Top Jets & Precision Measurements of the Top Quark Mass

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Goal: measure the top mass as accurately as possible!

Why?

- It is a parameter in the standard model
- It couples strongly to the Higgs: probes new physics
- It is the dominant source of uncertainty in EWPOs
- Higgs mass uncertainty is limited by top mass
  \[ \delta m_t \sim \delta m_h. \]

(from J. Erler)
What Mass is being Measured?

What is a top mass?

- Top mass is a parameter in the QCD Lagrangian
  - This parameter must be related to an observable
- However, top is a colored quark: cannot define on-shell mass
  - Some observable must be calculated: subject to perturbative and non-perturbative uncertainties

Top Mass is dependent on the observable and the scheme in which that observable is calculated
Which Mass is being Measured?

\[ M_t = 170.9 \pm 1.1\text{(stat)} \pm 1.5\text{(syst)} \text{ GeV/c}^2 \]
What makes for a good top quark mass observable?

- Well defined relation to a short distance mass
- Good signal to background ratio

**Example:** Threshold scan: \( \delta m_t^{th} \sim 100 \text{MeV} \)  

(Peskin & Strassler; Hoang, Manohar, Stewart, Teubner,...)

- count number of \( t\bar{t} \) events
- color singlet state
- background is non-resonant
- physics well understood
  (renormalons, summations)
Top quark mass observable at the LHC - Jet Reconstruction

Expt. Issues:

1) Determining parton momentum
2) Combinatorics
3) Jet energy scale
4) Underlying events
5) Initial & Final state radiation
6) b-Jets, b-fragmentation
7) MC dependence
8) b-tagging efficiency
9) Background & Statistics
Top quark mass observable at the LHC - Jet Reconstruction

Th. Issues:

1) Definition of jet observable with a clear relation to the Lagrangian mass
2) Color reconnection and soft gluon interactions
3) Summing large logs: \( Q \gg m_t \gg \Gamma_t \)
4) Final state radiation
5) Parton Distribution Functions
6) Beam remnant
7) Initial state radiation
8) Underlying events

These effects can be studied in: \( e^+e^- \rightarrow t\bar{t} + X \) for \( Q \gg m_t \)
Jet Observable Sensitive to Top Mass

Focus on the **dijet region** where the top and antitop jets have invariant masses close to the top mass.

The top and antitop jets are defined to have the invariant masses: \( M_t, \ M_{\bar{t}} \)

The jet invariant mass condition is characterized as:

\[
\hat{s}_{t,\bar{t}} \equiv \frac{s_{t,\bar{t}}}{m} \equiv \frac{M_{t,\bar{t}}^2 - m^2}{m} \sim \Gamma \ll m
\]

The observable of interest is the double differential jet invariant mass distribution:

\[
\frac{d\sigma}{dM_t^2 \ dM_{\bar{t}}^2}
\]
A Precise Definition: Hemisphere Masses

The jet masses are defined to be the invariant mass of all particles in each hemisphere perpendicular to the thrust axis

\[ M_t^2 - m^2 \sim m\Gamma \]

\[ M_t^2 - m^2 \sim m\Gamma \]
Relevant Energy Scales

- Center of mass energy $Q \sim 1\text{TeV}$
- Top quark mass $m \sim 174\text{GeV}$
- Top quark width $\Gamma \sim 2\text{GeV}$
- Confinement Scale $\Lambda_{QCD} \sim 500\text{MeV}$

$$\Gamma \frac{m}{Q} \sim \Lambda_{QCD}$$

Disparate energy scales $\rightarrow$ Effective Field Theory!
Effective theory of very energetic particles moving through a soft background.

Energy much greater than mass: light-like.

Soft particles have energy \( \sim m_t \) : “jiggle” collinear particles.

Expansion of QCD around the light-cone in powers of \( m_t/Q \):

\[
p^\mu = Q n^\mu + k^\mu \quad n^\mu = (1, 0, 0, 1) \quad n^2 = n_0^2 - n_3^2 = 0
\]

\[
k^\mu \sim m_t \ll Q
\]
SCET

Split QCD into two sectors:

- **Soft**
  - Describes interactions among the soft particles
  - Interactions between soft and collinear particles

- **Collinear**
  - Describes interactions among the energetic particles

Mathematical expressions:

\[
\mathcal{L}_s = \overline{\psi}_s i\slashed{\nabla}_s \psi_s
\]

\[
\mathcal{L}_c = \overline{\xi}_n \left\{ i\slashed{n} \cdot \slashed{D}_c + i\slashed{\nabla}_c^\perp \frac{1}{i\slashed{n} \cdot \slashed{D}_c} i\slashed{\nabla}_c^\perp + g n \cdot A_s \right\} \frac{\slashed{\eta}}{2} \xi_n
\]
SCET: Soft decoupling

Before:

\[ \mathcal{L}_c = \bar{\xi}_n \left\{ i n \cdot D_c + i \mathcal{D}_c^\perp \frac{1}{i n \cdot D_c} i \mathcal{D}_c^\perp + g n \cdot A_s \right\} \frac{\gamma}{2} \xi_n \]

Field redefinition by eikonal phase:

\[ \xi_{n,p} \rightarrow Y_n \xi_{n,p} , \quad A_{n,p}^\mu \rightarrow Y_n A_{n,p}^\mu Y_n^\dagger \]

\[ Y_n(x) = \bar{\mathcal{P}} \exp \left( -i g \int_0^\infty ds \ n \cdot A_s(ns+x) \right) \]

After:

\[ \mathcal{L}_c^{(0)} = \bar{\xi}_n \left\{ i n \cdot D_c + i \mathcal{D}_c^\perp \frac{1}{i n \cdot D_c} i \mathcal{D}_c^\perp \right\} \frac{\gamma}{2} \xi_n \]
Heavy Quark Effective Theory

- Describes Interactions of a heavy quark with soft degrees of freedom.
- Decompose heavy quark momentum:
  \[ p^\mu = m v^\mu + k^\mu \]
  \[ k^\mu \sim \Lambda_{QCD} \ll m \]
- The HQET Lagrangian is given as an expansion in inverse powers of the heavy quark mass:
  \[ \mathcal{L} = \overline{h}_v (i v \cdot D - \delta m + \frac{i}{2 \Gamma}) h_v \]
The QCD Cross-Section
The cross-section in QCD has the general form:

$$\sigma = \sum_X (2\pi)^4 \delta^4(p_e + p_\bar{e} - p_X) \sum_{ij} L_{\mu\nu}^{(ij)} \langle 0 | J_i^{\mu}(0) | X \rangle \langle X | J_j^{+\nu}(0) | 0 \rangle$$

The sum over final states $X$ is restricted to contain a top jet and an anti-top jet with invariant masses close to the top mass.

The top quark currents are produced by photon and Z exchange:

$$J_i^{\mu}(x) = \bar{\psi}(x)\Gamma_i^{\mu}\psi(x), \quad \Gamma_\gamma^{\mu} = \gamma^\mu, \quad \Gamma_Z^{\mu} = g^V\gamma^\mu + g^A\gamma^\mu\gamma_5$$
Step 2: Matching QCD Current onto SCET

- Restrict the final state phase space to **high energy** top quark pairs by matching the QCD current onto the SCET current:

\[
J_i^\mu(0) = \int d\omega \, d\bar{\omega} \, C(\omega, \bar{\omega}, \mu) J_i^\mu(\omega, \bar{\omega}, \mu)
\]

\[
\mathcal{J}_i^\mu(\omega, \bar{\omega}, \mu) = \bar{\chi}_{n,\omega}(0) \Gamma_i^\mu \chi_{\bar{n},\bar{\omega}}(0), \quad \chi_{n,\omega}(0) = \delta(\omega - \bar{\mathcal{P}})(W^\dagger \xi_n)(0)
\]

- By momentum conservation \( C(-Q, Q, \mu) \equiv C(Q, \mu) \)

- The **hard modes** of QCD are integrated out.
Matching QCD onto SCET at One Loop

\[ C(\mu) = 1 + \frac{\alpha_s C_F}{4\pi} \left[ 3 \ln \frac{-Q^2}{\mu^2} - \ln^2 \frac{-Q^2}{\mu^2} - 8 + \frac{\pi^2}{6} \right] \]

Note that the logs in the Wilson coefficient vanish by choosing the matching scale at: \( \mu = Q \)
The SCET Cross-Section
The SCET Cross-Section

After matching the QCD current onto SCET, the cross-section has the general form:

$$\sigma = \sum_{\tilde{n}} \sum_{X_n X_{\tilde{n}} X_s} (2\pi)^4 \delta^4(q - P_{X_n} - P_{X_{\tilde{n}}} - P_{X_s}) \sum_i L^{(i)}_{\mu\nu} \int d\omega \ d\tilde{\omega} \ d\omega' \ d\tilde{\omega}' \times C(\omega, \tilde{\omega}) C^* (\omega', \tilde{\omega}') \langle 0 | \chi_{\tilde{n}, \tilde{\omega}} \Gamma_{j}^{\nu} \chi_{n, \omega'} | X_n X_{\tilde{n}} X_s \rangle \langle X_n X_{\tilde{n}} X_s | \chi_{n, \omega} \Gamma_{i}^{\mu} \chi_{\tilde{n}, \tilde{\omega}} | 0 \rangle$$

The complete set of states in SCET involve only soft and collinear degrees of freedom.

$$|X\rangle = |X_n X_{\tilde{n}} X_s\rangle = |X_n\rangle \otimes |X_{\tilde{n}}\rangle \otimes |X_s\rangle$$

$$\sigma = K_0 \sum_{\tilde{n}} \sum_{X_n X_{\tilde{n}} X_s} (2\pi)^4 \delta^4(q - P_{X_n} - P_{X_{\tilde{n}}} - P_{X_s}) \langle 0 | \chi_{\tilde{n}} \chi_{n} | X_s \rangle \langle X_s | Y_{n}^{+} Y_{\tilde{n}}^{+} | 0 \rangle \times \int d\omega \ d\tilde{\omega} \ |C(\omega, \tilde{\omega})|^2 \langle 0 | \hat{n} \chi_{n} | X_n \rangle \langle X_n | \chi_{n, \omega} | 0 \rangle \langle 0 | \chi_{\tilde{n}} | X_{\tilde{n}} \rangle \langle X_{\tilde{n}} | \tilde{n} \chi_{\tilde{n}, \tilde{\omega}} | 0 \rangle.$$

- Hard Wilson Coeff.
- Collinear: $n$
- Collinear: $\tilde{n}$
Implement **Hemisphere mass** definition and make the invariant mass restrictions explicit.

In the hemisphere scenario the SCET cross section takes the form:

\[
\frac{d^2 \sigma}{dM_t^2 dM^2} = \sigma_0 \ H_Q(Q, \mu) \int_{-\infty}^{\infty} d\ell^+ d\ell^- \ J_n(s_t - Q \ell^+, \mu) J_{\bar{n}}(s_{\bar{t}} - Q \ell^-, \mu) S_{\text{hemi}}(\ell^+, \ell^-, \mu)
\]

**Hard Wilson Coefficient**

**Top Jet Function**

**Anti-Top Jet Function**

**Soft Cross Talk Function**

**Calculable perturbative top and antitop jet functions**

\[
J_n(Q r^+_n - m^2) = -\frac{1}{2\pi Q} \int d^4 x \ e^{ir_n \cdot x} \ \text{Disc} \ \langle 0 | T \{ \chi_n, Q(0) \hat{p} \chi_n(x) \} | 0 \rangle
\]

\[
J_{\bar{n}}(Q r^-_{\bar{n}} - m^2) = \frac{1}{2\pi Q} \int d^4 x \ e^{ir_{\bar{n}} \cdot x} \ \text{Disc} \ \langle 0 | T \{ \bar{\chi}_{\bar{n}}(x) \hat{p} \bar{\chi}_{\bar{n}, -Q(0)} \} | 0 \rangle
\]

**Universal nonperturbative soft function**

\[
S_{\text{hemi}}(\ell^+, \ell^-, \mu) = \frac{1}{N_c} \sum_{X_s} \delta(\ell^+ - k^{+a}_s) \delta(\ell^- - k^{-b}_s) \langle 0 | \bar{Y}_{\bar{n}} Y_n(0) | X_s \rangle \langle X_s | Y^+_n Y^{\dagger}_{\bar{n}}(0) | 0 \rangle
\]
Running between $Q$ and $m$ is local and only affects normalization.
Matching onto HQET

Recall:

Need to match SCET jet functions onto HQET and run below m.

\[
\left( \frac{d\sigma}{ds_t ds_i} \right)_{\text{hemi}} = \sigma_0 H_Q(Q, \mu) \int d\ell^+ d\ell^- J_n(s_t - Q\ell^+, m, \Gamma, \mu) J_{\bar{n}}(s_{\bar{t}} - Q\ell^-, m, \Gamma, \mu) S_{\text{hemi}}(\ell^+, \ell^-, \mu)
\]

Order m invariant mass fluctuations remain

Match and run in HQET
Boosted HQET
Matching the Differential Cross-Section in the Peak Region

**SCET cross section:**

\[
\frac{d^2\sigma}{dM_t^2 dM_i^2} = \sigma_0 H_Q(Q, \mu) \int_{-\infty}^{\infty} d\ell^+ d\ell^- \ J_n(s_t - Q\ell^+, \mu) J_n(s_i - Q\ell^-, \mu) S_{\text{hemi}}(\ell^+, \ell^-, \mu)
\]

**bHQET cross section:**

\[
\frac{d^2\sigma}{dM_t^2 dM_i^2} = \sigma_0 H_Q(Q, \mu_m) H_m(m, \frac{Q}{m}, \mu_m, \mu) \int_{-\infty}^{\infty} d\ell^+ d\ell^- \ B_+(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu) B_+(\hat{s}_i - \frac{Q\ell^-}{m}, \Gamma, \mu) \tilde{S}_{\text{hemi}}(\ell^+, \ell^-, \mu)
\]
The SCET and BHQET Soft Hemisphere Functions

SCET Soft Hemisphere Function:

\[ S_{\text{hemi}}(\ell^+, \ell^-, \mu) = \frac{1}{N_c} \sum_{X_s} \delta(\ell^+ - k_{s}^{+a}) \delta(\ell^- - k_{s}^{-b}) \langle 0 | \bar{Y}_n \ Y_n(0) | X_s \rangle \langle X_s | Y_n^\dagger \bar{Y}_n^\dagger (0) | 0 \rangle \]

\[ \ell \sim m \]

bHQET Soft Hemisphere Function:

\[ \tilde{S}_{\text{hemi}}(\ell^+, \ell^-, \mu) = \frac{1}{N_c} \sum_{X_s} \delta(\ell^+ - k_{s}^{+a}) \delta(\ell^- - k_{s}^{-b}) \langle 0 | \bar{Y}_n \ Y_n(0) | X_s \rangle \langle X_s | Y_n^\dagger \bar{Y}_n^\dagger (0) | 0 \rangle \]

\[ \ell \sim \Lambda_{\text{QCD}} \]

Match Soft Hemisphere Function:

\[ S_{\text{hemi}}(\ell^+, \ell^-, \mu_m) = H_S(m, \mu_m) \tilde{S}_{\text{hemi}}(\ell^+, \ell^-, \mu_m) \]

\[ H_S(m, \mu_m) = 1 + O(\alpha_s(\mu_m)^2) \]
The SCET and BHQET Jet Functions

The SCET jet functions are given by:

\[ J_n(Qr_n^+ - m^2) = \frac{-1}{2\pi Q} \int d^4x \ e^{i r_n \cdot x} \ \text{Disc} \left< 0|T\{\bar{\chi}_n(Q)\hat{\rho}\chi_n(x)\}|0 \right>, \]

\[ J_n(Qr_n^- - m^2) = \frac{1}{2\pi Q} \int d^4x \ e^{i r_n \cdot x} \ \text{Disc} \left< 0|T\{\bar{\chi}_n(x)\hat{\rho}\chi_n,-Q(0)\}|0 \right>. \]

The BHQET jet function are given by:

\[ B_+(2v_+ \cdot k) = \frac{-1}{8\pi N_c m} \int d^4x \ e^{i k \cdot x} \ \text{Disc} \left< 0|T\{\bar{h}_{v_+}(0)W_n(0)W_n(0)h_{v_+}(x)\}|0 \right>, \]

\[ B_-(2v_- \cdot k) = \frac{1}{8\pi N_c m} \int d^4x \ e^{i k \cdot x} \ \text{Disc} \left< 0|T\{\bar{h}_{v_-}(x)W_{\bar{n}}(x)W_{\bar{n}}(0)h_{v_-}(0)\}|0 \right>. \]

Matching:

\[ J_n(m\hat{s}, \Gamma, \mu_m) = T_+(m, \mu_m) \ B_+(\hat{s}, \Gamma, \mu_m) \]

\[ J_\bar{n}(m\hat{s}, \Gamma, \mu_m) = T_-(m, \mu_m) \ B_-(\hat{s}, \Gamma, \mu_m) \]
One Loop Matching of SCET onto BHQET

SCET

BHQET

Matching:

\[ T_{\pm}(\mu, m) = 1 + \frac{\alpha_s C_F}{4\pi} \left( \ln^2 \frac{m^2}{\mu^2} - \ln \frac{m^2}{\mu^2} + 4 + \frac{\pi^2}{6} \right). \]

Logs in the Wilson coefficient vanish by for: \( \mu = m \)
The Differential Cross-Section in the Peak Region

\[
\left( \frac{d^2 \sigma}{dM_t^2 \, dM_{\ell}^2} \right)_{\text{hemi}} = \sigma_0 \, H_Q(Q, \mu_m) \, H_m \left( m, \frac{Q}{m}, \mu_m, \mu \right)
\]

\[
\times \int_{-\infty}^{\infty} d\ell^+ \, d\ell^- \quad B_+ \left( \hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu \right) \quad B_- \left( \hat{s}_\ell - \frac{Q\ell^-}{m}, \Gamma, \mu \right) \quad S_{\text{hemi}}(\ell^+, \ell^-, \mu)
\]

Evolution and decay of top quark close to mass shell

Non-perturbative Cross talk

\[
H_m \left( m, \mu_m \right) = T_+ (m, \mu_m) \, T_- (m, \mu_m) \, H_S (m, \mu_m)
\]
The Differential Cross-Section at Tree Level

Jet functions are Breit Wigner distributions at tree level:

$$B_{\pm}^{\text{tree}}(\hat{s}, \Gamma) = \frac{1}{\pi m} \frac{\Gamma}{\hat{s}^2 + \Gamma^2}$$

Non-Perturbative Soft Hemisphere Function is Universal. Extract it from Massless Dijet data \(^{(Korchemsky \& Sterman)}\)

$$S_{\text{hemi}}^{M1}(\ell^+, \ell^-) = \theta(\ell^+)\theta(\ell^-) \frac{\mathcal{N}(a, b)}{\Lambda^2} \left( \frac{\ell^+\ell^-}{\Lambda^2} \right)^{a-1} \exp \left( \frac{-(\ell^+)^2 - (\ell^-)^2 - 2b\ell^+\ell^-}{\Lambda^2} \right)$$

$$a = 2, \quad b = -0.4, \quad \Lambda = 0.55 \text{ GeV}$$

$$\left( \frac{d^2\sigma}{dM_t^2 \, dM_{\ell_t}^2} \right)_{\text{hemi}} = \sigma_0 \, H_Q(Q, \mu_m) \, H_m \left( m, \frac{Q}{m}, \mu_m, \mu \right) \times \int_{-\infty}^{\infty} d\ell^+ \, d\ell^- \, B_+ \left( \frac{\hat{s}_{\ell} - \frac{Q\ell^+}{m}}{m}, \Gamma, \mu \right) \, B_- \left( \frac{\hat{s}_{\ell} - \frac{Q\ell^-}{m}}{m}, \Gamma, \mu \right) \, S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

Tree level BWs

Shape function
The Differential Cross-Section at Tree Level

\[
\left( \frac{d^2 \sigma}{dM_t^2 \, dM_{\tilde{t}}^2} \right)_{\text{hemi}} = \sigma_0 \, H_Q(Q, \mu_m) \, H_m \left( m, \frac{Q}{m}, \mu_m, \mu \right) \\
\times \int_{-\infty}^{\infty} \, d\ell^+ \, d\ell^- \, B_+ \left( \hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu \right) \, B_- \left( \hat{s}_{\tilde{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu \right) \, S_{\text{hemi}}(\ell^+, \ell^-, \mu)
\]

\( Q = 745 \text{ GeV} \)

\( \Gamma = 1.43 \text{ GeV} \)

\( m_t = 172 \text{ GeV} \)

- Non-Perturbative effects **shift** the perk to higher energies and **broaden** the distribution
- Naive Breit-Wigner not even valid at tree level.
The Differential Cross-Section Beyond Tree Level
Running only affects the normalization, if the boundary is common

If $B_{\pm}(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu)$ and $\tilde{S}_{\text{hemi}}(\ell^+, \ell^-, \mu)$ live at different scales, then the shape changes
The Differential Cross-Section Beyond Tree Level: Running

\[
\left( \frac{d^2 \sigma}{dM_t^2 \, dM_t^2} \right)_{\text{hemi}} = \sigma_0 \, H_Q(Q, \mu_m) H_m \left( m, \frac{Q}{m}, \mu_m, \mu \right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^- \, B_+ \left( \hat{s}_t - \frac{Q \ell^+}{m}, \Gamma, \mu \right) B_- \left( \hat{s}_t - \frac{Q \ell^-}{m}, \Gamma, \mu \right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)
\]

- Peaked at \( \hat{s} - \frac{Q \ell}{m} = 0 \) and contains \( \ln \left( \frac{\hat{s} - \frac{Q \ell}{m}}{\mu} \right) \)

- \( \mu \sim \hat{s} - \frac{Q \ell}{m} \sim \frac{Q}{m} \Lambda_{\text{QCD}} \)

- Contains \( \ln \left( \frac{\ell}{\mu} \right) \) \( \rightarrow \) \( \mu \sim \Lambda_{\text{QCD}} \)

- Run Jet Functions to \( \mu_\Gamma \sim \Gamma \sim \frac{Q}{m} \Lambda_{\text{QCD}} \) and the Soft Function to \( \mu_\Delta \sim \Lambda_{\text{QCD}} \): changes the distributions shape!
The Differential Cross-Section Beyond Tree Level: Running

Matching & matrix elements

H_Q(Q, \mu_h)

H_m(m, \mu_m)

B_±(\Gamma, \mu_\Gamma)

S(\Lambda, \mu_\Lambda)

U_HQ

U_{Hm}

U_S

RGE for bHQET cross-section
The Differential Cross-Section Beyond Tree Level: Renormalons

\[ B_{+}(2v_{+} \cdot k) = \frac{-1}{8\pi N_{c} m} \int d^{4}x \ e^{i k \cdot x} \ \text{Disc} \langle 0|T\{\bar{h}_{v_{+}}(0)W_{n}(0)W_{n}^{\dagger}(x)h_{v_{+}}(x)\}|0\rangle \]

In the pole mass scheme the HQET self energy, c), develops a linear sensitivity to IR momenta: “Renormalon problem” (Beneke & Braun)

\[ \mathcal{O}(\Lambda_{QCD}) \] ambiguity in \( m_{\text{pole}} \)

In the pole mass scheme \( B_{\pm} \) is poorly behaved in perturbation theory: leads to an unstable peak position

We need a new mass definition
Switching Mass Schemes in bHQET

**Top HQET**

\[ \mathcal{L}_+ = \bar{h}_{v_+} (iv_+ \cdot D_+ - \delta m + \frac{i}{2} \Gamma) h_{v_+}, \]

- Pole mass scheme: \( \delta m = 0 \)
- \( \overline{\text{MS}} \) scheme: \( \delta \bar{m} \sim \alpha_s \bar{m} \gg \Gamma \).

**Anti-Top HQET**

\[ \mathcal{L}_- = \bar{h}_{v_-} (iv_- \cdot D_- - \delta m + \frac{i}{2} \Gamma) h_{v_-} \]

Note this breaks power counting!

- Short distance mass must respect bHQET power counting
  \( \delta m \sim \hat{s}_t \sim \hat{s}_{\bar{t}} \sim \Gamma \)
- Define a short distance top jet mass:
  \[ \frac{dB_+ (\hat{s}, \mu, \delta m_J)}{d\hat{s}} \bigg|_{\hat{s}=0} = 0 \]
  keeps the peak fixed!

“Right” power-counting

\[ m_J (\mu) = m_{\text{pole}} - \Gamma \frac{\alpha_s (\mu)}{3} \left[ \ln \left( \frac{\mu}{\Gamma} \right) + \frac{3}{2} \right] \]
Short Distance Top Jet Mass

In the jet mass scheme the NLO jet function is modified:

\[
\tilde{B}_\pm(\hat{s}, \mu) = B_\pm(\hat{s}, \mu) + \frac{1}{\pi m_J} \frac{(4 \hat{s} \Gamma) \delta m_J}{(\hat{s}^2 + \Gamma^2)^2}
\]

Perturbative Stability of Peak Position at NLO.

One loop shift in the pole scheme is 300 MeV!
Soft Function Models and Perturbative Corrections

\[
\left( \frac{d^2 \sigma}{dM_t^2 \, dM_t^2} \right)_{\text{hemi}} = \sigma_0 \, H_Q(Q, \mu_m) \, H_m \left( m, \frac{Q}{m}, \mu_m, \mu \right)
\]

\[
\times \int_{-\infty}^{\infty} d\ell^+ \, d\ell^- \, B_+ \left( \hat{s}_t - \frac{Q \ell^+}{m}, \Gamma, \mu \right) \, B_- \left( \hat{s}_t - \frac{Q \ell^-}{m}, \Gamma, \mu \right) \, \tilde{S}_{\text{hemi}}(\ell^+, \ell^-, \mu)
\]

\( \tilde{S}_{\text{hemi}} \) is nonperturbative so we must introduce a model function with parameters that can be fit to data.

Desirable features for the Soft Function Model

- Simple to implement
- Has correct limiting behavior
  - Includes perturbative corrections
  - Free of perturbative ambiguities
Operator Product Expansion of the Soft Function

\[
\left( \frac{d^2 \sigma}{dM_t^2 \, d\ell_t^2} \right)_{\text{hemi}} = \sigma_0 \, H_Q(Q, \mu_m) \, H_m(m, \frac{Q}{m}, \mu_m, \mu) \\
\times \int_{-\infty}^{\infty} d\ell^+ d\ell^- \, B_+ \left( \hat{s}_t - \frac{Q \ell^+}{m}, \Gamma, \mu \right) \, B_- \left( \hat{s}_t - \frac{Q \ell^-}{m}, \Gamma, \mu \right) \tilde{S}_{\text{hemi}}(\ell^+, \ell^-, \mu)
\]

Consider the region close to the peak but parametrically away from the peak: \( \Gamma \ll \hat{s} \ll m_t \) (for example \( \hat{s} \sim \sqrt{\Gamma m_t} \))

\( B_{\pm} \) are peaked around values where their argument vanishes

\[
\ell \sim \frac{m}{Q} \hat{s} \gg \Lambda_{\text{QCD}}
\]

In this region we can carry out an OPE on \( \tilde{S}_{\text{hemi}} \) in powers of \( \frac{\Lambda_{\text{QCD}}}{\hat{s}} \)

The leading term in the OPE is 1, and the entire cross section is perturbatively calculable!

We must reproduce the perturbative result in this regime.
Soft Function Models and Perturbative Corrections

\[ \tilde{S}(\ell^+, \ell^-, \mu) = \int_{-\infty}^{+\infty} d\tilde{\ell}^+ \int_{-\infty}^{+\infty} d\tilde{\ell}^- S_{\text{part}}(\ell^+ - \tilde{\ell}^+, \ell^- - \tilde{\ell}^-, \mu, \delta_i) S_{\text{mod}}(\tilde{\ell}^+, \tilde{\ell}^-) \]

\[ S_{\text{mod}}(\ell^+, \ell^-) = \theta(\ell^+)\theta(\ell^-) \frac{\mathcal{N}(a, b)}{\Lambda^2} \left( \frac{\ell^+\ell^-}{\Lambda^2} \right)^{a-1} \exp \left( -\frac{(\ell^+)^2 - (\ell^-)^2 - 2b\ell^+\ell^-}{\Lambda^2} \right) \]

This contains the correct perturbative corrections

And associated with these perturbative corrections there is a renormalon ambiguity \( \delta_i \): renders the model useless

\[ \text{Corresponds to an } \mathcal{O}(\Lambda_{QCD}) \text{ambiguity in the partonic threshold where } \ell - \tilde{\ell} = 0 \]
A Better Soft Function Model

Introduce a non-zero threshold

\[ S_{\text{mod}}(\tilde{\ell}^+, \tilde{\ell}^-) \rightarrow S_{\text{mod}}(\tilde{\ell}^+ - \Delta, \tilde{\ell}^- - \Delta) \]

\[ \Delta \] Corresponds to a minimum hadronic energy

Remove the renormalon by letting: \[ \Delta = \bar{\Delta} + \delta \]

Where \( \bar{\Delta} \) is renormalon free

However, \( \delta(\mu) \) so \( \bar{\Delta}(\mu) \)

\[ \mu \frac{d}{d\mu} \bar{\Delta}(\mu) = 2L_\Delta \frac{C_F \alpha_s(\mu)}{\pi} \]
μ₀ = 1 to μ = 1.5, 4.0, 7.0 GeV (red, green, magenta)

μΓ = μΔ
Perturbative Results

The $\mu_\Gamma$ and $\mu_\Delta$ scale dependence

$\mu_\Gamma = 3.3, 5, 7.5$ GeV

$\mu_\Gamma = \mu_\Delta = 1$ GeV

$\mu_\Delta = 0.8, 1.0, 1.2$ GeV

$\mu_\Gamma / \mu_\Delta = Q / m = 5$
\[ \mu_m = 86, 172, 344 \text{ GeV} \]

\[ \mu_Q = 430, 860, 1720 \text{ GeV} \]
Diagonal Differential Distribution

\[ M_t = M_{\bar{t}} \]
NLL Invariant Mass Distribution

\[
\frac{d^2 \sigma}{dM_t \ dM_t^-}
\]

\(M_t\) (GeV)

\(M_t^-\) (GeV)

\(m_J\)

172

174

176

178

180

0.00

0.01

0.02

0.00
Conclusions

- Well defined relation between the top mass parameter in the Lagrangian and the measured mass
  - Give a mass definition that is free of perturbative ambiguities

- Well defined relation between the parameterization of nonperturbative physics and the observable
  - Correct limiting behavior in OPE region
  - Includes appropriate perturbative contribution
  - Free of perturbative ambiguities

- Peak position is shifted away from the short distance top mass value by both perturbative effects (running) and non-perturbative effects (soft function)
  - Shifts on order of 1 GeV

- All these effects must be precisely understood to carry our precision measurements in an hadronic environment