U(1) Gauge Extensions of the Standard Model

Ernest Ma
Physics and Astronomy Department
University of California
Riverside, CA 92521, USA
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Anomaly Freedom of the Standard Model

Gauge Group: $SU(3)_{C} \times SU((2)_{L} \times U(1)_{Y}$.

Consider the fermion multiplets:

$(u, d)_{L} \sim (3, 2, n_{1}), u_{R} \sim (3, 1, n_{2}), d_{R} \sim (3, 1, n_{3})$,

$(\nu, e)_{L} \sim (1, 2, n_{4}), e_{R} \sim (1, 1, n_{5})$.

Bouchiat/Iliopolous/Meyer(1972): The SM with

$n_{1} = 1/6, n_{2} = 2/3, n_{3} = -1/3, n_{4} = -1/2, n_{5} = -1$,

is free of axial-vector anomalies, i.e.

$[SU(3)]^{2}U(1)_{Y} : 2n_{1} - n_{2} - n_{3} = 0$.

$[SU(2)]^{2}U(1)_{Y} : 3n_{1} + n_{4} = 0$.

$[U(1)_{Y}]^{3} : 6n_{1}^{3} - 3n_{2}^{3} - 3n_{3}^{3} + 2n_{4}^{3} - n_{5}^{3} = 0$. 
It is also free of the mixed gravitational-gauge anomaly, $U(1)_Y : 6n_1 - 3n_2 - 3n_3 + 2n_4 - n_5 = 0$.

Geng/Marshak (1989), Minahan/Ramond/Warner (1990): Above 4 equations $\Rightarrow n_1(4n_1 - n_2)(2n_1 + n_2) = 0$.

$n_2 = 4n_1 \Rightarrow \text{SM}; \ n_2 = -2n_1 \Rightarrow \text{SM} \ (u_R \leftrightarrow d_R)$;

$n_1 = 0 \Rightarrow n_4 = n_5 = n_2 + n_3 = 0$.

Here $e_R \sim (1, 1, 0)$ may be dropped. $(u, d)_L, (\nu, e)_L$ have charges $(1/2, -1/2)$ and $(u_R, d_R)$ have charges $(n_2, -n_2)$. Pairing $(u, d)_L$ with $(u_R, d_R)$ with a Higgs doublet $\Rightarrow n_2 = 1/2$. As for $(\nu, e)_L$, a Higgs triplet $(s^+, s^0, s^-)$ will pair $\nu_L$ with $e_L$ to form a Dirac fermion.
$B - L$

Add one $\nu_R$ per family, then $U(1)_{B-L}$ is possible.

$U(1)_{B-L} : (3)(2)[(1/3) - (1/3)] + (2)[(-1) - (-1)] = 0.$

$[SU(3)]^2U(1)_{B-L} : (1/2)(2)[(1/3) - (1/3)] = 0.$

$[SU(2)]^2U(1)_{B-L} : (1/2)[(3)(1/3) + (-1)] = 0.$

$[U(1)_Y]^2U(1)_{B-L} : (3)[2(1/6)^2 - (2/3)^2 - (-1/3)^2](1/3) + [2(-1/2)^2 - (-1)^2](-1) = 0.$

$U(1)_Y[U(1)_{B-L}]^2 : (3)[2(1/6) - (2/3) - (-1/3)](1/3)^2 + [2(-1/2) - (-1)](-1)^2 = 0.$

$U(1)_{B-L}^3 : (3)(2)[(1/3)^3 - (1/3)^3] + (2)[(-1)^3 - (-1)^3] = 0.$
$U(1)_{B-L}$ is of course very well-known. The fact that it requires $\nu_R$ may be a hint of $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and $SO(10)$. Consider now the SM with 3 families, where $L_e$, $L_\mu$, $L_\tau$, and $B$ are apparently separately conserved. However, $[SU(2)]^2 B$, $[SU(2)]^2 L_{e,\mu,\tau}$ are also separately anomalous, so that the conservation of each is violated by instantons and sphalerons, whereas the linear combination $n_B B + n_e L_e + n_\mu L_\mu + n_\tau L_\tau$ is safe, if

(A) $n_B = 0$, $n_e + n_\mu + n_\tau = 0$; or

(B) $n_B = 1$, $n_e + n_\mu + n_\tau = -3$. 
\[ Le - L_\mu \text{ and } B - 3L_\tau \]

To have a gauge $U(1)$ extension of the SM, the 6 equations for anomaly freedom must be satisfied for a given choice of $n_B, n_e, n_\mu, n_\tau$.

$U(1)_{B-L} : n_B = 1$ and $n_e = n_\mu = n_\tau = -1$.

He/Joshi/Lew/Volkas(1991) : The SM with 3 families and without any $\nu_R$ admits 3 possible $U(1)$ gauge extensions: $Le - L_\mu, Le - L_\tau, L_\mu - L_\tau$.

Ma/Roy/Roy(2002) : For example, the gauge boson for $L_\mu - L_\tau$ would decay equally to $\mu^+\mu^-$ and $\tau^+\tau^-$ but not $e^+e^-$ or quarks. Muon $g - 2$ is a constraint.
Ma(1998): Add just one $\nu_R$ with $L_\tau = 1$, then $U(1)_X = B - 3L_\tau$ is anomaly-free and can be gauged. To break $U(1)_X$ spontaneously, a neutral scalar singlet $\chi^0 \sim (1, 1, 0; 6)$ is used, which also gives $\nu_R$ a large Majorana mass, thereby making $\nu_\tau$ massive.

The $X$ boson decays into quarks and $\tau$ but not $e$ or $\mu$.

Add scalar doublet $(\eta^+, \eta^0) \sim (1, 2, 1/3; -3)$ and singlet $\chi^- \sim (1, 1, -1; -3)$, then $\bar{\nu}_R \nu_\tau \langle \phi^0 \rangle$ and $\bar{\nu}_R \nu_{e, \mu} \langle \eta^0 \rangle$ ⇒ one linear combination of $\nu_e, \nu_\mu, \nu_\tau$ gets a tree-level mass, and the others get radiative masses via the Zee mechanism.
U(1) Gauge Extensions of the Standard Model (int08)
Ma/Roy(1998) : The $X$ boson is not constrained to be very heavy because it does not couple to $e$ or $\mu$. It can be produced easily at the LHC because it has quark couplings. Its decay into $\tau^+\tau^-$ is also a good signature. 

$$\frac{\Gamma(X \rightarrow \tau^+\tau^-)}{\Gamma(X \rightarrow \bar{q}q)} = \frac{9}{2}$$

$B - L$ may come from $SU(4) \times SU(2)_L \times SU(2)_R$ with $Q = T_{3L} + T_{3R} + (1/2)(B - L)$ and $SU(4)$ breaking to $SU(3)_C \times U(1)_{B-L}$.

$B - 3L_\tau$ may come from $SU(10) \times SU(2)_L \times U(1)_{Y'}$ with $Q = T_{3L} + Y' + (1/5)(B - 3L_\tau)$ and $SU(10)$ breaking to $[SU(3)_C]^3 \times U(1)_{B-3L_\tau}$. 
\[ \mathbf{U}(1)_\Sigma \]

Ma(2002): Add 3 copies of \((\Sigma^+, \Sigma^0, \Sigma^-)_R\) so that neutrinos get mass via seesaw mechanism (III), instead of (I), i.e. \(\nu_R\). Is there a U(1) gauge symmetry like \(B - L\) as in the case of \(\nu_R\) for each family? Under \(\mathbf{U}(1)_\Sigma\), let \((u, d)_L \sim n_1, u_R \sim n_2, d_R \sim n_3, (\nu, e)_L \sim n_4, e_R \sim n_5,\) and \(\Sigma_R \sim n_6\). Axial-vector anomaly freedom requires

\[
\begin{align*}
[\mathbf{SU}(3)]^2 \mathbf{U}(1)_\Sigma &: 2n_1 - n_2 - n_3 = 0. \\
[\mathbf{U}(1)_Y]^2 \mathbf{U}(1)_\Sigma &: n_1 - 8n_2 - 2n_3 + 3n_4 - 6n_5 = 0. \\
\mathbf{U}(1)_Y[\mathbf{U}(1)_\Sigma]^2 &: n_1^2 - 2n_2^2 + n_3^2 - n_4^2 + n_5^2 = \\
&= (3n_1 + n_4)(7n_1 - 4n_2 - 3n_4)/4 = 0.
\end{align*}
\]
$n_4 = -3n_1 \Rightarrow U(1)_Y$, so $n_2 = (7n_1 - 3n_4)/4$ will be assumed from now on. In that case, $n_3 = (n_1 + 3n_4)/4$ and $n_5 = (-9n_1 + 5n_4)/4$.

$[SU(2)]^2 U(1)_\Sigma : 3n_1 + n_4 - 4n_6 = 0$.

Mixed gravitational-gauge anomaly $U(1)_\Sigma :$

$6n_1 - 3n_2 - 3n_3 + 2n_4 - n_5 - 3n_6 = 3(3n_1 + n_4 - 4n_6)/4 = 0$.

$[U(1)_\Sigma]^3 : 6n_1^3 - 3n_2^3 - 3n_3^3 + 2n_4^3 - n_5^3 - 3n_6^3 = 3(3n_1 + n_4)^3/64 - 3n_6^3 = 0$.

Hence $n_6 = (3n_1 + n_4)/4$ satisfies all 3 conditions. If a fermion multiplet $(1, 2p + 1, 0; n_6)$ is used, the only solutions are $p = 0 \ [U(1)_{B-L}]$ and $p = 1 \ [U(1)_\Sigma]$. 
The scalar sector of this $U(1)_{\Sigma}$ model consists of two Higgs doublets $(\phi_1^+, \phi_1^0)$ with charge $(9n_1 - n_4)/4$ which couples to charged leptons, and $(\phi_2^+, \phi_2^0)$ with charge $(3n_1 - 3n_4)/4$ which couples to up and down quarks as well as to $\Sigma$. To break the $U(1)_{\Sigma}$ gauge symmetry spontaneously, a singlet $\chi$ with charge $-2n_6$ is added, which also allows the $\Sigma$’s to acquire Majorana masses at the $U(1)_{\Sigma}$ breaking scale.

Adhikari/Erler/Ma(2008): The new gauge boson $X$ may be accessible at the LHC. Its decay branching ratios could determine the parameter $r = n_4/n_1 = \tan \phi$. 

$U(1)$ Gauge Extensions of the Standard Model (int08) back to start
U(1) Gauge Extensions of the Standard Model (int08)
Adhikari/Erler/Ma(2008):
Assume one $N_R$ and two $\Sigma_R$, then
(1) $S_{1R} \sim (3n_1 + n_4)/4$, $S_{2R} \sim -5(3n_1 + n_4)/8$,
(2) $\Phi_3 \sim (9n_1 - 5n_4)/8$ are needed, where $\Phi_3$ links
($\nu, l)_L$ with $N_R$ and $\Sigma_{1R,2R}$.

The structure of this model allows an exactly conserved
$Z_2$ remnant of $U(1)_\Sigma$, under which $N, \Sigma, S_{2R}, \Phi_3$, and
$\chi_3 \sim -3(3n_1 + n_4)/8$ are odd, and all other fields even.
Scotogenic radiative neutrino mass is then possible. The
lightest scalar odd under $Z_2$, e.g. $\text{Re}(\chi_3)$ or $\text{Im}(\chi_3)$, is a
good dark-matter candidate.
$U(1)$ Gauge Extensions of the Standard Model (int08)
Supersymmetric $U(1)_X$

SM $\rightarrow$ MSSM has 3 well-known issues:

(1) $m_\nu = 0$ not realistic, (2) $B$ and $L_i$ conserved only by assumption, and (3) $\mu \hat{\phi}_1 \hat{\phi}_2$ adjusted with $\mu \sim M_{SUSY}$.

Each has a piecemeal solution, but is there one unifying explanation? Ma(2002): New $U(1)_X$ with 3 copies of

$(\hat{u}, \hat{d}) \sim (3, 2, 1/6; n_1)$, $\hat{u}^c \sim (3^*, 1, -2/3; n_2)$,
$\hat{d}^c \sim (3^*, 1, 1/3; n_3)$, $(\hat{\nu}, \hat{\bar{e}}) \sim (1, 2, -1/2; n_4)$,
$\hat{\bar{e}}^c \sim (1, 1, 1; n_5)$, $\hat{\bar{N}}^c \sim (1, 1, 0; n_6)$, and 1 copy of

$\hat{\phi}_1 \sim (1, 2, -1/2; -n_1 - n_3)$, $\hat{\phi}_2 \sim (1, 2, 1/2; -n_1 - n_2)$

with $n_1 + n_3 = n_4 + n_5$ and $n_1 + n_2 = n_4 + n_6$. 
Add Higgs singlet superfield $\hat{\chi} \sim (1, 1, 0; 2n_1 + n_2 + n_3)$. Then $\mu \hat{\phi}_1 \hat{\phi}_2$ is replaced by $\hat{\chi} \hat{\phi}_1 \hat{\phi}_2$ and $\langle \chi \rangle \neq 0$ breaks $U(1)_X$. Add 2 copies of singlet up quarks: $\hat{\tilde{U}} \sim (3, 1, 2/3; n_7)$, $\hat{\tilde{U}}^c \sim (3^*, 1, -2/3; n_8)$, and 1 copy of singlet down quarks: $\hat{\tilde{D}} \sim (3, 1, -1/3; n_7)$, $\hat{\tilde{D}}^c \sim (3^*, 1, 1/3; n_8)$, with $n_7 + n_8 = -2n_1 - n_2 - n_3$
so that $\hat{\chi} \hat{\tilde{U}} \hat{\tilde{U}}^c$ and $\hat{\chi} \hat{\tilde{D}} \hat{\tilde{D}}^c \Rightarrow M_U$ and $M_D$.
So far there are 8 numbers and 3 constraints. Consider next the 5 AVV conditions:

$$[SU(3)]^2 U(1)_X: 2n_1 + n_2 + n_3 + n_7 + n_8 = 0.$$
\([SU(2)]^2U(1)_X: 3(3n_1 + n_4) + (-n_1 - n_3) + (-n_1 - n_3) = 7n_1 - n_2 - n_3 + 3n_4 = 0.\)

\([U(1)_Y]^2U(1)_X: -n_1 + 7n_2 + n_3 + 3n_4 + 6n_5 + 6n_7 + 6n_8 = -7n_1 + n_2 + n_3 - 3n_4 = 0.\)

Using \(n_1, n_2, n_4, n_7\) as independent, \(U(1)_Y[U(1)_X]^2:\)
\[3n_1^2 - 6n_2^2 + 3n_3^2 - 3n_4^2 + 3n_5^2 + 3n_7^2 + 3n_8^2 - (n_1 + n_3)^2 + (n_1 + n_2)^2 = 6(3n_1 + n_4)(2n_1 - 4n_2 - 3n_7) = 0.\]

If \(3n_1 + n_4 = 0, U(1)_X = U(1)_Y,\)
hence the solution \(2n_1 - 4n_2 - 3n_7 = 0\)
is chosen from now on.
Using \( n_1, n_4, n_6 \) as independent, \( n_2 = -n_1 + n_4 + n_6 \), 
\( n_3 = 8n_1 + 2n_4 - n_6 \), \( n_5 = 9n_1 + n_4 - n_6 \), \( n_7 = 2n_1 - (4/3)n_4 - (4/3)n_6 \), \( n_8 = -11n_1 - (5/3)n_4 + (4/3)n_6 \).

\[ [U(1)_X]^3 : \quad 3[6n_1^3 + 3n_2^3 + 3n_3^3 + 2n_4^3 + n_5^3 + n_6^3] \]
\[ + 3(3n_7^3 + 3n_8^3) + 2(-n_1 - n_3)^3 + 2(-n_1 - n_2)^3 \]
\[ + (2n_1 + n_2 + n_3)^3 = \]
\[ -36(3n_1 + n_4)(9n_1 + n_4 - 2n_6)(6n_1 - n_4 - n_6) = 0. \]

Sum of 11 cubic terms has been factorized exactly!

Two possible solutions are

\( (A) \ n_6 = (9n_1 + n_4)/2, \quad (B) \ n_6 = 6n_1 - n_4. \)
$L$ conservation is automatic if (A) $9n_1 + 5n_4 \neq 0$ or (B) $3n_1 + 4n_4 \neq 0$.

$B$ conservation is automatic if (A) $7n_1 + 3n_4 \neq 0$ or (B) $3n_1 + 2n_4 \neq 0$.

$(A) = (B) \Rightarrow n_1 = n_4 = 1$, $n_2 = n_3 = n_5 = n_6 = 5$, $n_7 = n_8 = -6$ and $U(1)_X$ is orthogonal to $U(1)_Y$. However, mixed gravitational-gauge anomaly $= \text{sum of } U(1)_X \text{ charges } = 6(3n_1 + n_4) \neq 0$.

Add singlet superfields in units of $(3n_1 + n_4)$: 1 with charge 3, 3 ($S^c$) with charge $-2$, and 3 ($N$) with charge $-1$. 
Neutrino mass here is Dirac from pairing $\nu$ with $N^c$. If $n_6 = 3n_1 + n_4$, i.e. (A) $n_4 = 3n_1$ or (B) $n_4 = 3n_1/2$, then the singlets $S^c$ and $N$ are exactly right to allow the neutrinos to acquire naturally small seesaw Dirac masses. In the basis $(\nu, S^c, N, N^c)$, the $12 \times 12$ neutrino mass matrix is

$$M_\nu = \begin{pmatrix}
0 & 0 & 0 & m_1 \\
0 & 0 & m_2 & 0 \\
0 & m_2 & 0 & M \\
m_1 & 0 & M & 0
\end{pmatrix}$$

with $m_\nu = -m_1m_2/M$. 
Some Remarks

Many other U(1) gauge extensions of the SM and MSSM have been proposed. Recent studies include Dreiner/Luhn/Murayama/Thormeier(2007), Lee/Matchev/Wang(2008), Ma(2008), Chen/Jones/Rajaraman/Yu(2008).

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As the LHC era begins, one of the first easily detectable signals of new physics will be the possible existence of a new gauge boson coupling to both quarks (for production) and leptons (for detection).

There are many candidates for this honor, such as the various $U(1)$ remnants of $E_6 \rightarrow SU(3) \times SU(2) \times U(1)$, including $B - L$.

There are also other less familiar contenders, such as $L_e - L_\mu$, $B - 3L_\tau$, $U(1)_\Sigma$ with $(\Sigma^+, \Sigma^0, \Sigma^-)$ as the seesaw anchor, and supersymmetric $U(1)_X$ with a host of desirable properties.