Connecting LHC Measurements
(of Supersymmetry)
to High Scale Theories

David Morrissey,
Harvard University

with Gordy Kane, Piyush Kumar, and Manuel Toharia


INT Workshop, University of Washington, October 1, 2008
Motivation

• The LHC has turned on!

• Useful data taking is to start this spring.

• What do we (as theorists) hope to get out of the LHC?
  
  A SM Higgs boson?

  Some new particles?

  A deeper understanding of fundamental physics?

• For this, we must connect LHC measurements to underlying theories.
Step 1: Parameter Extraction from Data

- “the LHC inverse problem”
- A non-trivial and difficult problem . . .
- Standard Techniques: peaks, edges, endpoints. \(\text{(i.e. ATLAS TDR)}\)
- Recent Progress: spin determination, \(m_{T^2}\), jet/event shapes, . . .
- Suppose the LHC has discovered new physics, and these techniques have allowed us to construct a compelling low-scale model and get Lagrangian parameters.

- What next?
Step 2: Model Interpretation

- **What next?** Depends on the model . . .

- **Warped, UED, Little Higgs, Strong Coupling**
  - possibly non-perturbative completion
  - build a bigger collider?

- **Supersymmetry (or some new particles and fine-tuning)**
  - test high-scale theories against data by running down (model dependent)
  - run perturbatively up to higher scales (model independent)

- **We will focus on low-to-high running in SUSY.**

- The apparent unification of couplings in the MSSM suggests that this might be a sensible approach.
Obstacles to Running Up

- Renormalization group (RG) running is needed to extrapolate the parameter values at $M_{ew}$ to $M_{GUT}$ or $M_{mess}$.

- Obstacles:
  1. Extracting SUSY parameters from data is challenging.
     
     \[ \text{events} \rightarrow \text{masses/couplings} \rightarrow \text{Lagrangian parameters} \]

  2. Experimental and theoretical uncertainties in parameter values at $M_{ew}$ can become magnified by the RG flow.
     
     Some combinations of parameters are better than others.

  3. New intermediate scale physics can modify the predictions one would get assuming a grand desert.
     
     Unification strongly constrains the possibilities.
Outline

• We consider two classes of obstacles to RG running.

• Sensitivity to input uncertainties within the MSSM:
  MSSM running and the $S$ term.

• New intermediate scale physics:
  1. complete GUT multiplets at an intermediate scale.
  2. heavy right-handed neutrinos for seesaw neutrino masses.
  3. modifications due to the hidden sector.

• For both cases, we examine the effects on the low-to-high RG running of various Snowmass (SPS) mSUGRA points.
Assumptions

1. We work to two loops in the RG equations.

2. Only third generation Yukawas are taken into account.

3. Flavour universality:

   \[
   m_{ij}^2 = \begin{pmatrix} m_{12}^2 & 0 & 0 \\ 0 & m_{12}^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}, \quad \text{etc.} \ldots
   \]

4. We run between \( M_{ew} \approx 100 \, \text{GeV} \) and \( M_{GUT} \approx 2 \times 10^{16} \, \text{GeV} \).
MSSM Running and the $S$ Term
The $S$ Term

- The one-loop MSSM RG evolution of the soft masses is given by [e.g. Martin+Vaughn '94]

\[
(16\pi^2) \frac{d m_i^2}{d t} = \bar{X}_i - \sum_{a=1}^{3} 8 g_a^2 C_i^a |M_a|^2 + \frac{6}{5} g_1^2 Y_i S,
\]

where

\[
S = Tr(Y m^2) = m_{Hu}^2 - m_{Hd}^2 + tr(m_Q^2 - 2m_U^2 + m_E^2 + m_D^2 - m_L^2).
\]

- $S = 0$ in mSUGRA and in “general” gauge-mediated models. [Meade, Seiberg, Shih '08]

- $S$ evolves homogeneously. At one loop,

\[
(16\pi^2) \frac{d S}{d t} = -2 b_1 g_1^2 S, \quad b_1 = -\frac{33}{5}.
\]

- If $S$ vanishes at one scale, it vanishes at all scales. If $S$ is non-zero at $M_{ew}$, it runs large in the UV.
• If $S$ grows very large, it can dominate the scalar mass running.

• The net size of the effect is

$$\Delta m_i^2(t) = \frac{Y_i}{Tr(Y^2)} \left[ \frac{g_1^2(t)}{g_1^2(t_0)} - 1 \right] S(t_0).$$

• Problem: we don’t know $S(t_0)$ unless we know all the soft masses.
**$S$ and a Hypercharge FI Term**

- The $S$ term is related to a hypercharge Fayet Iliopoulos (FI) term.
- Consider the MSSM augmented by such a FI term, $\xi$:

\[
\mathcal{L} = \frac{1}{2} D_1^2 + \xi D_1 + D_1 \sqrt{\frac{3}{5}} g_1 \sum_i \bar{\phi}_i Y_i \phi^i - \sum_i \tilde{m}_i^2 |\phi^i|^2 + \ldots
\]

\[\to - \frac{1}{2} \xi^2 - \frac{3}{5} g_1^2 \left( \sum_i Y_i |\phi^i|^2 \right)^2 - \sum_i \left( \tilde{m}_i^2 + \sqrt{\frac{3}{5}} g_1 Y_i \xi \right) |\phi^i|^2 + \ldots\]

- The net effect is to shift the soft masses:

\[m_i^2 = \tilde{m}_i^2 + g_Y Y_i \xi.\]

- Only the shifted masses $m_i^2$ are observable, not $\tilde{m}_i^2$ or $\xi$ individually.
Running $\xi$

- There are two convenient ways to do the RG running:

1. Run the shifted masses $m_i^2$ alone. ($D_1$ eliminated.)
   - The running of $m_i^2$ is the same as before.

2. Run $\tilde{m}_i^2$ and $\xi$ separately. ($D_1$ uneliminated.)
   - The RG running of $\tilde{m}_i^2$ is the same as $m_i^2$, but without the $S$ term.
   - $\xi$ evolves at one-loop according to [Jack, Jones, Parsons '00]
     \[
     \frac{d\xi}{dt} = \frac{\xi}{g_1} \frac{dg_1}{dt} + \frac{2 g_1}{16\pi^2} \sqrt{\frac{3}{5}} Tr(Y \tilde{m}^2). 
     \]
   - This is inhomogeneous - $Tr(Y \tilde{m}^2) \neq 0$ generates a $\xi$.

- The $S$ term in the running of $m_i^2$ corresponds to the running of the FI term in the uneliminated formalism.

- $\xi$ doesn’t affect the other soft parameters until three loop order.
Uncertainties due to $S$

- The running of the soft masses can be very sensitive to the value of $S$.

- Since $S$ depends on all the soft masses, there can be a large uncertainty in its value.
  
  *e.g. 1.* One of the soft masses is undetermined.
  
  *e.g. 2.* Some of the soft masses have large uncertainties.

- In terms of $\tilde{m}_i^2$ and $\xi$, there is a theory ambiguity.
  For each set $\{m_i^2\}$, there is an equivalence class of possible $\{\tilde{m}_i^2, \xi\}$.

- An invariant combination in both cases is

  $$Y_im_j^2 - Y_jm_i^2.$$  

  for any $i \neq j$.

- Even without running, $S \neq 0$ provides interesting information.  
  (mSUGRA+$\xi$? [de Gouvêa, Murayama, Friedland ’98])
Example: SPS-5

• Input values at $M_{GUT} = 2 \times 10^{16}$ GeV:

  $m_0 = 150$ GeV, $m_{1/2} = 300$ GeV, $A_0 = -1000$ GeV,

  $\tan \beta = 5$, $\text{sgn}(\mu) = +1$.

• At the low scale $M_{\text{ew}},$

  $M_a \simeq 700$ GeV, $\mu \simeq 650$ GeV, $m_\tilde{q} = 400-600$ GeV

• For this spectrum, many parameters related to the Higgs sector will be hard to pin down at the LHC:

  $m^2_{H_d}, m^2_{H_u}, \tan \beta, \ldots$

• $m^2_{H_d} \simeq (235 \text{ GeV})^2$ in mSUGRA.

  Look at the effect of setting $m^2_{H_d} \rightarrow (1000 \text{ GeV})^2$. 
Dashed $\Rightarrow$ mSUGRA running; $m^2_{H_d}(M_{ew}) = (235 \text{ GeV})^2$.

Solid $\Rightarrow$ running up with $m^2_{H_d}(M_{ew}) = (1000 \text{ GeV})^2$. 
The $S$ term does not obscure family mixing information.
Other S-term Tools

- We have focussed on $S = S_Y$ for hypercharge.

- For every non-anomalous $U(1)$ there is a similar $S$ term.

- MSSM (+ singlets) $\Rightarrow$ only possibilities are $Y$ and $B - L$.
  \[
  S_{B-L} = Tr(Q_{BL}m^2)
  \]
  \[
  = tr(2m_Q^2 - m_U^2 - m_D^2 - 2m_L^2 + m_E^2)
  \]

- RG running:
  \[
  (16\pi^2) \frac{dS_{B-L}}{dt} = \frac{3}{5} Tr(Q_{B-L}Y) g_1^2 S_Y.
  \]

  $\Rightarrow S_{B-L}$ can be another useful invariant if $S_Y = 0$. 
The Upshot

- Scalar soft masses can be sensitive to the value of $S(M_{ew})$.
- Since $S$ depends on all MSSM soft masses, it is hard to pin down.
- This is particularly relevant to the slepton soft masses:
  - They can perhaps be deduced from LHC data.
  - Their running is less sensitive to uncertainties in $m_t$, $\alpha_s$.
  - They are very sensitive to $S \neq 0$ since $|Y| = 1/2, 1$.
- Uncertainties due to $S$ cancel out in $Y_i m_j^2 - Y_j m_i^2$.
- $S = S_Y$ and $S_{B-L}$ can also be useful probes.
New Intermediate Scale Physics
Life in the Desert

• A grand desert is not the only possibility consistent with unification.

• If the new physics consists of gauge singlets or complete GUT multiplets, unification will be about as good as in the MSSM.

• Examples:
  – Gauge singlets for a \( \mu \) term, or to induce small neutrino masses.
  – Gauge-mediated models often contain several GUT multiplets.
  – Extended gauge structures associated with the GUT group.
• New intermediate scale physics can modify the high scale predictions one would get assuming a grand desert.

![Graph showing functions g₁, g₂, and g₃](image)

- In many cases, certain combinations of parameters are not affected by the new physics.
- In other cases, the new physics can be inferred from low-scale measurements. *(i.e. heavy singlet neutrinos and lepton flavour violation.)*
Primary SUSY Questions

- We are chiefly interested in the source of SUSY breaking.

- Priorities:
  1. Pattern of gaugino masses at $M_{mess}$.
  2. Relative size of gaugino and scalar masses at $M_{mess}$.
     ($U(1)_R$ breaking)
  3. Relative sizes of $m^2_{\tilde{f}_{1,2}}$ and $m^2_{\tilde{f}_3}$.
     (Flavour)

- Strategy:
  Running of soft terms $\leftrightarrow$ symmetries broken by couplings.
  [Arkani-Hamed, Giudice, Luty, Rattazzi '98; Nelson+Strassler '01]
Case 1:
Complete GUT Multiplets
Complete GUT Multiplets

- Consider the MSSM augmented by $N_5$ sets of $5 \oplus \bar{5}$ multiplets,

$$W \supset \tilde{\mu} \cdot 5 \cdot \bar{5},$$

with $\tilde{\mu} \gg M_{ew}$.

- We assume that all other superpotential couplings are small.

- At one-loop, this preserves unification and its scale $M_{GUT}$, but increases the value of $g(M_{GUT})$, 

![Graph showing the behavior of $g_1$, $g_2$, and $g_3$ with $N_5 = 7$.]
Priority 1. Gaugino Masses

- Gaugino masses are robust.

- $M_a/g_a^2$ remains RG-invariant to one-loop.

- For $M_{mess} \geq M_{GUT}$, we know the gaugino mass ratios at $M_{GUT}$ if there is unification.
Priorities 2. + 3. Scalar Soft Masses

- **Priority 2:** Larger $N_5$ reduces $M_a$ relative to $m_i^2$ at the low scale.
  \[ \Rightarrow \text{Priority 2. is challenging.} \]

- **Priority 3:** Relative Yukawa mass splitting is modified for $N_5 \neq 0$:
  \[
  (16\pi^2) \frac{d m_i^2}{d t} = \tilde{X}_i - \sum_{a=1}^{3} 8 \ g_a^2 \ C_i^a \ |M_a|^2 + \frac{6}{5} \ g_1^2 \ Y_i \ S,
  \]
  with $\tilde{X}_i \sim |y_i|^2 \left( m_i^2 + m_j^2 + m_{H_{u,d}}^2 + |A_i|^2 \right)$.

  \[ \Rightarrow \text{interference between } \tilde{X}_i \text{ and the gaugino sources} \]
Some clues about the relative splitting can be obtained from certain useful mass combinations: [Ibañez+Lopez '84]

\[
\begin{align*}
m_{A_3}^2 &= 2m_{L_3}^2 - m_{E_3}^2 \\
m_{B_3}^2 &= 2m_{Q_3}^2 - m_{U_3}^2 - m_{D_3}^2 \\
m_{X_3}^2 &= 2m_{H_u}^2 - 3m_{U_3}^2 \\
m_{Y_3}^2 &= 3m_{D_3}^2 + 2m_{L_3}^2 - 2m_{H_d}^2
\end{align*}
\]

These run only due to gauge interactions in the MSSM. They correspond to (approximate) anomalous symmetries.

Compare \(m_{A_3}^2\), \(m_{B_3}^2\) to

\[
\begin{align*}
m_{A_{1,2}}^2 &= 2m_{L_{1,2}}^2 - m_{E_{1,2}}^2 \\
m_{B_{1,2}}^2 &= 2m_{Q_{1,2}}^2 - m_{U_{1,2}}^2 - m_{D_{1,2}}^2
\end{align*}
\]

Compare \(m_{X_3}^2\) and \(m_{Y_3}^2\) to

\[
\begin{align*}
m_{X_{1,2}}^2 &= 2m_{L_{1,2}}^2 - 3m_{U_{1,2}}^2 \\
m_{Y_{1,2}}^2 &= 3m_{D_{1,2}}^2
\end{align*}
\]
e.g. \[ m_0 = 300 \text{ GeV}, \ m_{1/2} = 700 \text{ GeV}, \]
\[ A_0 = 0, \ \tan \beta = 10, \]
\[ N_5 = 3, \ \tilde{\mu} = 10^4 \text{ GeV}. \]

Running soft scalar masses without including GUT multiplets leads to misleading conclusions.
$m_{B_1}^2$ vs. $m_{B_3}^2$

- The combinations $m_{B_1}^2$ and $m_{B_3}^2$ have the same one-loop running.

- Equality at $M_{GUT}$ implies $m_{B_1}^2 \simeq m_{B_3}^2$ at all scales.
$m^2_{X_1}$ vs. $m^2_{X_3}$

- Similar story to $m^2_{B_i}$.
Case 2: Right-Handed Neutrinos
Right-Handed Neutrinos

- Type-I SeeSaw:

\[
W \supset y_\nu \, L \cdot H_u \, N^c + \frac{1}{2} \, M_N \, N^c N^c \n\]

\[
\rightarrow -\frac{1}{2} \, y_\nu^t \, M_N^{-1} \, y_\nu \, (L \cdot H_u) \, (L \cdot H_u),
\]

generates small neutrino masses.

- The neutrino Yukawa couplings can be large,

\[
y_\nu \sim \frac{1}{\sin \beta} \left( \frac{M_N}{10^{14} \text{ GeV}} \right)^{1/2} \left( \frac{m_\nu}{0.1 \text{ eV}} \right)^{1/2}.
\]

- This modifies the running of \( m_L^2 \) and \( m_{H_u}^2 \) above \( M_N \):

\[
\Delta \frac{d m_i^2}{d t} = |y_\nu|^2 \left( m_{H_u}^2 + m_L^2 + m_N^2 + |A_\nu|^2 \right).
\]
e.g. $M_N = 10^{14} \text{ GeV}$
Priorities 1., 2., 3.

- **Priority 1**: Gaugino masses are not affected.

- **Priority 2**: Squark soft masses, $m^2_{E_i}$ are fairly insensitive.

- **Priority 3**: Only $m^2_{B_i}$ remains a useful mass combination.
  The approximate symmetries related to $m^2_{A_i}$, $m^2_{X_i}$, $m^2_{Y_i}$ are broken by the neutrino Yukawa and mass couplings.

- LFV constraints generally force $y_\nu$ to be small.
Case 3:

Additional Running from Hidden Sector Dynamics
Running from the Hidden Sector

- Soft SUSY breaking \( \sim F/M_{mess} \).

- Hidden sector dynamics below \( M_{mess} \) also contributes to the running of the soft terms.  
  \[ \text{[Luty+Sundrum '97; Nelson+Strassler '00; Cohen,Roy,Schmaltz '06; ...]} \]

- Net effects are often similar to extra gauge multiplets:
  - Gaugino masses are robust.
  - Scalar masses are shifted overall relative to gaugino masses.
  - Relationships between scalar masses are scrambled.

- Useful information can still be obtained from nice mass combinations.  
  \[ \text{[Cohen,Roy,Schmaltz '06; Meade,Seiberg,Shih '08]} \]
Summary

• Extracting Lagrangian parameters from LHC data will be challenging. Running these parameters up will also require some thinking.

• Low-scale uncertainties can get magnified by the RG running. → slepton masses and the $S$ term.

• New intermediate scale physics can change the predictions one would obtain assuming a grand desert. → new physics that preserves unification can have a large effect.

• Even with these challenges, it will still be possible to test specific models against LHC data.

• Certain combinations of soft parameters are better than others.