Minimal Lepton Flavor Violation

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Motivation, rationale, MFV

• New physics is required to explain the fine tuning puzzle.

• It must involve new dynamics that become relevant at an energy scale Λ. This new dynamics likely involves new fields, new particles and new interactions.

• The new dynamics scale Λ must not be much higher than the electroweak scale (the higher the scale the more severe the fine tuning).

• Quarks and leptons contribute to quadratic divergence in higgs mass
  • divergences depend on quark/lepton masses
  • new dynamics must have flavor dependence

• New flavor dependent dynamics at a scale Λ_F not far above the electroweak scale is a disaster:
  • either Λ_F ~10^{6-7} GeV, or
  • fine tune coefficients of dangerous operators (those giving large flavor changing neutral “currents”)

• Unless: avoid large FCNC automatically “⇒” Minimal Flavor Violation
• MFV is NOT the only possibility
  • *e.g.,* NMFV and generally theories with quark mass suppression

• But MFV is fairly minimal
  • good if you want to estimate the minimal effect of this new physics in flavor changing processes
  • more predictive, patterns

• Let’s gain some understanding by example. Consider $K_L \rightarrow \pi \nu \nu$
In the SM:

\[ \mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} c^\ell \bar{s}_L \gamma_\mu d_L \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.} \]

\[ C^\ell = \frac{\alpha X \left( \frac{m_t}{M_W} \right)}{2\pi \sin^2 \theta_W} V_{ts}^* V_{td} \]

1 loop factor, \( X \sim 1 \)

\[ V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \]

\[ V_{\text{CKM}} \approx \begin{pmatrix} 1 - \frac{1}{2} \lambda^2 & \lambda & A\lambda^3(\bar{\rho} - i\bar{\eta}) \\ -\lambda(1 + iA^2\lambda^4\bar{\eta}) & 1 - \frac{1}{2} \lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2(1 + i\lambda^2\bar{\eta}) & 1 \end{pmatrix} + \mathcal{O}(\lambda^6). \]

\[ \lambda \simeq 0.22 \quad |V_{ts}V_{td}| \sim \lambda^5 \]
\[ H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} C^\ell \bar{s}_L \gamma_\mu d_L \bar{\nu}_L \gamma^\mu \nu^\ell_L + \text{h.c.} \]

New physics

\[ H_{\text{eff}} = \frac{1}{\Lambda_F^2} \sum_{\ell=e,\mu,\tau} C^\ell_{\text{new}} \bar{s}_L \gamma_\mu d_L \bar{\nu}_L \gamma^\mu \nu^\ell_L + \text{h.c.} \]

Assume sensitivity to fractional deviation \( r \) from SM rate, with \( C^\ell_{\text{new}} \sim 1 \)

\[ 1 + r \sim \left| 1 + \frac{(M_W/\Lambda)^2}{A^2 \lambda^5/(16\pi^2)} \right|^2 \]

For example, \( r = 4\% \) gives sensitivity to \( \Lambda_F \sim 10^6 \text{ GeV} \)

But if new physics has same CKM factors in \( C^\ell_{\text{new}} \), then

\[ 1 + r \sim \left| 1 + \frac{(M_W/\Lambda)^2}{1/(16\pi^2)} \right|^2 \]

And now \( r = 4\% \) gives sensitivity to \( \Lambda_F \sim 10^{3-4} \text{ GeV} \)
Minimal Flavor Violation (MFV)

• Premise: Unique source of flavor braking
• Quark sector in SM, in absence of masses has large flavor (global) symmetry:
  \[ G_F = SU(3)^3 \times U(1)^2 \]
• In SM, symmetry is only broken by Yukawa interactions, parametrized by couplings \( \lambda_U \) and \( \lambda_D \)
  \[ \mathcal{L}_{\text{Yuk}} = H \bar{q}_L \lambda_U u_R + \tilde{H} \bar{q}_L \lambda_D d_R \]
• MFV: all breaking of \( G_F \) must transform as these
• When going to mass eigenstate basis, all mixing is parametrized by CKM and GIM is automatic
• Approach: via effective field theory: at low energies only SM fields
How does this work?
Consider $K_L \rightarrow \pi \nu \bar{\nu}$

Recall, $G_F$ breaking from: $\mathcal{L}_{\text{Yuk}} = H \bar{q}_L \lambda_U u_R + \bar{H} \bar{q}_L \lambda_D d_R$

Implications of $G_F$? use spurion method:

\begin{align*}
q_L &\rightarrow V_L q_L \\
u_R &\rightarrow V_u \nu_R \\
d_R &\rightarrow V_d d_R
\end{align*}

\begin{align*}
\lambda_U &\rightarrow V_L \lambda_U V_u^\dagger \\
\lambda_D &\rightarrow V_L \lambda_D V_d^\dagger
\end{align*}

Effective lagrangian \( \mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2} \sum C_i O_i \) among the operators have, for example

\[
O = \bar{q}_L (\lambda_U \lambda_U^\dagger) \gamma_\mu q_L \bar{\nu}_L \gamma_\mu \nu_L
\]

In mass basis \( \Rightarrow \left( \sum_{q=u,c,t} V_{qs}^* V_{qd} \frac{m_q^2}{v^2} \right) \bar{s}_L \gamma_\mu d_L \bar{\nu}_L \gamma_\mu \nu_L \)

As needed it includes the factor

\[
|V_{ts}^* V_{td}| m_t^2 / v^2 \approx A^2 \lambda^5 \approx 5 \times 10^{-4}
\]
Minimal Lepton Flavor Violation (MLFV)

- What motivation for MLFV?
  - Aping quark sector
  - GUT's
- If leptons acquire Dirac masses (like quark sector) then copy from above. Uninteresting: flavor violation in charge lepton sector proportional to tiny neutrino masses
- Alternative: Majorana masses.
  - Appealing: rationale for small neutrino masses: see-saw mechanism
  - Different from quark sector. So... What are the prediction (for charged lepton $\Delta F \neq 0$ processes) from MLFV in see-saw models?
Note: LN vs LF

• Distinguish Lepton Number (LN) violating interactions from Lepton Flavor (LF) violating interactions

• LN is a $U(1)$ symmetry, assigning unit charge to all leptons (like baryon number for quarks)
  • Majorana mass breaks LN

• LF is an $SU(3)$ symmetry, mixing different flavors
  • It commutes with $U(1)_{LN}$, i.e., preserves the LN charge
We want to consider LFV at a ‘low scale’ (few TeV?), while for see-saw want LNV at an intermediate scale

\[ \Lambda_{LF} \ll \Lambda_{LN} \ll M_{\text{planck}} \]

- Two approaches. Field content below \( \Lambda_{LF} \) scale is three families of \( L_i \) and \( e_{Ri} \) (plus \( H \) and gauge). Then:
  - Minimal: majorana mass is from non-renormalizable LN breaking interaction
  - Extended: include very heavy \( \nu_{Ri} \) insofar as it dictates MFV coupling, but then integrate out
MLFV: Minimal Field Content

Assumptions:

1. The breaking of the $U(1)_{LN}$ is independent from the breaking of lepton flavor $G_{LF}$, with large $\Lambda_{LN}$ (associated with see-saw)
2. There are only two irreducible sources of $G_{LF}$ breaking, $\lambda_e$ and $g_\nu$, defined by

$$\mathcal{L}_{\text{Sym.Br.}} = -\lambda^{ij}_e \bar{e}_R^i \left(H^\dagger L_j^i\right) - \frac{1}{2\Lambda_{LN}} g^{ij}_\nu \left(\bar{L}_L^c i \tau_2 H \right) \left(H^T \tau_2 L_L^j\right) + \text{h.c.}$$

(Note: one can also study 2 higgs model case. Note here)

Ex: SUSY Triplet Model, A. Rossi, PRD66(2002)075003
MLFV: Extended Field Content

Recall, now we include RH neutrinos, flavor group has additional SU(3)_{\nu R} factor

Assumptions:

1. The right handed neutrino mass is flavor neutral, i.e., it breaks SU(3)_{\nu R} to O(3)_{\nu R}. Denote $M_{\nu}^{i j} = M_{\nu} \delta^{i j}$ (this makes it still “minimal”)
2. The right handed neutrino mass is the only source of LN breaking and $M_{\nu} \gg \Lambda_{LFV}$
3. Remaining LF-symmetry broken only by $\lambda_e$ and $\lambda_{\nu}$ defined by

$$L_{Sym. Br.} = -\lambda_{e}^{i j} \bar{e}_{R}^{i} (H^{\dagger} L_{j}^{j}) + i \lambda_{\nu}^{i j} \bar{\nu}_{R}^{i} (H^{T} \tau_{2} L_{j}^{j}) + h.c.$$

Note: after integrating out $\nu_{R}$ we have the same lagrangian as for minimal field content, with $g_{\nu} = (\lambda_{\nu})^{T} \lambda_{\nu}$

The distinction is then in what operators are allowed

Ex: SUSY with RH degenerate N’s,
Implementation of MLFV

- Want to add all possible terms to the lagrangian consistent with assumptions (and usual stuff: Lorentz invariance, gauge symmetry, locality, ...)
- Need characterization of terms that are allowed
- As before, use spurion method. In minimal field content case:

\[ L_L \rightarrow V_L L_L \quad e_R \rightarrow V_R e_R \]
\[ \lambda_e \rightarrow V_R \lambda_e V_L^\dagger \quad g_\nu \rightarrow V_L^* g_\nu V_L^\dagger \]

and define \( \Delta \equiv g_\nu^\dagger g_\nu \) with transformation \( \Delta \rightarrow V_L \Delta V_L^\dagger \)

- Extended field content case

\[ L_L \rightarrow V_L L_L \quad e_R \rightarrow V_R e_R \quad \nu_R \rightarrow O_\nu \nu_R \]
\[ \lambda_e \rightarrow V_R \lambda_e V_L^\dagger \quad \lambda_\nu \rightarrow O_\nu \lambda_\nu V_L^\dagger \]

and define \( \Delta = \lambda_\nu^\dagger \lambda_\nu \) \( \Delta \rightarrow V_L \Delta V_L^\dagger \)

Also \( \delta = \lambda_\nu^T \lambda_\nu \) with \( \delta \rightarrow V_L^* \delta V_L^\dagger \) just like \( g_\nu \) for minimal content
Then write all operators of dimension 5, 6, ... consistent with assumptions.

For \( \mu \to e\gamma, \mu + N \to e + N' \), need two lepton field ops:

**Ops with LL**

\[ O_{LL}^{(1)} = \bar{L}_L \gamma^\mu \Delta L_L \ H^\dagger i D_\mu H \]
\[ O_{LL}^{(2)} = \bar{L}_L \gamma^\mu \tau^a \Delta L_L \ H^\dagger \tau^a i D_\mu H \]
\[ O_{LL}^{(3)} = \bar{L}_L \gamma^\mu \Delta L_L \ \bar{Q}_L \gamma_\mu Q_L \]
\[ O_{LL}^{(4d)} = \bar{L}_L \gamma^\mu \Delta L_L \ \bar{d}_R \gamma_\mu d_R \]
\[ O_{LL}^{(4u)} = \bar{L}_L \gamma^\mu \Delta L_L \ \bar{u}_R \gamma_\mu u_R \]
\[ O_{LL}^{(5)} = \bar{L}_L \gamma^\mu \tau^a \Delta L_L \ \bar{Q}_L \gamma_\mu \tau^a Q_L \]

**Ops with RL**

\[ O_{RL}^{(1)} = g' H^\dagger \bar{e}_R \sigma^{\mu\nu} \lambda_e \Delta L_L B_{\mu\nu} \]
\[ O_{RL}^{(2)} = g H^\dagger \bar{e}_R \sigma^{\mu\nu} \tau^a \lambda_e \Delta L_L W_{a\mu\nu} \]
\[ O_{RL}^{(3)} = (D_\mu H)^\dagger \bar{e}_R \lambda_e \Delta D_\mu L_L \]
\[ O_{RL}^{(4)} = \bar{e}_R \lambda_e \Delta L_L \bar{Q}_L \lambda D d_R \]
\[ O_{RL}^{(5)} = \bar{e}_R \sigma^{\mu\nu} \lambda_e \Delta L_L \bar{Q}_L \sigma_{\mu\nu} \lambda D d_R \]
\[ O_{RL}^{(6)} = \bar{e}_R \lambda_e \Delta L_L \bar{u}_R \lambda_U^\dagger i \tau^2 Q_L \]
\[ O_{RL}^{(7)} = \bar{e}_R \sigma^{\mu\nu} \lambda_e \Delta L_L \bar{u}_R \sigma_{\mu\nu} \lambda_U^\dagger i \tau^2 Q_L \]

We have neglected \( \Delta^2 \)

We have neglected \( \sim (\lambda_e)^2 \), hence no RR operators

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For $\mu \rightarrow eee$ need, in addition, four lepton operators

\[ O^{(1)}_{4L} = \bar{L}_L \gamma^\mu \Delta L_L \bar{L}_L \gamma_\mu L_L \]
\[ O^{(2)}_{4L} = \bar{L}_L \gamma^\mu \tau_\alpha \Delta L_L \bar{L}_L \gamma_\mu \tau_\alpha L_L \]
\[ O^{(3)}_{4L} = \bar{L}_L \gamma^\mu \Delta L_L \bar{e}_R \gamma_\mu e_R \]
\[ O^{(4)}_{4L} = \delta_{nj} \delta^*_m \bar{L}_i^L \gamma^\mu L_j^L \bar{L}_m^L \gamma^\mu L_n^L \]
\[ O^{(5)}_{4L} = \delta_{nj} \delta^*_m \bar{L}_i^L \gamma^\mu \tau_\alpha L_j^L \bar{L}_m^L \gamma^\mu \tau_\alpha L_n^L \]
Up to dimension 6 operators, the new interactions are

$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2_{\text{LFV}}} \sum_{i=1}^{5} \left( c_{LL}^{(i)} O_{LL}^{(i)} + c_{4L}^{(i)} O_{4L}^{(i)} \right) + \frac{1}{\Lambda^2_{\text{LFV}}} \left( \sum_{j=1}^{2} c_{RL}^{(j)} O_{RL}^{(j)} + \text{h.c.} \right)$$

with coefficients naively $c \sim 1$

We can now study the phenomenology of MLFV with minimal field content.

Useful to look at parameters first
Use $G_{LF}$ symmetry to rotate to the mass eigenstate basis ($\nu = \text{Higgs vev}$)

$$\lambda_e = \frac{m_\ell}{\nu} = \frac{1}{\nu} \text{diag}(m_e, m_\mu, m_\tau)$$

$$g_\nu = \frac{\Lambda_{LN}}{\nu^2} U^* m_\nu U^\dagger = \frac{\Lambda_{LN}}{\nu^2} U^* \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) U^\dagger$$

$U$ is the PMNS matrix. It is determined from neutrino mixing:

$$U \approx \begin{pmatrix} ce^{i\alpha_1/2} & se^{i\alpha_2/2} & s_{13}e^{-i\delta} \\ -se^{i\alpha_1/2}/\sqrt{2} & ce^{i\alpha_2/2}/\sqrt{2} & 1/\sqrt{2} \\ se^{i\alpha_1/2}/\sqrt{2} & -ce^{i\alpha_2/2}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

Here $c \equiv \cos \theta_{\text{sol}}$, $s \equiv \sin \theta_{\text{sol}}$, $\theta_{\text{sol}} \simeq 32.5^\circ$

$s_{13}$ is poorly known, $s_{13} < 0.3$

oops! sorry: two different $\delta$
Hence, amplitudes are given in terms of
- $\Lambda_{LN}$ or $M_\nu$ and $\Lambda_{LFV}$ (actually only ratio $\Lambda_{LN}/\Lambda_{LFV}$ or $M_\nu/\Lambda_{LFV}$)
- Coefficients, $c$, of order 1
- Low energy measured (or measurable) masses and mixing angles
- In particular, the following two combinations appear in the operators:

$$\Delta = \begin{cases} 
\frac{\Lambda_{LN}^2}{v^4} U m_\nu^2 U^\dagger & \text{minimal field content} \\
\frac{M_\nu}{v^2} U m_\nu U^\dagger & \text{extended field content, CP limit}
\end{cases}$$

$$\delta = \delta^T = \begin{cases} 
\frac{\Lambda_{LN}}{v^2} U^* m_\nu U^\dagger & \text{minimal field content} \\
\frac{M_\nu}{v^2} U^* m_\nu U^\dagger & \text{extended field content}
\end{cases}$$

Note: recall in extended case $\Delta = \lambda_\nu^\dagger \lambda_\nu$, which is not directly related to mass. It is in CP limit, which we assume for simplicity (and further minimality).
MLFV: Phenomenology
\[ B_{\ell_i \rightarrow \ell_j(\gamma)} = 10^{-50} \left( \frac{\Lambda_{LN}}{\Lambda_{LFV}} \right)^4 R_{\ell_i \rightarrow \ell_j(\gamma)}(s_{13}, \delta; c^{(i)}) \]

- since \( \Delta \propto U(m_\nu)^2 U^\dagger \), only differences of \( m^2 \) enter; these are measured!
- \( s_{13} \) and \( \delta \) unknown PMNS parameters (scan on \( \delta \))
- choose \( c^{(i)} \) of order one for the estimate
- ratio of scales can be large:
  \[ \text{perturbative } g_\nu \Rightarrow \Lambda_{LN} \lesssim 3 \times 10^{13} (1 \text{ eV}/m_\nu) \text{ GeV} \]
  so \( \Lambda_{LFV} \sim 1 \text{ TeV} \Rightarrow \Lambda_{LN}/\Lambda_{LFV} \lesssim 10^{10} \]
Predictive: $\ell \rightarrow \ell'$ $\gamma$ patterns are independent of unknown input parameters (scales cancel in ratios, in this case $c^{(i)}$s cancel too, and all other parameters are from long distance)
If \( s_{13} \) is small, look at tau modes.

Here \( \Lambda_{LN}/\Lambda_{LFV} = 10^{10} \) and \( c_{RL}^{(1)} - c_{RL}^{(2)} = 1 \)

Belle and BaBar have bounds (summer ‘05) of a few \( \times 10^{-7} \) for \( \text{Br}(\tau \rightarrow \ell \gamma) \) and \( \text{Br}(\tau \rightarrow \ell \ell \ell) \)
\( \mu \to e\gamma, \mu\text{-to-}e \) conversion and their relatives II: extended field content

- Replace \( \Lambda_{LN}^2/\Lambda_{LFV}^2 \) by \( v M_\nu/\Lambda_{LFV}^2 \)
- Now \( \Delta \propto U m_\nu U^\dagger \) so amplitudes depend on overall neutrino mass scale (ie, lightest neutrino mass)

\[
B_{\ell_i \to \ell_j (\gamma)} = 10^{-25} \left( \frac{v M_\nu}{\Lambda_{LFV}^2} \right)^2 \widehat{R}_{\ell_i \to \ell_j (\gamma)}(s_{13}, m_\nu^{\text{lightest}}, c^{(i)})
\]

perturbative \( \lambda_\nu \Rightarrow M_\nu \lesssim 10^{13} \) GeV; with \( \Lambda_{LFV} \geq 1 \) TeV, \[
\frac{v M_\nu}{\Lambda_{LFV}^2} \leq 10^9
\]
One final note: results depend on hierarchy of neutrino masses, normal \((m_{\nu 1} \sim m_{\nu 2} \ll m_{\nu 3})\) vs. inverted \((m_{\nu 1} \ll m_{\nu 2} \sim m_{\nu 3})\)

\[
\frac{(\nu M_\nu)}{\Lambda_{\text{LFV}}^2} = 5 \times 10^7
\]

\[
c_{RL}^{(1)} - c_{RL}^{(2)} = 1
\]

shading: \(0 \leq m_{\nu}^{\text{lightest}} \leq 0.02\) eV
$3l$ Decays: $4L$ operators

$$\frac{\Gamma_{\mu \to 3e}}{\Gamma_{\mu \to e\nu\bar{\nu}}} = \left[ |a_+|^2 + 2|a_-|^2 - 8\text{Re}(a_0^*a_-) - 4\text{Re}(a_0^*a_+) + 6|a_0|^2 \right] \left\{ \left( \frac{\Lambda_{\text{LN}}}{\Lambda_{\text{LFV}}} \right)^4 |a_{e\mu}|^2 \right\} \text{ minimal}$$

$$\left\{ \left( \frac{\nu M_{\nu}}{\Lambda_{\text{LFV}}^2} \right)^2 |b_{e\mu}|^2 \right\} \text{ extended}$$

$$a_+ = \sin^2 \theta_w (c_{LL}^{(1)} + c_{LL}^{(2)}) + c_{4L}^{(3)}$$

$$a_- = (\sin^2 \theta_w - \frac{1}{2})(c_{LL}^{(1)} + c_{LL}^{(2)}) + c_{4L}^{(1)} + c_{4L}^{(2)} + \frac{2\delta_{e\mu}\delta_{ee}}{\Delta_{e\mu}} (c_{4L}^{(4)} + c_{4L}^{(5)})$$

$$a_0 = 2e^2(c_{RL}^{(1)} - c_{RL}^{(2)})^*$$

![Graphs showing $|a_{e\mu}|^2$ and $|b_{e\mu}|^2$ vs $s_{13}$ for different values of $\delta$.](image)
\[ \Gamma_{\tau \rightarrow e\mu\bar{\mu}} = \Gamma_{\tau \rightarrow e\nu\bar{\nu}} \frac{v^4 |\Delta e\tau|^2}{\Lambda_{LFV}^4} \left[ |a_+|^2 + |\tilde{a}_-|^2 - 4 \text{Re}[a_0^*(a_+ + \tilde{a}_-)] + 12\tilde{I}|a_0|^2 \right] \]

\[ \Gamma_{\tau \rightarrow \mu\mu\bar{e}} = \Gamma_{\tau \rightarrow e\nu\bar{\nu}} \frac{v^4 |2\delta e\tau \delta_{\mu\mu}|^2}{\Lambda_{LFV}^4} \left| c_L^{(4)} + c_L^{(5)} \right|^2 \]
GUTs

• GUTs connect MFV in quark and lepton sectors
  • Better motivation for MLFV
  • New effects (e.g., LFV even for Dirac neutrino)
  • Includes thoroughly studied models (e.g., SUSY-GUTs)
MFV-GUTs in a nut-shell

three families of left handed fields:

\[ \psi_i \sim \bar{5} \quad \chi_i \sim 10 \quad N_i \sim 1 \quad i = 1, 2, 3 \]

\[(d^c_R, L_L) \quad (Q_L, u_R^c, e_R^c)\]

In the absence of masses, symmetric under \[SU(3)_{\bar{5}} \times SU(3)_{10} \times SU(3)_1\]

Include symmetry breaking (here with one higgs):

\[ \lambda^{ij}_5 \psi^T_i \chi_j H_5^* + \lambda^{ij}_{10} \chi^T_i \chi_j H_5 \quad \text{gives bad mass relations for light families}\]

\[ \frac{1}{M} \left( \lambda'_{5}^{ij} \right)^T \psi^T_i \Sigma \chi_j H^*_5 \]

\[ \Sigma \sim 24; M \text{ large; freedom to fix mass relations}\]

\[ \lambda_u \propto \lambda_{10}, \lambda_d \propto (\lambda_5 + \epsilon \lambda'_5), \lambda_e^T \propto (\lambda_5 - \frac{3}{2} \epsilon \lambda'_5), \epsilon = \frac{M_{\text{GUT}}}{M} \]

\[ \lambda^{ij}_1 N^T_i \psi_j H_5 + M^{ij}_R N^T_i N_j \quad \text{neutrino masses (Dirac+Majorana)} \]

spurion transformation laws:

\[ Q_L \rightarrow V_{10} Q_L \quad \lambda_{10} \rightarrow V^*_{10} \lambda_{10} V^T_{10} \]
\[ u_R \rightarrow V^*_{10} u_R \quad \lambda_5 \rightarrow V^*_{5} \lambda_5 V^T_{10} \]
\[ d_R \rightarrow V^*_{5} d_R \quad \lambda'_{5} \rightarrow V^*_{5} \lambda'_{5} V^T_{10} \]
\[ L_L \rightarrow V_{5} L_L \quad \lambda_1 \rightarrow V^*_{10} \lambda_1 V^T_{5} \]
\[ e_R \rightarrow V^*_{10} e_R \quad M_R \rightarrow V^*_{1} M_R V^T_{1} \]

\[ \rightarrow \text{connect lepton to quark MFV} \]
get old mixing structures (to be included in composite operators), like

**quarks:** \( Q_L \lambda_u^\dagger \lambda_u Q_L \), \( \bar{d}_R \lambda_d^T \lambda_u^\dagger \lambda_u Q_L \)

**leptons:** \( \bar{L}_L \lambda_1^\dagger \lambda_1 L_L \), \( \bar{e}_R \lambda_e^\dagger \lambda_1 L_L \)

but also get interesting new ones, like

**quarks:** \( \bar{Q}_L (\lambda_e \lambda_e^\dagger)^T Q_L \), \( \bar{d}_R \lambda_e^T (\lambda_e \lambda_e^\dagger)^T Q_L \), \( \bar{d}_R (\lambda_e^\dagger \lambda_e)^T d_R \), \( \bar{d}_R (\lambda_1^\dagger \lambda_1)^T d_R \)

**leptons:** \( \bar{L}_L (\lambda_d^\dagger \lambda_d)^T L_L \), \( \bar{e}_R (\lambda_d^\dagger \lambda_d)^T L_L \), \( \bar{e}_R (\lambda_1^\dagger \lambda_1)^T L_L \)

going over to quark/lepton mass basis, introduce two new mixing matrices \( C = V_{eR}^T V_{dL} \), \( G = V_{eL}^T V_{dR} \)

so get, for example

\[
\begin{align*}
\bar{e}_R \lambda_u^\dagger \lambda_u e_R & \quad \bar{e}_R \left[ C \Delta (q) C^\dagger \right]^* e_R \\
\bar{e}_R \lambda_u^\dagger \lambda_d^T L_L & \quad \bar{e}_R \left[ C \Delta (q) \bar{\lambda}_d G^\dagger \right]^* \bar{e}_L \\
\bar{e}_R \lambda_u^\dagger \lambda_e L_L & \quad \bar{e}_R \left[ C \Delta (q) C^\dagger \right]^* \lambda_e \bar{e}_L
\end{align*}
\]

where \( \Delta_{ij}^{(q)} \equiv V_{CKM}^\dagger \bar{\lambda}_u^2 V_{CKM} = \frac{m_i^2}{v^2} (V_{CKM})^*_{3i} (V_{CKM})_{3j} + \mathcal{O}(m_{c,u}^2/m_t^2) \)
quick example (probably out of time by now):

\[ \tau \to \mu \gamma, \quad \tau \to e \gamma \quad & \quad \mu \to e \gamma \]

\[ \Delta L_{\text{eff}} = \frac{v}{\Lambda^2} \bar{e}_R \left[ c_1 \lambda_e \lambda_1^\dagger \lambda_1 + c_2 \lambda_u \lambda_u^\dagger \lambda_e + c_3 \lambda_u \lambda_u^\dagger \lambda_d^T \right] \sigma^{\mu\nu} e_L F_{\mu\nu} \]

just like MLFV above

Generalizes Barbieri & Hall \( (\lambda'_5 = 0, C = G = 1) \)

New mixing structures

Independent of \( M_\nu \)

Hierarchical

Large: for \( \Lambda = 10\) TeV

\[ \text{Br}(\mu \to e \gamma) \sim 10^{-12} \]

\[ C = V_{eR}^T V_{dL} \]

\[ G = V_{eL}^T V_{dR} \]

\[ \left( \frac{m_t^2}{v^2} \right) \times \left\{ \begin{array}{l} \lambda^2 (m_\tau/v), \quad (\tau \to \mu) \\ \lambda^3 (m_\tau/v), \quad (\tau \to e) \\ \lambda^5 (m_\mu/v), \quad (\mu \to e) \end{array} \right\} \]

\( \lambda = 0.22 \)
Conclusion

(I think the case is made that)

Non-tuned theories of flavor with LHC-scale new-physics may reasonably be expected to exhibit charged lepton LFV (lepton flavor violation) at levels that are accessible through experiments like MEG and the next generation, the proposed PRIME@j-parc and mu2e

Not water tight, only compelling. What could go wrong?
- Accidents (small coefficients $c$)
- No GUTs, “Light” right handed neutrinos (or purely Dirac)

Leptogenesis? Ask Vincenzo
More slides
Part of loop graph (W is virtual).
For any one intermediate quark amplitude is

\[ M_W^D F \left( \frac{m_q^2}{M_W^2} , \mu/M_W \right) \]

Sum over intermediate quarks and expand

\[ \sum_q V_{qd} V_{qs}^* F \left( \frac{m_q^2}{M_W^2} \right) \approx \sum_q V_{qd} V_{qs}^* \left[ F(0) + \frac{m_q^2}{M_W^2} F'(0) + \cdots \right] \]

For first term use \[ \sum_q V_{qd} V_{qs}^* = 0 \] and for second \[ \sum_{q \neq u} V_{qd} V_{qs}^* = -V_{ud} V_{us}^* \]

\[ \Rightarrow \sum_q m_q^2 V_{qd} V_{qs}^* = \sum_{q \neq u} (m_q^2 - m_u^2) V_{qd} V_{qs}^* \]

(jump back)
Decays of/to hadrons

Hopelessly small!

\[ \pi^0 \rightarrow \mu^+ e^- \]  
\[ \Upsilon \rightarrow \tau \mu \]  
\[ \tau \rightarrow \pi \mu \]  

\[ \text{Br} \]  
\[ 10^{-25} \]  
\[ 10^{-20} \]  
\[ 10^{-15} \]
• We have also explored the effects of deleting a class of operators.
• For example: assume 4L operators are not present
• Can we get 3l decays? Yes, through loops
• Need care in loops of light quarks: chiral lagrangian does the job
• Result: amplitude is ~0.1 of 4L ops (large logs)
• Equivalently, these give a ~20% correction to rate
• Patterns are similar to those from 4L