Supersymmetry at the LHC

What is supersymmetry?

Present data & SUSY

SUSY at the LHC

C. Balázs, L. Cooper, D. Carter, D. Kahawala
What shall we find at the LHC?

Experimental data & theoretical hints ⇒ new physics beyond the SM @ LHC

The new physics will hopefully explain the origin of

- mass
- light Higgs
- dark matter
- baryons
- gauge unification
- naturalness
- gravity
- and more.

But ...
... we already have theories that explain... well... "everything"!

For example:

in the supersymmetric framework we can explain the origin of

- **mass**: SUSY breaking & radiative dynamics → EWSB
- **light Higgs**: $m_{h}^{\text{tree}} \leq m_{Z}$ & loop corrections → $m_{h} \leq 135$ GeV
- **dark matter**: conserved $R = (-1)^{3(B-L)+2S}$ → LSP stable, neutral WIMP
- **baryons**: GUT-, lepto-, baryogenesis → baryon-antibaryon asymmetry
- **gauge unification**: sparticle loops → unification at $M_{\text{GUT}} \sim 10^{16}$ GeV
- **naturalness**: Higgsinos → Higgs mass protected by chiral symmetry
- **gravity**: gauged supersymmetry → supergravity

and more.

But ...
Surprisingly, this question can be answered.

Even more surprisingly, this question can be answered quite precisely!

But to obtain a quantitative answer, first we have to know:

- What we mean by the theory
- A bit of statistics
- What present experiments tell us about the theory
- What the LHC data might look like

I will illustrate this logic using supersymmetry.
What do we mean by supersymmetry?

Superfields

<table>
<thead>
<tr>
<th>spin</th>
<th>Higgs</th>
<th>matter</th>
<th>force</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\varphi^\mu$</td>
<td>$\psi^\mu$</td>
<td>$\lambda^\mu$</td>
</tr>
<tr>
<td>$1/2$</td>
<td>$\tilde{\varphi}_\alpha$</td>
<td>$\psi_\alpha$</td>
<td>$\tilde{\lambda}_\alpha$</td>
</tr>
<tr>
<td>0</td>
<td>$\varphi$</td>
<td>$\tilde{\psi}$</td>
<td>$\lambda$</td>
</tr>
</tbody>
</table>

Supersymmetry

\[ b^+ |O\rangle = |\tilde{\psi}\rangle, \quad f^+_\alpha |O\rangle = |\psi_\alpha\rangle \]

\[ Q^+_\alpha \equiv b^- f^+_\alpha, \quad Q^-_\alpha \equiv b^+ f^-_\alpha \]

\[ Q^+_\alpha |\tilde{\psi}\rangle = |\psi_\alpha\rangle, \quad Q^-_\alpha |\psi_\alpha\rangle = |\tilde{\psi}\rangle \]

\[ H = \sum_{i=1}^{2} (p_i^2 + \omega^2 x_i^2)/2 = \omega ([b^+, b^-] + \{f^+_\alpha, f^-_\alpha\})/2 = \omega \{Q^+_\alpha, Q^-_\alpha\}/2 \]

\[ [H, Q^\pm_\alpha] = 0 \]
The MSSM

Standard fields → superfields

Kinetic Lagrangian & gauge interactions ⇐ super- & gauge symmetry

Superpotential

\[ W_{\text{MSSM}} = -y_u \hat{H}_u \cdot \hat{Q} \hat{U} + y_d \hat{H}_d \cdot \hat{Q} \hat{D} + y_e \hat{H}_d \cdot \hat{L} \hat{E} - \mu \hat{H}_u \cdot \hat{H}_d \]

Scalar potential \[ V_{\text{MSSM}} = V_{\text{MSSM}}(H_u, H_d, ...) \]

No new parameters (compared to the SM with 2 Higgs doublets)

Lightest spartner (dark matter) candidates (neutral WIMFs (n,N,U)MSSM)

<table>
<thead>
<tr>
<th>spin\mass</th>
<th>(M_{3/2})</th>
<th>(M_1)</th>
<th>(M_2)</th>
<th>(M_1')</th>
<th>(\mu_1)</th>
<th>(\mu_2)</th>
<th>(m_S)</th>
<th>(m_a)</th>
<th>(m_{\tilde{\nu}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>(G)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3/2</td>
<td>(\tilde{G})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>(B)</td>
<td>(W^0)</td>
<td>(B')</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/2</td>
<td>(\tilde{B})</td>
<td>(\tilde{W}^0)</td>
<td>(\tilde{B}')</td>
<td>(\tilde{H}_u)</td>
<td>(\tilde{H}_d)</td>
<td>(\tilde{S}_i)</td>
<td>(\tilde{a})</td>
<td>(\nu_i)</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(H_u)</td>
<td>(H_d)</td>
<td>(S_i)</td>
</tr>
</tbody>
</table>
Supersymmetry breaking

SUSY might be beautiful, attractive and smart, but... she's broken!

SUSY breaking is parametrized by the soft Lagrangian

- all possible term allowed by the symmetries but free of quadratic divergences:

\[
\mathcal{L}_{\text{soft}} = -y_u A_u H_u \cdot \tilde{Q} \tilde{U} + y_d A_d H_d \cdot \tilde{Q} \tilde{D} + y_e A_e H_d \cdot \tilde{L} \tilde{E} - \mu B H_u \cdot H_d + \text{hc} + \frac{1}{2} M_i \tilde{\chi}_i^\alpha \tilde{\chi}_i^\alpha + \mu_i^2 \left| H_i \right|^2 + m_i^2 \tilde{\psi}_i \tilde{\psi}_i.
\]

- \(O(100)\) soft parameters

Top-down approach: \textit{mSuGra} (CMSSM), GMSB, AMSB, \textit{inoMSB}, ...

- assumptions: hidden sector, SSB mediation, unification, universality, ...

- scenario is parametrized in terms of a few high (GUT) scale parameters

- all soft parameters at \(M_{\text{EW}}\) are determined dynamically by RGE flow

Example: \textit{mSuGra} \(M_i = M_{1/2}, A_j = A_0, m_i = M_0\)

\[\mu_1, \mu_2, \mu, B \leftarrow \text{Higgs potential extrema}, \nu^2 = \langle H_1 \rangle^2 + \langle H_2 \rangle^2: \tan\beta \text{ & } \text{sgn}(\mu)\]
The statistics bit (that I promised you earlier)

Until now, we defined a theory (supersymmetry) and its parameters

Now, we assume: the theory is correct & a set of experimental data exists, and define the probability that under these conditions the theoretical parameters have certain values: \( P(\text{parameters}|\text{theory},\text{data}) \)

According to Reverend Bayes

\[
P(\text{parameters}|\text{theory},\text{data}) = \frac{P(d|t,p)*P(p|t)}{P(d|t)}
\]

- \( P(d|t,p) \) = the likelihood that a theory \( t \) with parameters \( p \) produces data \( d \)
- \( P(p|t) \) = the probability that the parameters have values \( p \) in the theory \( t \)
- \( P(d|t) \) = the chance that the set of data \( d \) is predicted by a theory \( t \)

\[
P(d|p,t) = \prod_i \exp(-\chi_i^2/2)/\sqrt{2\pi\sigma_i}
\]

\[
\chi_i^2 = (d_i - t_i(p))^2/\sigma_i^2
\]

In many cases \( P(p|t) \) and \( P(d|t) \) are uniform and then \( P(p|t,d) \sim P(d|t,p) \)
Likelihood

Theoretical parameters ($p$) are constrained by exp'tal data available today ($d_i$)

for mSuGra $p = \{M_0, M_{1/2}, A_0, \tan\beta, \text{sign}(\mu)\}$

To quantify experimental constraints we can simply use the $\sqrt{\log\text{-likelihood}}$:

$$\chi^2 = \sqrt{\sum_i (d_i - t_i(p))^2 / \sigma_i^2}, \quad \text{where } i \text{ runs over:}$$

- LEP  chargino & Higgs mass
- Tevatron  $\text{Br}(B_s \rightarrow \ell^+ \ell^-)$
- HFAG  $\text{Br}(b \rightarrow s \gamma)$
- $g-2$  anomalous magnetic moment of $\mu$
- WMAP  neutralino relic abundance
- CDMS  spin-independent neutralino–proton elastic recoil
mSuGra likelihood

mSugra with $\tan\beta = 45$, $A_0 = 0$, $\mu < 0$
mSuGra likelihood

\[ P(p|t,d) = P(d|t,p) \times P(p|t)/P(d|t) \]
Problems with MSSM

\( \mu \) problem

- \( W \supset \mu \hat{H}_1 \hat{H}_2 \) unnatural \( \leftarrow \) EW size for \( \mu \) is not justified

Electroweak fine-tuning (little hierarchy) problem

- tension between a light \( h^0 \) and LEP limits on spartner (\( \tilde{t} \)) masses:
  - electroweak precision data demand \( m_h \) close to 114.4 GeV, but
    \[
    m_h = \cos^2(2\beta) m_Z^2 + m_{EW}^2 \left( \log \left( \frac{m_{SUSY}^2}{m_{\tilde{t}}^2} \right) + \frac{X_t^2}{m_{SUSY}^2} \left( 1 - \frac{X_t^2}{12 m_{SUSY}^2} \right) \right)
    \]
  - \( m_{SUSY} = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \), \( X_t = A_t - \mu \cot(\beta) \)

Dark matter fine-tuning problem

- \( f \sim \max_i \left( \frac{1}{\Omega} \frac{d\Omega}{dP_i} \right) \) large in most constrained MSSM scenarios

--- list your own problems with the MSSM here ---
Singlet extensions of the MSSM

Easy to fix these MSSM problems while keeping positive features intact!

The root of the $\mu$ and fine-tuning problems is the Higgs sector

- extending the EWSB sector of the MSSM, these problems are alleviated
- in the $(n,N,S)$MSSM the $W \supset \mu \hat{H}_1 \hat{H}_2$ dynamically generated by
  
  $W \supset \lambda \hat{S} \hat{H}_1 \hat{H}_2$

- no dimensionful parameters are present in the superpotential
- all these fields ($H_i$ and $S$) acquire a vev.s at the weak scale
- electroweak fine tuning can also be alleviated

Caveat: new dimensionful soft parameters associated with the singlet
  it can be argued that the $\mu$ problem is only deferred to SUSY breaking
NmSuGra

Discreet symmetries of super- & Kahler potentials: $\mathbb{Z}_3 \times \mathbb{Z}_2^{MP}$

solve domain wall problem

Superpotential $\mathcal{W} = \mathcal{W}_{\text{MSSM}} + \lambda \hat{S} \hat{H}_1 \hat{H}_2 + \frac{k}{3} \hat{S}^3$ \quad Next-to-minimal MSSM

Scalar potential $\mathcal{V} = \mathcal{V}_{\text{MSSM}} + m_S^2 |S|^2 + \left( \lambda A_\lambda S H_1 H_2 + \frac{k}{3} A_k S^3 + \text{h.c.} \right)$

New parameters $\langle S \rangle, \lambda, \kappa, A_\lambda, A_k, m_S$

SUSY breaking $\text{mSuGra} \rightarrow \text{universality}$: fixes $A_k = A_\lambda = A_0$

9 parameters left $M_0, M_{1/2}, A_0, \langle H_1 \rangle, \langle H_2 \rangle, \langle S \rangle, \lambda, \kappa, m_S$

3 minimization eq. & $v^2 = \langle H_1 \rangle^2 + \langle H_2 \rangle^2$ eliminates 4 para &

$\tan \beta = \langle H_1 \rangle / \langle H_2 \rangle$, $\mu = \lambda \langle S \rangle$ exchanges $\beta$ and $\mu$ with 2 para $\rightarrow$

5 free parameters (and a sign): $M_0, M_{1/2}, A_0, \tan \beta, \lambda, \text{sign}(\mu)$

A single parameter extension of mSuGra, without new dimensionful parameters

C. Balázs, Monash U. Melbourne | SUSY@LHC.nb
Role of $g_{\mu} - 2$

Large contribution to $\chi^2$ for high $M_0$ and $M_{1/2}$ from the muon anomalous magnetic moment when the theoretical error is taken at face value

$$\Delta a_{\mu} = a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = 29.5 \pm 8.8 \times 10^{-10} \quad 3.4 \sigma \text{ discrepancy}$$

- Main theoretical uncertainty from measurement of $\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})$
- Optical theorem: hadronic vacuum polarization (HVP) $\sim \sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})$
- The HVP contribution can also be calculated using $\tau$ decay spectral functions.
- In the past there was a significant discrepancy between the $e^+e^-$ and $\tau$ decay based data.
- Most recent calculations of the HVP contribution are in better agreement.
- Isospin-breaking corrections reduce the difference between these two sets of data (lowering the $\tau$-based determination), and a new analysis of the pion form factor claims that the $\tau$ and $e^+e^-$ data are more consistent.
- High precision measurements of BaBar will hopefully, improve the accuracy.
Dark matter

The LSP is the lightest neutralino

\[ \tilde{Z}_1 = N_{11} \tilde{B} + N_{12} \tilde{W}_3 + N_{13} \tilde{H}_1 + N_{14} \tilde{H}_2 + N_{15} \tilde{S} \]

\( \tilde{Z}_1 \) is mostly bino-like as in mSuGra, but all type of dark matter occurs:

- bino-, wino-, higgsino- and singlino-like dark matter

All the familiar neutralino (co-)annihilation mechanisms are found:

- bulk region,
- focus point type region (1),
- Higgs resonances (via all five neutral Higgses 2),
- stau co-annihilation (3),
- stop co-annihilation (4).
Dark matter

![Graph showing the distribution of dark matter particles with respect to certain parameters.](image-url)
Discovery of NmSuGra

A given NmSuGra model point is discoverable at

- **LEP** if $m_{h^0} < 114.4$ GeV or $m_{\tilde{W}_1} < 103.5$ GeV
  - $m_{h^0}$ limit is relaxed for non-SM-like Higgses

- **LHC** if $m_{\tilde{g}} < 1.75$ TeV or $\sqrt{m_{\tilde{\ell}_1} m_{\tilde{\ell}_2}} < 2$ TeV
  - this is based on detailed mSuGra analysis

- **CDMS1T** if $\sigma_{p\tilde{Z}_1}^{SI}(m_{\tilde{Z}_1})$ is larger than the estimated 1 ton CDMS limit

Assuming this, the bulk of the NmSuGra para-space is discoverable!

The remaining para-space is disfavored at more than 99% confidence level

- this is dominated by $g_\mu - 2$
Discovery of NmSuGra

\[
\left( \frac{N_{11}^2 + N_{12}^2}{1 - N_{11}^2 - N_{12}^2} \right)
\]

\(\Omega h^2\)

LEP reach

LHC reach

CDMS1T reach
After the discovery (mSuGra)
Summary

- The Bayesian approach allows us to calculate the likelihood of discovery of various parameter regions for any (well defined) theories.

- The constrained models of supersymmetry have been tested and the approach is found to produce robust results.

- Present experiments prefer moderate values of the dimensionful \( (N)\text{mSuGra} \) parameters \( M_0, M_{1/2}, \) and \( |A_0| \) (at 95 % CL).

- If \( M_0, M_{1/2}, \) and \( |A_0| \) are indeed moderate, then the LHC and a ton equivalent of CDMS is guaranteed to discover \( (N)\text{mSuGra} \) under the condition that the \( g_\mu -2 \) discrepancy prevails.

- The \( \text{mSuGra} \) and \( \text{NmSuGra} \) models can be discovered at the LHC at 95 % CL.