Dispersive $\gamma Z$ correction to the QWEAK measurement

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Outline

Precision electroweak physics: points of interest

Elastic electron scattering off the proton: kinematics and observables reduction for the forward kinematics

Dispersion calculation of the γZ box in forward regime
  Optical theorem - connection with PV DIS
  Crossing behavior (C-,P- parity) - dispersion relations

Input for DR - model the PV DIS structure functions

Uncertainties due to t-dependence

Vector analyzing power: possible check for t-dependence

Summary & Outlook
Precision electroweak physics: tests of Standard Model

Weak mixing angle: central role in SM

\[
\begin{align*}
W_\pm^\mu &= \frac{1}{\sqrt{2}} (A_1^\mu \pm iA_2^\mu) \\
Z^\mu &= \cos \theta_W A_3^\mu + \sin \theta_W B^\mu \\
A^\mu &= -\sin \theta_W A_3^\mu + \cos \theta_W B^\mu \\
g &= -\frac{e}{\sin \theta_W} \frac{M_W}{M_Z} = \cos \theta_W \\
g' &= -\frac{e}{\cos \theta_W} \frac{M_W}{M_Z} = \sin \theta_W
\end{align*}
\]

\[
\begin{align*}
\sin \theta_W (M_Z^2) &= 0.23113 \pm 0.00015 \quad \text{PDG} \\
\sin^2 \theta_W (0) &= 0.23807 \pm 0.00017 \quad \text{Erler et al. '04}
\end{align*}
\]

Weak charge of the proton in SM \( q_W^P (0) = 1 - 4 \sin^2 \theta_W \approx 0.05 \)

QWEAK experiment: 4% determination of weak charge of the proton (2% exp. + 2% theory) - 0.3% determination of the weak angle

Deviation from SM value: New Physics at low energies

Agreement with SM value: constraints on NP

4% precision estimate relies on 2% theoretical uncertainty - do we know it that well?
Radiative corrections

\[ Q_W^p = 1 - 4 \sin^2 \hat{\theta}_W(0) + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z} \]

Existing estimates: Marciano, Sirliin ‘83,84 (APV) - Ramsey-Musolf ‘99 (proton)

**WW, ZZ-box:** dominated by large momenta in the loop

\[ \sim \int d^4 l \ldots \frac{1}{(l^2 - M_{WW,ZZ}^2)^2} \]

Purely short-range contribution - calculable perturbatively

\[ \Box_{WW} = \frac{7\hat{\alpha}}{4\pi \hat{s}^2} \lesssim 26\% \quad \Box_{ZZ} = \frac{\hat{\alpha}}{4\pi \hat{s}^2 c^2} \left( \frac{9}{4} - 5\hat{s}^2 \right) \left( 1 - 4\hat{s}^2 + 8\hat{s}^4 \right) \lesssim 3\% \]

**\( \gamma Z \)-box**

\[ \sim \int d^4 l \ldots \frac{1}{l^2} \frac{1}{l^2 - M_Z^2} \]

no reason to be dominated by large loop momenta

Cancellation between the box and crossed box

\[ \bar{u}[\gamma_\alpha l \gamma_\beta (g^e_V + g^e_A \gamma_5) - \gamma_\beta (g^e_V + g^e_A \gamma_5) l \gamma_\alpha] u T^{\alpha\beta}(l) D(l^2) \]

\[ = 2i \epsilon_{\alpha\lambda \beta \mu} l^\lambda \bar{u} \gamma^\mu (g^e_V \gamma_5 + g^e_A) u T^{\alpha\beta}(l) D(l^2), \]

Large axial term - exact cancellation for \( l \gg k \)

QWEAK: \( k \sim 1 \text{ GeV} \) - is it still small?
Elastic electron-proton scattering: kinematics & observables

\[ s = (P + K)^2 \]
\[ u = (P - K)^2 \]
\[ t = \Delta^2 = -Q^2 \]
\[ s + u + t = 2M^2 - 2m^2 \approx 2M^2 \]
\[ \nu = \frac{PK}{M} = \frac{s - M^2 + \Delta^2/2}{2M} \]

Elastic electron-proton scattering amplitude (massless electron)

\[ T = \frac{e^2}{-t} \bar{u}(k')\gamma^\mu u(k)\bar{N}(p') \left[ f_1 \gamma^\mu + f_2 i\sigma^{\mu\alpha} \frac{\Delta^\alpha}{2M} \right] N(p) + \frac{e^2}{-t} f_3 \bar{u}(k')\gamma^\mu\gamma^5 u(k)\bar{N}(p')\gamma^\mu\gamma^5 N(p) - \frac{G_F}{2\sqrt{2}} \bar{u}(k')\gamma^\mu\gamma_5 u(k)\bar{N}(p')\gamma^\mu\gamma_5 N(p) \]

Near forward direction: further reduction

\[ \bar{N}\gamma^\mu N = \bar{N} \left[ \frac{P^\mu}{M} + i\sigma^{\mu\alpha} \frac{\Delta^\alpha}{2M} \right] N \rightarrow 2P^\mu \]
\[ \bar{N}\gamma^\mu\gamma_5 N = \rightarrow -2P^\mu\lambda_z(p) \]
\[ f_3 = f_3^\gamma + f_3^Z = \mathcal{O}(t) \]

Polarized forward electron scattering

\[ T(t \rightarrow 0) = \frac{e^2}{-t} 2\tilde{f}_1 \bar{u}(k')\slashed{p} u(k) - \frac{G_F}{\sqrt{2}} \tilde{f}_4 \bar{u}(k')\slashed{p}\gamma_5 u(k) + \mathcal{O}(t) \]
\[ \tilde{f}_i(\nu) = f_i(\nu, t = 0) \]
Elastic electron-proton scattering: kinematics & observables

Unpolarized cross section

\[
\left( \frac{d\sigma}{d\Omega_{lab}} \right) = \frac{4\alpha^2 \cos^2 \frac{\theta}{2} E^3}{t^2} \frac{E}{E} |\tilde{f}_1|^2 (1 + \mathcal{O}(t))
\]

Parity violating asymmetry

\[
A^{PV}(t \to 0) = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} = \frac{G_F t}{4\pi\alpha\sqrt{2}} \frac{\text{Re}(\tilde{f}_1^* \tilde{f}_4)}{|\tilde{f}_1|^2}
\]

One-Boson-Exchange (OBE)

\[
\tilde{f}_1^{OBE} = 1 \\
\tilde{f}_4^{OBE} = g_A Q_W^p
\]

Including radiative corrections

\[
\tilde{f}_1 = 1 + \mathcal{O}(\alpha t) \\
\tilde{f}_4 = Q_W^p [g_A^e + \delta_{RC}(\nu) + \delta_{\gamma Z}(\nu)] + \mathcal{O}(\alpha t, G_F, \ldots)
\]

Parity violating asymmetry - to NLO

\[
A^{PV} = \frac{G_F t}{4\pi\alpha\sqrt{2}} Q_W^p \left[ 1 + \text{Re} \delta_{RC} + \text{Re} \delta_{\gamma Z}(\nu) \right] + \mathcal{O}(t^2)
\]
Forward dispersion relations for $\delta_{YZ}$

Imaginary part - box only

$$\text{Im} T_{\gamma Z} = -\frac{G_F}{\sqrt{2}} \frac{e^2}{(2\pi)^3} \int \frac{d^3k_1}{2E_1} \frac{l_{\mu \nu} \cdot W^{\mu \nu}}{Q^2(1 + Q^2/M_Z^2)}$$

$$Q^2 = -q^2 = -(k - k_1)^2 \geq 0$$

Exchanged bosons are always spacelike for Im part

Leptonic tensor

$$l_{\mu \nu} = \bar{u}(k') \gamma_{\nu} k_1 \gamma_{\mu}(g_V^e + g_A^e \gamma_5) u(k)$$

Hadronic tensor - from optical theorem

$$W^{\mu \nu} = 2\pi \left\{-g^{\mu \nu} \tilde{F}_1 + \frac{P_\mu P_\nu}{Pq} \tilde{F}_2 + i\epsilon^{\mu \nu \alpha \beta} \frac{P_\alpha q_\beta}{Pq} \tilde{F}_3 \right\}$$

Tensor contraction

$$\text{Im} \delta_{YZ}(\nu) = \alpha \int_{W_\pi^2}^s \frac{dW^2}{s - M^2} \int_{0}^{Q_{max}^2} \frac{dQ^2}{1 + \frac{Q^2}{M_Z^2}} \left\{ g_A^e \frac{1}{Pq} \left[ \frac{Pq}{Pk} \tilde{F}_1 + \left( \frac{2Pk_1}{Q^2} - \frac{P^2}{2Pk} \right) \tilde{F}_2 \right] - g_V^e \frac{1}{Pq} \left( \frac{P, k + k_1}{2Pk} \right) \tilde{F}_3 \right\}$$

On-shell intermediate states - limitations onto phase space in the loop

$$Q_{max}^2 = \frac{(s - M^2)(s - W^2)}{s}$$

$$(M + m_\pi)^2 \leq W^2 \leq s$$
Forward dispersion relations for $\delta_{YZ}$

Analyticity + Crossing -> Dispersion Relations

$$\text{Re} \delta_{YZ}(\nu) = \frac{1}{\pi} \int_{\nu}^{\infty} d\nu' \left[ \frac{1}{\nu' - \nu} \pm \frac{1}{\nu' - \nu} \right] \text{Im} \delta_{YZ}(\nu')$$

Crossing: partial CP-transformation

$$e^-(k) \rightarrow e^-(k')$$
$$p \rightarrow p'$$

$$e^+(k') \rightarrow e^+(k)$$
$$p \rightarrow p'$$

C-parity:

$$l^{\mu\nu}_{\gamma Z, \text{direct}} \sim J_{\text{em}}^\mu (J_V^\nu + J_A^\nu) \rightarrow l^{\mu\nu}_{\gamma Z, \text{crossed}} \sim (-J_{\text{em}}^\mu) (-J_V^\nu + J_A^\nu)$$

CP is conserved - different parity behavior for (em,V) and (em,A) parts

$$\text{Re} \delta_{YZ_A}(\nu) = \frac{2\nu}{\pi} \int_{\nu}^{\infty} \frac{d\nu'}{\nu'^2 - \nu^2} \text{Im} \delta_{YZ_A}(\nu')$$

Marciano&Sirlin mechanism for APV:

$$\text{Re} \delta_{YZ_A}(0) = 0 \quad \text{Re} \delta_{YZ_V}(0) \sim \alpha \frac{g_V^e}{Q^p_W} \lesssim 1\%$$

QWEAK: $\nu = 1.165 \text{ GeV}$
PV DIS structure functions over a wide range in $Q^2$ and $W^2$ - no data available (yet)

$$\begin{align*}
W_{\gamma Z}^{\mu \nu} &= (2\pi)^3 \delta(p_X - P - q) \sum_X \left[ \langle N | V_\mu^Z | X \rangle \langle X | J_\nu^\gamma | N \rangle + \langle N | J_\mu^\gamma | X \rangle \langle X | V_\nu^Z | N \rangle \right] \\
W_{\gamma \gamma}^{\mu \nu} &= (2\pi)^3 \delta(p_X - P - q) \sum_X \langle N | J_\mu^\gamma | X \rangle \langle X | J_\nu^\gamma | N \rangle
\end{align*}$$

Both cases ~ vector x vector - might be similar

No direct comparison to data possible - this similarity needs to be checked in a model

$\Delta(1232)$ resonance - isospin decomposition

$$\langle \Delta | J_{NC}^\mu | p \rangle = \langle \Delta | (2 - 4 \sin^2 \theta_W) J_{em}^\mu - J_{I=0}^\mu | p \rangle \approx \langle \Delta | J_{em}^\mu | p \rangle$$

Matsui, Sato, Lee, PRC’05

High energy part - Regge (Color Dipole Picture)

$$\begin{align*}
\gamma^* \gamma^* : & \sum_q e_q^2 | V_{qq}(\omega, Q^2, k_\perp) |^2 \times A_{qqN}(k_\perp) \\
\gamma^* Z : & \sum_q e_q g_q^V | V_{qq}(\omega, Q^2, k_\perp) |^2 \times A_{qqN}(k_\perp)
\end{align*}$$

$u, d, s : \sum_q e_q^2 = \frac{2}{3}$

$u, d, s : \sum_q e_q g_q^V = \frac{2}{3} (1 + [1 - 4 \sin^2 \theta_W]) \approx \frac{2}{3}$

Similarity is not unnatural both in low and high energy regime
Input to dispersion relations for $\delta_{YZ}$

Use e.-m. DIS structure functions for the estimate

$$\sigma_T = \frac{4\pi^2\alpha}{Pq} F_1 \quad \sigma_L = \frac{4\pi^2\alpha}{Pq} \left[ \left( \frac{1}{2x} + \frac{M^2}{Pq} \right) F_2 - F_1 \right]$$

Total cross sections: resonance + Regge

Match the two components to fit the real photon data

$$\sigma_{\gamma p}(W^2, 0) = \sum_R \frac{\sigma_R \Gamma_R \Gamma_R^\gamma M_R^2}{(W^2 - M_R^2)^2 + M_R^2 \Gamma_R^2} + \sigma_{\gamma p}^{Regge}(W^2)$$

Bianchi et al., PRC’96

$R = P33,D15,P11,S11,F35,F37$

$$\sigma_{\gamma p}^{Regge}(W^2) = f_{thr} \left[ 145 \mu b (W^2) \alpha_P^{-1} + 63.5 \mu b (W^2) \alpha_P^{-1} \right]$$

$$f_{thr} \rightarrow \begin{cases} 0, & W^2 \rightarrow (M + m_\pi)^2 \\ 1, & W^2 \rightarrow \infty \end{cases}$$
Input to dispersion relations for $\delta_{yz}$

Q2 dependence: different for high energy and resonance contributions

Resonances: transition form factors $\sim$ dipole form with $\Lambda \sim 1$ GeV

$$F_T(Q^2) = \frac{1}{(1 + Q^2/\Lambda^2)^2}$$

$$F_L(Q^2) = \frac{Q/\Lambda}{(1 + Q^2/\Lambda^2)^{2.5}}$$

Regge: from CDP/VDM picture

approximate scaling

$$\sigma^{Regge}_{L,T}(W^2, Q^2) = \sigma^{Regge}_{\gamma p}(W^2) \frac{I_{L,T}(\eta, \eta_0)}{I_T(\eta_0, \eta_0)}$$

$$\eta = \frac{m_0^2 + Q^2}{\Lambda^2(W^2)}$$

$$\eta_0 = \eta(Q^2 = 0)$$

$I_{L,T}(\eta, \eta_0)$ analytical functions

$$\sigma_{T,L}(W^2, Q^2) = \sum_R \frac{\sigma_R \Gamma_R \Gamma_R^\gamma}{(W^2 - M_R^2)^2 + M_R^2 \Gamma_R^2} F_{T,L}(Q^2) + \sigma^{Regge}_{T,L}(W^2, Q^2)$$
Dispersion relations for $\delta_{yz}$ - results for the proton

\[ \text{Dispersion correction to QWEAK} \sim 5.5-6\% \]

\[ \text{The dispersion correction to PVES is energy-dependent} \]
- negligibly small for APV, not so small for QWEAK
Present estimate $\sim 6\%$ is model-dependent, since no PV DIS data for $F_1, F_2$ are available
Has to be studied carefully (to 25-30\%) to fit into 2.2\% theory uncertainty for QWEAK
Dispersion correction $\delta_{YZ}$ for spin-0 nuclei

How does the dispersion correction affect PVES on nuclear target?

$$T(t \to 0) = \frac{e^2}{2t} 2\tilde{f}_1 \bar{u}(k')\slashed{p} u(k) - \frac{G_F}{\sqrt{2}} \tilde{f}_4 \bar{u}(k')\slashed{p} \gamma_5 u(k) + \mathcal{O}(t)$$

$$\tilde{f}_1^{OBE} = Z$$
$$\tilde{f}_4^{OBE} = g_A^e [Z Q_W^p - N]$$

Naive scaling

$$\delta_{YZ}[A, N] \sim \delta_{\gamma Z}^p \frac{A}{Z Q_W^p - N} Q_W^p$$

6-10 times smaller than for the proton

Additional uncertainty:
- nuclear structure, nuclear form factors

Work in progress...
Additional uncertainties: \( t \)-dependence

**Optical theorem - in exact forward direction only**

To go to finite \( t \) - correct the input for off-forward kinematics

\[
\frac{d\sigma}{dt} \approx \left[ \frac{d\sigma}{dt} \right]_{t=0} e^{Bt} \quad \Rightarrow \quad \sigma_{\gamma A}(t \neq 0) \approx \sigma_{\gamma A}(t = 0) e^{Bt/2}
\]

_Afanasev, Merenkov ‘04; Gorchtein ‘06, ’07 - for the proton target_

**Experimental Compton slope:**

- **Proton** \( B_p = 7.5 \pm 0.5 \text{ GeV}^{-2} \)  
  **Bauer et al. ‘78, ’79**
- **Helium** \( B_{4\text{He}} = 32.95 \pm 1.91 \text{ GeV}^{-2} \)  
  **Aleksanian et al., ‘87**

Additional uncertainty if connecting low and moderate \( t \) data - need other observables?

**Vector analyzing power (Mott asymmetry, Beam normal spin asymmetry)**

\[
A_n = \frac{\sigma_{\perp+} - \sigma_{\perp-}}{\sigma_{\perp+} + \sigma_{\perp-}} \sim \frac{2\text{Im} T_{\gamma\gamma} T^*_{\gamma\gamma}}{|T_{\gamma}|^2}
\]

An is zero in forward direction

\[
A_{\text{inelast}}^n \approx -\frac{1}{4\pi^2} \frac{m_e}{E_{\text{lab}}} \frac{M}{\sqrt{s}} \frac{A e^{-BQ^2/2}}{F_C(Q^2)} \tan \frac{\theta_{cm}}{2} \int_0^{E_{\text{lab}}} d\omega \sigma_{\gamma p}(\omega) \ln \left[ \frac{Q^2}{m_e^2} \left( \frac{E_{\text{lab}}}{\omega} - 1 \right)^2 \right]
\]
Vector analyzing power on spin-0 target - results

High energies: cross section is roughly constant

\[ \sigma_R \approx 110 \mu\text{barn} \]

\[ A_{n}^{\text{inelast}} \approx A_n^0 e^{-BxQ^2} \tan \left( \frac{\theta_{cm}}{2} \right) \ln \left( \frac{Q^2}{m_e^2} \right) - 2 \]

\[ A_n^0 = -\frac{m_e E_{\text{lab}} \sigma_R}{8\pi^2} \frac{M A}{\sqrt{s} Z} \approx \begin{cases} 
-4 \text{ppm}, & ^{208}\text{Pb and } E_{\text{lab}} = 850 \text{MeV} \\
-7 \text{ppm}, & ^{4}\text{He and } E_{\text{lab}} = 3 \text{GeV} 
\end{cases} \]

MG and CJH '08

Generally: the “corrected optical theorem” tends to underestimate the analyzing power

- --- Elastic - Coulomb distortion (Chuck)
- ------ Inelastic
- --- Elastic + Inelastic

\[ ^{4}\text{He}, \quad E_{\text{lab}} = 3 \text{GeV} \]

HAPPEX
Vector analyzing power vs. parity violating asymmetry

For electron beam polarized longitudinally not to 100% vector analyzing power represents a systematical background effect. Is modulated by sine of the out-of-plane angle - can be measured directly to reduce the uncertainty. PV asymmetry is independent of the azimuthal angle.

For An ~ 10 ppm and degree of transverse polarization ~ 1% the systematical effect for PV asymmetry ~ 0.1 ppm
Dispersion corrections to QWEAK: uncertainty due to $\gamma Z$ is larger than it was thought.

How precise can we calculate it?
- For now, only estimates with e.-m. DIS structure functions available.
- Any detailed PV DIS data for $F_{1,2}$ are welcome.

Departure from exact forward direction:
- Additional uncertainty due to Compton form factor.
- May use $A_n$ as a tool to check our understanding of this Compton slope.

Analyzing power - common systematic effect of PVES experiments:
- Is purely electromagnetic effect.
- Is $\sim 10$ times larger than the PV asymmetries.
- Is not fully understood theoretically.
- Can be separated experimentally by measuring $\sim \sin \phi$ modulated asymmetry.