Event Shapes
from Soft-Collinear Effective Theory

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Outline

- Introduction
- Event Shapes in $e^+e^-$ annihilation
- Effective Field Theories and SCET
- Energy Flow and Factorization in SCET
  - Operator-based definitions of jet observables
  - Conditions for breakdown of factorization
- Towards Jets and Event Shapes at LHC
Factorization

• Separate full cross-sections into calculable partonic functions and nonperturbative (noncalculable) but (hopefully) universal functions

\[ \sigma(e^+e^- \rightarrow n \text{ jets}) \sim \sigma(e^+e^- \rightarrow \text{ partons})\sigma(\text{partons} \rightarrow \text{ hadrons}) \]

\[ H \otimes J_1 \otimes J_2 \otimes S \]
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Does the observable factorize?
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Does the observable factorize?

Are the noncalculable functions universal?
Collinear and soft states

• Contrast factorization in $B$ physics...
  \[
  \sim \langle D | \bar{c}'_v \Gamma_h b_v | B \rangle \langle \pi | \bar{\chi}_n \Gamma_c \chi_n | 0 \rangle
  \]

• ... with jet physics
  \[
  \sim \langle J_1 | \bar{\chi}_n | 0 \rangle \Gamma \langle J_2 | \chi_{\bar{n}} | 0 \rangle \langle X_s | Y_n Y_{\bar{n}} | 0 \rangle
  \]
Collinear and soft states

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Collinear and soft states

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Factorization of states justified? a long-standing issue...
Event Shapes in $e^+e^-$ annihilation
Event Shapes

• Hadrons produced in high-energy collisions usually produced as jets

• Two-jet events are easily distinguished using event shapes

• Event shapes $e$ are number-valued observables, functions of all the final state momenta, such that two-jet events have $e \approx 0$
Event Shapes

- Hadrons produced in high-energy collisions usually produced as jets
- Two-jet events are easily distinguished using event shapes
- Event shapes $e$ are number-valued observables, functions of all the final state momenta, such that two-jet events have $e \approx 0$
Some event shapes

- **Thrust**
  \[ T = \frac{1}{Q} \max \hat{t} \sum_{i} \left| \hat{t} \cdot \mathbf{p}_i \right| \]
  \[ \tau = 1 - T \]
  Thrust axis is in the direction of the total hemisphere three-momentum \( \hat{t} = \hat{p}_A \)

- **Jet broadening**
  \[ B = \frac{1}{Q} \sum_{i} \left| \hat{t} \times \mathbf{p}_i \right| \]

- **Jet invariant masses**
  \[ M^2_A, M^2_B \]
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**Review:**
Dasgupta, Salam (2003)
Some event shapes

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  \[ M_A^2, M_B^2 \]

review: Dasgupta, Salam (2003)
Some event shapes

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Angularities

• Generalization of thrust:

\[ \tau_a = \frac{1}{Q} \sum_i E_i (\sin \theta_i)^a (1 - |\cos \theta_i|)^{1-a} \]

\[ \begin{align*}
  a = 0 & \quad \text{thrust} \\
  a = 1 & \quad \text{broadening}
\end{align*} \]

\[ \begin{align*}
  \text{infrared safety:} & \quad -\infty < a < 2 \\
  \text{factorizability:} & \quad -\infty < a < 1
\end{align*} \]

Jets with, e.g.,

\[ \tau_a < 0.25 \]

are narrower than:

\[ \begin{align*}
  a = 1 \\
  a = 0 \\
  a = -1
\end{align*} \]
Angularities

- Rapidity: $\eta = \frac{1}{2} \log \left( \frac{E + p_{\parallel}}{E - p_{\parallel}} \right)$

- Angularities can be expressed as
  $$\tau_a = \frac{1}{Q} \sum_i |p_i^T| e^{-|\eta_i|(1-a)}$$

- Other functions $f(\eta_i)$ define other event shapes
  - e.g. $C$-parameter
    $$f_C(\eta) = \frac{3}{\cosh \eta}$$
Thrust distribution

- Distribution of events in $t = 1-T$:
  - Dominated by two-jet-like events near $t = 0$.
  - Perturbative (partonic) prediction always shifted with respect to the data: universal power correction?

Gardi, Rathsman (2002)
Shift in average thrust

- Shift scales with center-of-mass energy $Q$ like $1/Q$:

- These *power corrections* come from nonperturbative soft radiation from the jets

Dasgupta, Salam (2003)
Effective Field Theories and SCET
Effective Field Theory

- Choose degrees of freedom relevant at the energy or distance scales at which one is working
- Expand full theory quantities in powers of small parameters, ratios of disparate scales
- Effective theory often possesses enhanced symmetries or simplifications not immediately evident in full theory
Soft-Collinear Effective Theory

- Effective field theory for $\Lambda_{QCD}$ fluctuations about lightlike trajectory $n = (1,0,0,1)$
- $p \sim Qn$
- $p_\perp \sim \sqrt{Q\Lambda_{QCD}}$
- $k \sim \Lambda_{QCD}$

- Simplifies demonstration of soft-collinear decoupling

Collinear and soft modes

\[ p^− \]

\[ \Lambda_{QCD} \]

\[ p^+ \]

\[ Q \]

- n-collinear
- hard
- soft
- \( n\)-collinear
Collinear and soft modes

\[ p^2 \sim Q^2 \]

\[ p^2 \sim Q \Lambda_{QCD} \]

\[ p^2 \sim \Lambda_{QCD}^2 \]
Interactions in SCET

- Collinear sector interactions:

- Soft parton interactions with each other same as in QCD

- Soft gluon interactions with collinear sector:
Soft-collinear decoupling

- **Collinear quark-soft gluon interaction:**
  \[ \mathcal{L} \ni \bar{\xi}_n i n \cdot D_s \frac{\vec{n} \cdot A_s}{2} \xi_n = \bar{\xi}_n i n \cdot \left( \partial_s + i g A_s \right) \frac{\vec{n} \cdot A_s}{2} \xi_n \]

- **Soft Wilson line:**
  \[ Y_n(z) = P \exp \left[ i g \int_0^\infty ds \, n \cdot A_s(ns + z) \right] \]

- **Field redefinition:**
  \[ \xi_n = Y_n \xi_n^{(0)} \]
  \[ A_n = Y_n A_n^{(0)} Y_n^\dagger \]

- **Removes coupling in Lagrangian:**
  \[ n \cdot D_s Y_n(z) = 0 \implies \mathcal{L}^{(0)} \ni \bar{\xi}_n^{(0)} i n \cdot \partial_s \frac{\vec{n} \cdot \xi_n^{(0)}}{2} \]

Bauer, Pirjol, Stewart (2001)
Soft-collinear decoupling

\[ -ig n^{\mu} T_A \frac{\gamma}{2} \]

\[ Y_n(z) = P \exp \left[ ig \int_0^\infty ds \ n \cdot A_s(ns + z) \right] \]
Energy Flow and Factorization in SCET

Event shape distribution

- Differential cross-section for $e^+e^-$ annihilation to hadrons with respect to event shape $e$:
  \[
  \frac{d\sigma}{de}(Q) = \frac{1}{2Q^2} \sum_N \langle N | j^\mu(0) | 0 \rangle L_\mu \left(2\pi\right)^4 \delta^4(Q - p_N) \delta(e - e(N))
  \]

- Match QCD current onto SCET two-jet operator:
  \[
  j^\mu_{QCD} = \bar{q}\gamma^\mu q \rightarrow j^\mu_{SCET} = C_2(\mu)[\bar{\xi}_n W_n^\dagger] \gamma^\mu [W_n \xi_n]
  \]

IR divergences same as in QCD, UV behavior differs
Event shape distribution

- Differential cross-section for $e^+e^-$ annihilation to hadrons with respect to event shape $e$:

$$
\frac{d\sigma}{de}(Q) = \frac{1}{2Q^2} \sum_N |\langle N| j_\mu^\ast(0)|0\rangle L_\mu|^2 (2\pi)^4 \delta^4(Q - p_N) \delta(e - e(N))
$$

- Decouple soft gluons from collinear fields

$$
j_{\text{SCET}}^\mu \rightarrow C_2(\mu)[\bar{\xi}_n W_n^\dagger \bar{Y}_n^\dagger] \gamma_\perp^\mu [Y_n W_n \xi_n]
$$

- Formerly also assumed factorization of final states:

$$
|N\rangle = |J_n\rangle \otimes |\bar{J}_n\rangle \otimes |X_s\rangle
$$
Event shape distribution

- Differential cross-section for $e^+e^-$ annihilation to hadrons with respect to event shape $e$

$$\frac{d\sigma}{de}(Q) = \frac{1}{2Q^2} \sum_N \langle N | j^\mu(0) | 0 \rangle L_\mu \left| 2(2\pi)^4 \delta^4(Q - p_N) \delta(e - e(N)) \right|^2$$

- Decouple soft gluons from collinear fields

$$j^\mu_{\text{SCET}} \rightarrow C_2(\mu)[\bar{\xi}_n W_n^\dagger \bar{Y}_n^\dagger] \gamma_\perp^\mu [Y_n W_n \xi_n]$$

- Formerly also assumed factorization of final states:

$$|N\rangle = |J_n\rangle \otimes |J_{\bar{n}}\rangle \otimes |X_s\rangle$$
Event shape operator

- Operator representation of $e(N)$ ...
  $$e(N) = \frac{1}{Q} \sum_{i \in N} |p_i^\perp|f_e(\eta_i)$$
  $$\hat{e}|N\rangle \equiv e(N)|N\rangle$$

- would eliminate sum over final states:
  $$\frac{d\sigma}{de} \sim \int d^4x \, e^{iQ \cdot x} \langle 0| j^*_\mu(x) \delta (e - \hat{e}) j^\mu(0)|0\rangle$$

- Define a transverse energy flow operator:
  $$\mathcal{E}_T(\eta)|N\rangle = \sum_{i \in N} |p_i^\perp|\delta(\eta - \eta_i)|N\rangle$$

- Then
  $$\hat{e} = \frac{1}{Q} \int_{-\infty}^{\infty} d\eta \, f_e(\eta) \mathcal{E}_T(\eta)$$
Energy flow

- Relate transverse energy flow to energy flow operator:

  \[ \mathcal{E}_T(\eta) = \frac{1}{\cosh^3 \eta} \int_0^{2\pi} d\phi \mathcal{E}(\hat{n}) \]

  \[ \mathcal{E}(\hat{n})|N\rangle = \sum_{i \in N} E_i \delta^2(\hat{n} - \hat{n}_i)|N\rangle \]

- Energy flow from energy-momentum tensor:

  \[ \mathcal{E}(\hat{n}) = \lim_{R \to \infty} \int_0^\infty dt \hat{n}_i T_{0i}(t, R\hat{n}) \]

Energy flow

\( \mathcal{E}(\hat{n}) \) measures energy flow through a patch of large sphere in direction \( \hat{n} \)

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Factorization

- Matrix elements in event shape distribution factorize due to collinear-soft decoupling:

\[ \frac{d\sigma}{de} \sim \sum_N \langle 0 | \delta(e - e(N)) | N \rangle \langle N | 0 \rangle \]
Factorization

- Matrix elements in event shape distribution factorize due to collinear-soft decoupling:

Express event shape as operator:

$$\frac{d\sigma}{de} \sim \sum_{N} \langle 0 | \begin{array}{c} \delta(e - \hat{e}(T_{\mu\nu})) \end{array} | N \rangle \langle N | \begin{array}{c} 0 \end{array} \rangle$$
Factorization

- Matrix elements in event shape distribution factorize due to collinear-soft decoupling:

Sum over complete set of hadronic states:

\[
\frac{d\sigma}{de} \sim \langle 0 | \delta(e - \hat{e}(T_{\mu\nu})) | 0 \rangle
\]
Factorization

- Matrix elements in event shape distribution factorize due to collinear-soft decoupling:

Decouple soft gluons through collinear field redefinition:

\[
\frac{d\sigma}{de} \sim \langle 0 | \delta(e - \hat{e}(T_{\mu\nu})) | 0 \rangle
\]

\[
T_{\mu\nu} = T^n_{\mu\nu} + T^\parallel_{\mu\nu} + T^s_{\mu\nu}
\]
Factorization

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Decouple soft gluons through collinear field redefinition:

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Partonic
E-M tensor
Factorization

• Matrix elements in event shape distribution factorize due to collinear-soft decoupling:

\[ \frac{d\sigma}{de} \sim \langle 0 | \delta(e_n - \hat{e}_n(T_{\mu\nu}^n)) | 0 \rangle \]
\[ \times \langle 0 | \delta(e_{\bar{n}} - \hat{e}_{\bar{n}}(T_{\mu\nu}^{\bar{n}})) | 0 \rangle \]
\[ \times \langle 0 | \delta(e_s - \hat{e}_s(T_{\mu\nu}^s)) | 0 \rangle \]

• Convolution of hard, jet, and soft functions:

\[ \frac{d\sigma}{de} = H(Q, \mu) \int de_J \sigma_J(e_J; \mu) S(e - e_J; \mu) \]
Hypothesis of Universal Soft Power Corrections

- Soft power corrections shift mean values of event shapes

\[ \langle e \rangle = \langle e \rangle_{PT} + c_e \frac{A}{Q} \]

- Stronger version: simple shift of full distribution

\[ \frac{d\sigma}{de}(e) = \frac{d\sigma}{de} \left( e - c_e \frac{A}{Q} \right) \bigg|_{PT} \]

- Based on perturbative behavior of single soft gluon emission and model of “effective IR coupling”

\[ c_a = \frac{2}{1 - a}, \quad c_C = 3\pi \]

\[ A \quad \text{universal nonperturbative parameter} \]

Dokshitzer, Webber (1995, 1997)

CL, Sterman (2007)
Hypothesis of Universal Soft Power Corrections

- Soft power corrections shift mean values of event shapes
  \[ \langle e \rangle = \langle e \rangle_{PT} + c_e \frac{A}{Q} \]
  \[ c_e \quad \text{observable dependent, calculable coefficient} \]
  \[ A \quad \text{universal nonperturbative parameter} \]

- Stronger version: simple shift of full distribution
  \[ \frac{d\sigma}{de} (e) = \frac{d\sigma}{de} \left( e - c_e \frac{A}{Q} \right) \bigg|_{PT} \]

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Hypothesis of Universal Soft Power Corrections

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Dokshitzer, Webber (1995, 1997)

CL, Sterman (2007)
When does factorization break down?

- Successful factorization means well-defined separation of soft and collinear contributions to cross-section.
- Jet and soft functions must separately be infrared-finite.
- For $a > 1$, distribution dominated by narrow jets in which soft and collinear modes not completely distinguished by SCETI. Jet and soft functions IR divergent.
- Following example illustrates NLO calculation of soft function, jet function, integrated over kinematically-allowed region $0 < \tau_a < 1$.

Hornig, CL, Ovanesyan (2008, in progress)
Calculation of soft function

Real emission:

Virtual:

Hornig, CL, Ovanesyan (2008, in progress)
Calculation of jet function

Real emission:
\[ \delta(\tau_a - \hat{\tau}_a) \]
\[ k - k_{\perp} Q^2_a = 0 \]
\[ \text{Hornig, CL, Ovanesyan (2008, in progress)} \]

Virtual:
\[ \bar{n} \]
\[ \int dk^- \int dk_{\perp}^2 \frac{1}{k^-} \frac{1}{(k_{\perp}^2)^{1+\epsilon}} \]
Calculation of soft function

Real emission:

\[ \delta(\tau_a - \hat{\tau}_a) \]

Virtual:

\[ \int dk^- \int dk^2_\perp \frac{1}{k^-} \frac{1}{(k^2_\perp)^{1+\epsilon}} \]

Hornig, CL, Ovanesyan (2008, in progress)
Calculation of soft function

Real emission:
\[ \delta(\tau_a - \hat{\tau}_a) \]

Virtual:
\[ \bar{n} k^2 - \int dk - \int dk^2_{\perp} \frac{1}{k^{-}} \frac{1}{(k^2_{\perp})^{1+\epsilon}} \]

Hornig, CL, Ovanesyan (2008, in progress)
Calculation of soft function

Real emission:
\[ \delta(\tau_a - \hat{\tau}_a) \]

Virtual:
\[ \bar{n} \]

\[ \frac{\int dk^- \int dk_{\perp}^2 \frac{1}{k^-} \frac{1}{(k_{\perp}^2)^{1+\epsilon}}}{(UV/IR) \quad \text{IR}} \]

Hornig, CL, Ovanesyan (2008, in progress)
Calculation of jet function

\[
\int d k^- \int d k^2_\perp \frac{1}{k^-} \frac{1}{(k^2_\perp)^{1+\epsilon}}
\]

Hornig, CL, Ovanesyan (2008, in progress)
Calculation of jet function

\[ \int dk^- \int dk^\perp \frac{1}{k^-} \frac{1}{(k^\perp)^{1+\epsilon}} \]

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Collinear and soft modes

\[ p^2 \sim Q^2 \]

\[ p^2 \sim Q \Lambda_{QCD} \]

\[ p^2 \sim \Lambda_{QCD}^2 \]

\[ a = 0 \]
Collinear and soft modes

Factorization now requires distinguishing collinear and soft modes by rapidity, not virtuality (i.e. transverse momentum)

Manohar, Stewart (2006)
Towards Jets and Event Shapes at LHC
Factorization in pp $\rightarrow$ Jets

- Hadrons in both initial and final state
- Initial state hadrons $\rightarrow$ PDFs, operator definitions of beam remnant and underlying event
- Final state jets $\rightarrow$ jet functions
- New soft functions connecting incoming and outgoing lines
- Factorization shown so far for subprocess $q\bar{q} \rightarrow q\bar{q}$, needs generalization to other partonic subprocesses.

Bauer, Hornig, Tackmann (2008)
Any jet algorithm can be described as acting on a list of final state momenta, that is, the energy-momentum flow.

Energy-momentum flow operator at heart of field theoretic treatment of jet algorithms.

Parameters of jet algorithm choose typical size of jets, setting the scale of jet functions, and determining the factorizability of jet cross sections.

Jet algorithms may be combined with event shapes to study internal structure of individual jets.
Angularities as measures of jet substructure


• A dependence of angularities distinguishes “diffuse” jets with energy spread out more evenly from jets whose constituents are sharply bunched

• Potential to distinguish light quark, heavy quark, and gluon jets; or QCD jets from W, Z jets

• Possible to construct other jet shapes, study factorization properties and distinguishing power
Angularities as measures of jet substructure

Angularity, $\tau_a$ (a = -2, z = 0.05, R = 0.4)

- $Z_{\text{Long. jets}}$
- QCD jets

Angularity ($\tau_2$)
Conclusions: The Future of Jets

- Achieved:
  - Operator-based definition for variety of jet observables
  - Proof of factorization of large class of event shapes, universality of soft power corrections
- Development of tools to gain control over perturbative and nonperturbative aspects of jets
- Precision physics with jets
  - Top mass: Fleming, Hoang, Mantry, Stewart, Jain, Scimemi
  - Strong coupling: Becher, Schwartz
- Deduce jet origin from substructure (energy profile)
- \( N^n\text{LL} \) resummed jet, event shape distributions
- Direct application here to jet physics at linear colliders, points towards jets in hadron collisions at LHC.