Latest Results on $\rho$ and $\delta$ from Muon Decay

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for the TWIST Collaboration
Muon decay spectrum

The energy and angle distributions of positrons following polarized muon decay obey the spectrum:

\[
\frac{d^2\Gamma}{x^2dx\,d(\cos\theta)} \propto (3 - 3x) + \frac{2}{3}\rho(4x - 3) + 3\eta\frac{x_0}{x}(1 - x) \\
+ P_{\mu}\xi_0\xi \cos\theta \left[(1 - x) + \frac{2}{3}\delta(4x - 3)\right]
\]

where \( x = \frac{E_e}{E_{e,\text{max}}} \)

[Radiative corrections not included]
Muon decay matrix element

- Most general local, derivative-free, lepton-number conserving muon decay matrix element:

\[ M = \frac{4G_F}{\sqrt{2}} \sum_{\gamma=S,V,T} g_{\epsilon\mu}^\gamma \langle \bar{\epsilon}_\epsilon | \Gamma_\gamma | (\nu_\epsilon)_n \rangle \langle (\bar{\nu}_\mu)_m | \Gamma_\gamma | \mu_\mu \rangle \]

- In the Standard Model, \( g_{LL}^V = 1 \), all others are zero

- Pre-TWIST global fit results (all 90% c.l.):

| \( |g_{RR}^S| < 0.066 \) | \( |g_{RR}^V| < 0.033 \) | \( |g_{RR}^T| \equiv 0 \) |
|---|---|---|
| \( |g_{LR}^S| < 0.125 \) | \( |g_{LR}^V| < 0.060 \) | \( |g_{LR}^T| < 0.036 \) |
| \( |g_{RL}^S| < 0.424 \) | \( |g_{RL}^V| < 0.110 \) | \( |g_{RL}^T| < 0.122 \) |
| \( |g_{LL}^S| < 0.550 \) | \( |g_{LL}^V| > 0.960 \) | \( |g_{LL}^T| \equiv 0 \) |
Muon decay parameters and coupling constants

$$\rho = \frac{3}{4} - \frac{3}{4} \left[ |g_{RL}^V|^2 + |g_{LR}^V|^2 + 2 |g_{RL}^T|^2 + 2 |g_{LR}^T|^2 \right] + \Re \left( g_{RL}^S g_{RL}^{T*} + g_{LR}^S g_{LR}^{T*} \right)$$

$$\eta = \frac{1}{2} \Re \left[ g_{RR}^V g_{LL}^{S*} + g_{LL}^V g_{RR}^{S*} + g_{RL}^V (g_{LR}^{S*} + 6 g_{LR}^{T*}) + g_{LR}^V (g_{RL}^{S*} + 6 g_{RL}^{T*}) \right]$$

$$\xi = 1 - \frac{1}{2} |g_{LR}^S|^2 - \frac{1}{2} |g_{RR}^S|^2 - 4 |g_{RL}^V|^2 + 2 |g_{LR}^V|^2 - 2 |g_{RR}^V|^2$$

$$+ 2 |g_{LR}^T|^2 - 8 |g_{RL}^T|^2 + 4 \Re \left( g_{LR}^S g_{RL}^{T*} - g_{RL}^S g_{LR}^{T*} \right)$$

$$\xi \delta = \frac{3}{4} - \frac{3}{8} |g_{RR}^S|^2 - \frac{3}{8} |g_{LR}^S|^2 - \frac{3}{2} |g_{RR}^V|^2 - \frac{3}{4} |g_{RL}^V|^2 - \frac{3}{4} |g_{LR}^V|^2$$

$$- \frac{3}{2} |g_{RL}^T|^2 - 3 |g_{LR}^T|^2 + \frac{3}{4} \Re \left( g_{LR}^S g_{RL}^{T*} - g_{RL}^S g_{LR}^{T*} \right)$$

SM

$$\rho = 0.7518 \pm 0.0026 \quad 3/4$$

$$\eta = -0.007 \pm 0.013 \quad 0$$

Prior to **TWIST**

$$P_{\mu}^\xi = 1.0027 \pm 0.0079 \pm 0.0030 \quad 1$$

$$\delta = 0.7486 \pm 0.0026 \pm 0.0028 \quad 3/4$$

$$P_{\mu}^\xi(\xi \delta / \rho) > 0.99682 \; (90\% \; c.l.) \quad 1$$

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Goal of *TWIST*

- Search for new physics that can be revealed by **order-of-magnitude improvements** in our knowledge of $\rho$, $\delta$, and $P_\mu \xi$

Two examples

- Model-independent limit on muon handedness

\[
Q^\mu_R = \frac{1}{2} \left[ 1 + \frac{1}{3} \xi - \frac{16}{9} \xi \delta \right]
\]

- Left-right symmetric models

\[
\frac{3}{4} - \rho = \frac{3}{2} \xi^2 \quad 1 - P_\mu \xi = 4 \left( \xi^2 + \xi \left( \frac{M_L}{M_R} \right)^2 + \left( \frac{M_L}{M_R} \right)^4 \right)
\]

- …..
What is required?

Must:
- Determine spectrum shape
  -- All three parameters
- Understand sources of muon depolarization
  -- $P_\mu$ and $\xi$ come as a product
- Measure forward-backward asymmetry
  -- For $P_\mu \xi$ and $\delta$

to within a few parts in $10^4$

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Surface muon beam
Detector array

- 56 low-mass high-precision planar chambers symmetrically placed around thin target foil (DME, CF$_4$/Isobutane)
- Measurement initiated by single thin scintillation counter at entrance to detector
- Beam stop position controlled by variable He/CO$_2$ gas degrader

Yu.Davydov et al. NIM A461(2001)68
R.Henderson et al. NIM A548(2005)306

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Typical events

• Use pattern recognition (in position and time) to sort hits into tracks, then fit to helix

• Must also recognize beam positrons, delta tracks, backscattering tracks
Physics data sets

- **Fall 2002**
  - Test data-taking procedures and develop analysis techniques
  - First physics results – $\rho$ and $\delta$
  - Graphite-coated Mylar target not suitable for $P_{\mu \xi}$

- **Fall 2004**
  - Al target (70 $\mu$m) and Time Expansion Chamber enabled first $P_{\mu \xi}$ measurement
  - Improved determinations of $\rho$ and $\delta$ recently published

- **2006-07**
  - Ag and Al target data
  - Larger data sets and better beam characterization
  - Achieve ultimate **TWIST** precision for $\rho$, $\delta$, and $P_{\mu \xi}$
Analysis method

• Extract energy and angle distributions for data:
  – Apply (unbiased) cuts on muon variables.
  – Reject fast decays and backgrounds.
  – Calibrate $e^+$ energy to kinematic end point at 52.83 MeV.

• Fit to identically derived distributions from simulation:
  – GEANT3 geometry contains virtually all detector components.
  – Simulate chamber response in detail.
  – Realistic, measured beam profile and divergence.
  – Extra muon and beam positron contamination included.
  – Output in digitized format, identical to real data.
2-d momentum-angle spectrum

Acceptance of the **TWIST** spectrometer

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Fitting the data distributions

• Decay distribution is linear in $\rho$, $\eta$, $P_{\mu \xi}$, and $P_{\mu \xi \delta}$, so a fit to first order expansion is exact.

• Fit data to simulated (MC) base distribution with hidden assumed parameters,

$$\lambda_{MC} = (\rho, \eta, P_{\mu \xi}, P_{\mu \xi \delta}, P_{\mu \xi \delta})$$

plus MC-generated distributions from analytic derivatives, times fitting parameters ($\Delta \lambda$) representing deviations from base MC. ($\eta$ is now fixed to global analysis value).

(graphic thanks to Blair Jamieson)
Validating the Monte Carlo with “upstream stops”
Fitting the 2002 data to determine $\rho$ and $\delta$

Normalized residuals \[\frac{(\text{Data-Fit})}{\text{sigma}}\] of the 2-d momentum-angle fit

Fit describes the data well, even when extrapolated far outside the fiducial region

Angle-integrated results

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First *TWIST* results for $\rho$ and $\delta$

- From Fall, 2002 run:
  - $\rho = 0.75080 \pm 0.00032$ (stat) $\pm 0.00097$ (syst) $\pm 0.00023$ ($\eta$)
    
    J. Musser et al., PRL *94*, 101805

  - $\delta = 0.74964 \pm 0.00066$ (stat) $\pm 0.00112$ (syst)
    
    A. Gaponenko et al., PRD *71*, 071101
Systematics in the first measurements

The same effects tend to dominate the systematic uncertainties for both $\rho$ and $\delta$.

### TABLE II. Contributions to the systematic uncertainty in $\rho$.
Average values are given for those denoted (av), which are considered set dependent when performing the weighted average of the data sets.

<table>
<thead>
<tr>
<th>Effect</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chamber response (av)</td>
<td>$\pm 0.00051$</td>
</tr>
<tr>
<td>Stopping target thickness</td>
<td>$\pm 0.00049$</td>
</tr>
<tr>
<td>Positron interactions</td>
<td>$\pm 0.00046$</td>
</tr>
<tr>
<td>Spectrometer alignment</td>
<td>$\pm 0.00022$</td>
</tr>
<tr>
<td>Momentum calibration (av)</td>
<td>$\pm 0.00020$</td>
</tr>
<tr>
<td>Theoretical radiative corrections [12]</td>
<td>$\pm 0.00020$</td>
</tr>
<tr>
<td>Track selection algorithm</td>
<td>$\pm 0.00011$</td>
</tr>
<tr>
<td>Muon beam stability (av)</td>
<td>$\pm 0.00004$</td>
</tr>
<tr>
<td>Total in quadrature</td>
<td>$\pm 0.00093$</td>
</tr>
<tr>
<td>Scaled total</td>
<td>$\pm 0.00097$</td>
</tr>
</tbody>
</table>

Systematic uncertainties typically determined from data sets with a possible problem exaggerated or by MC done with an exaggerated ‘defect’ put into detector.

### TABLE II. Contributions to the systematic uncertainty for $\delta$.
Average values are denoted by (ave), which are considered set-dependent when performing the weighted average of data sets.

<table>
<thead>
<tr>
<th>Effect</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spectrometer alignment</td>
<td>$\pm 0.00061$</td>
</tr>
<tr>
<td>Chamber response (ave)</td>
<td>$\pm 0.00056$</td>
</tr>
<tr>
<td>Positron interactions</td>
<td>$\pm 0.00055$</td>
</tr>
<tr>
<td>Stopping target thickness</td>
<td>$\pm 0.00037$</td>
</tr>
<tr>
<td>Momentum calibration (ave)</td>
<td>$\pm 0.00029$</td>
</tr>
<tr>
<td>Muon beam stability (ave)</td>
<td>$\pm 0.00010$</td>
</tr>
<tr>
<td>Theoretical radiative corrections [9]</td>
<td>$\pm 0.00010$</td>
</tr>
<tr>
<td>Upstream/downstream efficiencies</td>
<td>$\pm 0.00004$</td>
</tr>
</tbody>
</table>
Global Analysis

Use general form of interaction:

\[ M = \frac{4G_F}{\sqrt{2}} \sum_{\gamma=S,V,T} g_{\epsilon \mu}^\gamma \begin{pmatrix} \bar{e}_\epsilon \mid \Gamma_\gamma \mid (\nu_e)_n \end{pmatrix} \begin{pmatrix} (\bar{\nu}_\mu)_m \mid \Gamma_\gamma \mid \mu_\mu \end{pmatrix} \]

Global Analysis

\[ Q_{RR} = \frac{1}{4} |g_{RR}^S|^2 + |g_{RR}^V|^2, \]
\[ Q_{LR} = \frac{1}{4} |g_{LR}^S|^2 + |g_{LR}^V|^2 + 3|g_{LR}^T|^2, \]
\[ Q_{RL} = \frac{1}{4} |g_{RL}^S|^2 + |g_{RL}^V|^2 + 3|g_{RL}^T|^2, \]
\[ Q_{LL} = \frac{1}{4} |g_{LL}^S|^2 + |g_{LL}^V|^2, \]
\[ B_{LR} = \frac{1}{16} |g_{LR}^S + 6g_{LR}^T|^2 + |g_{LR}^V|^2, \]
\[ B_{RL} = \frac{1}{16} |g_{RL}^S + 6g_{RL}^T|^2 + |g_{RL}^V|^2, \]
\[ I_\alpha = \frac{1}{4} \left[ g_{LR}^V (g_{RL}^S + 6g_{RL}^T)^* + (g_{RL}^V)^* (g_{LR}^S + 6g_{LR}^T) \right] \]
\[ = (\alpha + i\alpha')/2A, \]
\[ I_\beta = \frac{1}{2} \left[ g_{LL}^V (g_{RR}^S)^* + (g_{RR}^V)^* g_{LL}^S \right] = -2(\beta + i\beta')/A \]

Constraints:

\[ 0 \leq Q_{\epsilon\mu} \leq 1, \quad \text{where } \epsilon, \mu = R, L, \]
\[ 0 \leq B_{\epsilon\mu} \leq Q_{\epsilon\mu}, \quad \text{where } \epsilon\mu = RL, LR, \]
\[ |I_\alpha|^2 \leq B_{LR} B_{RL}, \quad |I_\beta|^2 \leq Q_{LL} Q_{RR}, \]

Normalization:

\[ Q_{RR} + Q_{LR} + Q_{RL} + Q_{LL} = 1 \]

Note that \( Q_{LL} \approx 1 \)

(from Phys. Lett. 173B)

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Global Analysis

Relation to muon decay observables:

\[ \rho = \frac{3}{4} + \frac{1}{4}(Q_{LR} + Q_{RL}) - (B_{LR} + B_{RL}), \]

\[ \xi = 1 - 2Q_{RR} - \frac{10}{3}Q_{LR} + \frac{4}{3}Q_{RL} + \frac{16}{3}(B_{LR} - B_{RL}), \]

\[ \xi \delta = \frac{3}{4} - \frac{3}{2}Q_{RR} - \frac{7}{4}Q_{LR} + \frac{1}{4}Q_{RL} + (B_{LR} - B_{RL}), \]

\[ \xi' = 1 - 2Q_{RR} - 2Q_{RL}, \]

\[ \xi'' = 1 - \frac{10}{3}(Q_{LR} + Q_{RL}) + \frac{16}{3}(B_{LR} + B_{RL}), \]

rad. decay \{ \eta = \frac{1}{3}(Q_{LR} + Q_{RL}) + \frac{2}{3}(B_{LR} + B_{RL}), \}

\[ e^{+}_L \{ \eta = (\alpha - 2\beta)/A, \quad \eta'' = (3\alpha + 2\beta)/A. \]

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# Global Analysis

## 2005 Input:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>$0.7518 \pm 0.0026$</td>
</tr>
<tr>
<td></td>
<td>$0.75080 \pm 0.00105^a$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$0.7486 \pm 0.0038$</td>
</tr>
<tr>
<td></td>
<td>$0.74964 \pm 0.00130$</td>
</tr>
<tr>
<td>$P_\mu \xi$</td>
<td>$1.0027 \pm 0.0085^b$</td>
</tr>
<tr>
<td>$P_\mu \xi \delta / \rho$</td>
<td>$0.99787 \pm 0.00082^b$</td>
</tr>
<tr>
<td>$\xi'$</td>
<td>$1.00 \pm 0.04$</td>
</tr>
<tr>
<td>$\xi''$</td>
<td>$0.65 \pm 0.36$</td>
</tr>
<tr>
<td>$\bar \eta$</td>
<td>$0.02 \pm 0.08$</td>
</tr>
<tr>
<td>$\alpha / A$</td>
<td>$0.015 \pm 0.052^c$</td>
</tr>
<tr>
<td>$\beta / A$</td>
<td>$0.002 \pm 0.018^c$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$0.071 \pm 0.037^d$</td>
</tr>
<tr>
<td>$\eta''$</td>
<td>$0.105 \pm 0.052^d$</td>
</tr>
<tr>
<td>$\alpha' / A$</td>
<td>$-0.047 \pm 0.052^e$</td>
</tr>
<tr>
<td></td>
<td>$-0.0034 \pm 0.0219^f$</td>
</tr>
<tr>
<td>$\beta' / A$</td>
<td>$0.017 \pm 0.018^e$</td>
</tr>
<tr>
<td></td>
<td>$-0.0005 \pm 0.0080^f$</td>
</tr>
</tbody>
</table>

## 2005 Output:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fit Result ($\times 10^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{RR}$</td>
<td>$&lt;1.14(0.60 \pm 0.38)$</td>
</tr>
<tr>
<td>$Q_{LR}$</td>
<td>$&lt;1.94(1.22 \pm 0.53)$</td>
</tr>
<tr>
<td>$B_{LR}$</td>
<td>$&lt;1.27(0.72 \pm 0.40)$</td>
</tr>
<tr>
<td>$Q_{RL}$</td>
<td>$&lt;44(26 \pm 13)$</td>
</tr>
<tr>
<td>$B_{RL}$</td>
<td>$&lt;10.9(6.4 \pm 3.3)$</td>
</tr>
<tr>
<td>$Q_{LL}$</td>
<td>$&gt;955(973 \pm 13)$</td>
</tr>
<tr>
<td>$\alpha / A$</td>
<td>$0.3 \pm 2.1$</td>
</tr>
<tr>
<td>$\beta / A$</td>
<td>$2.0 \pm 3.1$</td>
</tr>
<tr>
<td>$\alpha' / A$</td>
<td>$-0.1 \pm 2.2$</td>
</tr>
<tr>
<td>$\beta' / A$</td>
<td>$-0.8 \pm 3.2$</td>
</tr>
</tbody>
</table>

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Reducing the leading systematics

• Issues that were unique to 2002 data
  – Stopping target thickness uncertainty
  – Chamber orientation uncertainty with respect to magnetic field

• Improvements in 2004 data
  – Chamber response
    • Improved gas system regulation and monitoring
    • Improved determination of foil geometry
    • Improved treatment of drift chamber behavior
  – Positron interactions better understood
  – Detector fully instrumented
  – Improved alignment techniques and understanding of uncertainties
  – New momentum calibration techniques (uncertainty is statistical)
  – Radiative corrections uncertainty evaluated
### Systematic uncertainties for 2004 data: ρ and δ

<table>
<thead>
<tr>
<th>Systematic uncertainties</th>
<th>ρ (×10⁴)</th>
<th>δ (×10⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2002</td>
<td>2004</td>
</tr>
<tr>
<td>Chamber response (ave)</td>
<td>5.1</td>
<td>2.9</td>
</tr>
<tr>
<td>Stopping target thickness</td>
<td>4.9</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>Positron interactions</td>
<td>4.6</td>
<td>1.6</td>
</tr>
<tr>
<td>Spectrometer alignment</td>
<td>2.2</td>
<td>0.3</td>
</tr>
<tr>
<td>Momentum calibration (ave)</td>
<td>2.0</td>
<td>2.9</td>
</tr>
<tr>
<td>Theoretical radiative correction</td>
<td>2.0</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>Other</td>
<td>1.2</td>
<td>1.1</td>
</tr>
<tr>
<td><strong>Total in quadrature</strong></td>
<td><strong>9.2</strong></td>
<td><strong>4.6</strong></td>
</tr>
</tbody>
</table>
Consistency Checks: $\rho$ and $\delta$

- Data sets for 2004 analysis
- $\Delta$’s from fits to MC
- No corrections applied
- Decay parameters in BB still hidden

$\Delta\rho$ vs set

$\Delta\delta$ vs set

$\chi^2 / \text{ndf}$

$\rho$ and $\delta$
Results to date

- From Fall, 2002 run:
  - $\rho = 0.75080 \pm 0.00032$ (stat) $\pm 0.00097$ (syst) $\pm 0.00023$ ($\eta$)
  - $\delta = 0.74964 \pm 0.00066$ (stat) $\pm 0.00112$ (syst)

- From Fall, 2004 run:
  - $\rho = 0.75014 \pm 0.00017$ (stat) $\pm 0.00044$ (syst) $\pm 0.00011$ ($\eta$)
  - $\delta = 0.74964 \pm 0.00030$ (stat) $\pm 0.00067$ (syst)

R. McDonald et al., PRD 78, 032010
### Global Analysis Results

<table>
<thead>
<tr>
<th></th>
<th>Pre-TWIST</th>
<th>2002 Data</th>
<th>2004 Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>g^S_{LR}</td>
<td>$</td>
<td>$&lt;0.125$</td>
</tr>
<tr>
<td>$</td>
<td>g^V_{LR}</td>
<td>$</td>
<td>$&lt;0.066$</td>
</tr>
<tr>
<td>$</td>
<td>g^T_{LR}</td>
<td>$</td>
<td>$&lt;0.036$</td>
</tr>
<tr>
<td>$Q^\mu_R$</td>
<td>$&lt;0.0051$</td>
<td>$&lt;0.0031$</td>
<td>$&lt;0.0024$</td>
</tr>
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</table>

90% confidence limits
## Final Uncertainty Goals

<table>
<thead>
<tr>
<th></th>
<th>Published</th>
<th>Final (est.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<tr>
<td></td>
<td>Systematics</td>
<td>Systematics</td>
</tr>
<tr>
<td>$\rho$</td>
<td>1.7</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>4.4</td>
<td>2.4</td>
</tr>
<tr>
<td>$\delta$</td>
<td>3.0</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>6.7</td>
<td>3.2</td>
</tr>
</tbody>
</table>

All values in units of $10^{-4}$

Final Publications in 2009
TWIST Collaboration
TWIST Participants

TRIUMF
Ryan Bayes *†
Yuri Davydov
Jaap Doornbos
Wayne Faszer
Makoto Fujiwara
David Gill
Alex Grossheim
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Jingliang Hu
John A. Macdonald ‡
Glen Marshall
Dick Mischke
Mina Nozar
Konstantin Olchanski
Art Olin y
Robert Openshaw
Tracy Porcelli §
Jean-Michel Poutissou
Renée Poutissou
Grant Sheffer
Bill Shin §§

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Valparaiso
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Shirvel Stanislaus

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* Graduate student
** Graduated
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§§ also Saskatchewan
‡ deceased

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Robert Tribble – INT, October, 2008