td>$&lt;10^{-13}$ SM*</td>
<td>0.03 QCD (CP conserving!)</td>
</tr>
<tr>
<td></td>
<td>$\sim 10^{-6}$ H*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\sim 10^{-3}$ SUSY*</td>
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<td><strong>Atlas reach</strong></td>
<td>$0.046$ at $5\sigma$ with $10\text{fb}^{-1}$ &amp;</td>
<td>$\sim 0.01$ at $2\sigma$ with $10\text{fb}^{-1}$ $</td>
</tr>
</tbody>
</table>


Summary

• We have presented examples of triple product correlations for the LHC that are really CP-odd and others that can be faked by unitarity phases.
• Since the LHC is a pp collider, our signals rely on the CP properties of the final state.
• We have estimated the asymmetry in a simple model of CP violation.
• More detailed phenomenological studies are under way.
• Other models that give large CP at LHC?