Spinodal decomposition:
A tool for seeing the phase transition?

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Thermodynamics of phase transitions

Spinodal decomposition

Nuclear spinodal fragmentation

The deconfinement transition?

Urgent challenges!
Familiar example: Nuclear matter

(One conserved charge)

*Nuclear equation of state* $p_T(\rho)$

![Graph showing the nuclear equation of state](graph.png)
Thermodynamics reminder

Statistical equilibrium in bulk matter

Control parameter(s) \{X\}:

\[ \begin{align*}
\text{Energy } E &= V\varepsilon \\
\text{Number } N &= V\rho \\
\text{Volume } V &\rightarrow \infty
\end{align*} \]

Entropy function \( S\{X\} \):

\[ S(E,N,V) = V\sigma(\varepsilon,\rho) \]

Derivative(s) \( \lambda_X = \partial_X S \):

\[ \begin{align*}
\beta &= 1/T = \partial_E S(E,N,V) = \partial_\varepsilon \sigma(\varepsilon,\rho) \\
\alpha &= -\mu/T = \partial_N S(E,N,V) = \partial_\rho \sigma(\varepsilon,\rho) \\
\pi &= p/T = \partial_V S(E,N,V) = \sigma - \beta\varepsilon - \alpha\rho
\end{align*} \]

Thermodynamic coexistence:

\[ \delta S_{\text{tot}} = 0 \Rightarrow (\partial_X \sigma)_1 = (\partial_X \sigma)_2 \]

\[ T_1 = T_2 \quad \& \quad \mu_1 = \mu_2 \quad \& \quad p_1 = p_2 \]

\[ \Rightarrow \text{common tangent!} \]

Thermodynamic (local) stability: \[ \delta^2 S_{\text{tot}} < 0 \]

\[ \Rightarrow \{\partial_X \partial_Y \sigma\} \text{ has only negative eigenvalues} \]
**Simplest example: No conserved charges**

### Entropy density: $\sigma(\varepsilon)$

- **Phase 1**: Blue line
- **Phase 2**: Red line
- **Mixture**: Black line
- **$1 \leftrightarrow 2$**: Green line

- Concave regions

### Pressure: $p(\varepsilon) = T\sigma - \varepsilon$

- **Phase 1**: Blue line
- **Phase 2**: Red line
- **Mixture**: Black line
- **$1 \leftrightarrow 2$**: Green line

- **Equation of State**

### Inverse temperature: $\beta(\varepsilon) = \partial\sigma/\partial\varepsilon$

- **Phase 1**: Blue line
- **Phase 2**: Red line
- **Mixture**: Black line
- **$1 \leftrightarrow 2$**: Green line

### Pressure: $p(T)$

- **Phase 1**: Blue line
- **Phase 2**: Red line
- **$1 \leftrightarrow 2$**: Green line

**INT - August 2008**
Schematic Equation of State for Compressed Baryonic Matter

Isentropic expansion: $\sigma/\rho$ constant

\[
\frac{\delta \varepsilon}{\varepsilon + p} = \frac{\delta \rho}{\rho}
\]
Phase diagram in different representations

$\varepsilon$

$\rho$

$\mu$

$T$

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Isentropic phase trajectories in different representations
Spinodal decomposition is a process by which a mixture of two materials can separate into distinct regions with different material concentrations. [This differs from nucleation in that spinodal phase separation occurs throughout the material, not just at nucleation sites.]

Phase separation may occur whenever a material finds itself in the thermodynamically unstable region of the phase diagram. The boundary of this unstable region (the binodal) is signaled by a common tangent of the thermodynamic potential. Inside the binodal boundary, the spinodal region is entered when the curvature of the potential turns negative. The binodal and spinodal meet at the critical point. It is when a material is brought into the spinodal phase region that spinodal decomposition can occur.

To reach the spinodal region of the phase diagram, the system must be brought through the binodal region where nucleation may occur. For spinodal decomposition to be realized, a very fast transition (a quench) is required to evolve the system from the stable region through the meta-stable nucleation region and well into the mechanically unstable spinodal phase region.

In the spinodal phase region, the thermodynamics favors spontaneous separation of the components. But large regions will change their concentrations only slowly due to the amount of material that must be moved, and small regions will shrink away due to the energy cost of the interface between the two different component materials. Thus domains of a characteristic spinodal length scale will be favored and since the growth is exponential, such domain sizes will come to dominate the morphology in the course of the associated spinodal time.
Spinodal Multifragmentation

Nuclear EoS:

1st order phase transition

Spinodal instability

Density undulations may be amplified

Spinodal pattern:

Pressures $P$ (MeV/fm$^3$) vs. Nucleon density $\rho$ (fm$^{-3}$)

Spinodal region:

Temperature $T$ (MeV) vs. Relative relative density $\rho/\rho_0$

Growth rates:

Growth rate $\gamma$ (fm$^{-1}$) vs. Wave number $k$ (fm$^{-1}$)

Highly non-statistical $\Rightarrow$ Good candidate signature

Ph Chomaz, M Colonna, J Randrup
*Nuclear Spinodal Fragmentation*

Fragments $\approx$ equal!
Spinodal decomposition in nuclear multifragmentation

32 MeV/A Xe + Sn (b=0) (select events with 6 IMFs)

Bin wrt
\[ \langle Z \rangle : \text{average IMF charge} \]
\[ \Delta Z : \text{dispersion in IMF charge} \]

Experiment (INDRA @ GANIL)
Borderie et al, PRL 86 (2001) 3252

Theory (Boltzmann-Langevin)
Chomaz, Colonna, Randrup, …

occurs!
**Idealized model for exploration of clumping:**

The expanding system decomposes into plasma blobs which hadronize thermally:

Somewhat similar scenarios have been considered by

- I.N. Mishustin, Phys Rev Lett 82 (4779) 1999: *Nonequilibrium phase transition in rapidly expanding matter*
Rapidity correlations

Each blob hadronizes thermally - in its own flow frame

1D: \( y_{12}(p_1, p_2) = |y_1(p_1) - y_2(p_2)| \)

3D: \( y_{12}(p_1, p_2) = \ln[\gamma_{12} + \sqrt{\gamma_{12}^2 - 1}] \)

\( m_1 m_2 \gamma_{12} = E_1 E_2 - p_1 \cdot p_2 \)

[J. Heavy Ion Physics 22 (2005) 69]
Kinetic energy per particle (in the $N$-body CM frame):

$$\kappa_N\{p_n\} = \frac{1}{N} \left[ \{P\{p_n\} \cdot P\{p_n\}\}^{\frac{1}{2}} - \sum_n m_n \right]$$

Distribution of $\kappa$:

$$P_N(\kappa) \equiv <\delta(\kappa - \kappa_N\{p_n\})>$$

Total four-momentum:

$$P\{p_n\} = \sum_n (E_n, p_n)$$

Correlation function:

$$C_N(\kappa) \equiv P_N(\kappa) / P_N^0(\kappa) - 1$$

Same event / Mixed events

**Higher-order correlations stand out more clearly!**

(but require larger samples)

[J. Heavy Ion Physics 22 (2005) 69]
Strangeness trapping

[V. Koch, A. Majumder, J. Randrup, Phys. Rev. C72, 064903 (2005)]

First-order phase transition => spinodal decomposition

The expanding system decomposes into plasma blobs which each contain a certain amount of strangeness:

The hadronization of each isolated blob conserves strangeness!

=> Kaon fluctuations are enhanced!

Already seen by NA49 at SPS?
More refined transport treatments are needed! most urgent, most difficult

What is expected in idealized matter  What occurs in a nuclear collision

large, stationary, in global equilibrium  small, fast evolving, out of equilibrium

Requirements: (some of them)

Must describe each phase separately: hadron gas & QGP
Must have an "interesting" EoS: 1st order & critical point
Must "work" in the phase-coexistence region

Microscopic transport models are inadequate:

deconfinement!
EoS is unknown!
no phase transition!

Ordinary fluid dynamics is inapplicable:

no inherent scale!
no interface energy!