The possible quasi-particle picture of the quark near $T_c$ and its effect of the dilepton production rate

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QCD phase diagram

Temperature $T$ [MeV]

Early universe

RHIC & LHC

Deconfinement & chiral transition

Critical point?

GSI SIS 200

Neutron stars

Net Baryon Density

Quarks and Gluons

Color Superconductor?
QGP near Tc - sQGP

- Phenomenological side
  - Collective flow
  - Success of hydrodynamic model in the analysis of RHIC data

- Theoretical side
  - J/Psi and eta_c above Tc in lattice calculation
  - Hadronic excitation in model calculations
    (Hatsuda & Kunihiro, Shuryak & Zahed)

Interesting structure of vacuum. It must change the properties of fundamental degrees of freedom = quarks and gluons

There are only a few works on quark property near Tc while gluonic sector is studied by many people (potential, thermal mass...
Quark near but above Tc

- [ph] quark number scaling of $v_2$ in RHIC
  - success of quark recombination models

2. [th] Success of 2-peak ansatz for LQCD result (at $T/T_c = 1.5$ ,3)
  - F.Karsch and M.Kitazawa (2007)

3. [th] Sharp peaks in the quark spectrum calculated by SD eq.
  - M.Harada, Y.Nemoto and S.Yoshimoto (2007); M.Harada, Y. Nemoto (2008)

These result suggest the quark quasi-particle near Tc
HTL approximation

(E. Braaten and R. D. Pisarski 1990)

Available for $T \gg m_f, p, \omega$

\[ S^{\text{HTL}}(\omega, p) = \left[ (\omega + i\eta)\gamma^0 - p \cdot \gamma - \Sigma^{\text{HTL}}(\omega + i\eta, p) \right]^{-1} \]

\[ \Sigma^{\text{HTL}}(\omega, p) = \frac{m_T^2}{p} Q_0 \left( \frac{\omega}{p} \right) \gamma^0 + \frac{m_T^2}{p} \left( 1 - \frac{\omega}{p} Q_0 \left( \frac{\omega}{p} \right) \right) \gamma \cdot \hat{p} \]

\[ Q_0 = (1/2) \ln(x + 1)/(x - 1) \quad m_T^2 = (1/8) g^2 T^2 \times C_F \]

- Two branches of dispersion relation
- One have minimum: `plasmino`
- The mass is proportional to $T$: `Thermal mass`
- Chiral symmetric mass

H.A. Weldon, PRD 40(1989)2410

First appear in V. V. Klimov, Yad. Fiz 33, 1734 (1981)
• Chiral Soft Mode

What is important near Tc is fluctuation of the order parameter, i.e., chiral soft mode, for the 2nd or nearly 2nd order P. T.

T. Hatsuda and T. Kunihiro, PRL55, 158 (‘85)

• Quark Spectrum Near And Above Tc

ρ_+ [GeV^{-1}]

\[ T = 1.1T_c \]

3-peak structure

Understood form Level Mixing

In NJL and Yukawa Model

\[ m_q = 0 \] (chiral limit)


• But real quarks have finite masses

- (constituent quark mass, thermal mass, current mass)

We investigate the effect of mass in a Yukawa model
Spectral functions with finite $m_f$

$m_f/m_b = 0$

$m_f/m_b = 0.1$

With finite $m_f$, 3-peak structure gradually ceases because the negative energy excitation is suppressed for finite $m_f$.

M. Kitazawa, T. Kunihiro, KM and Y. Nemoto
Finite $m_f$ NJL model (Y. Nemoto, M. Kitazawa, T. Kunihiro)

$\mu = 0, T = T_{PC}$

The pseudo-critical line is determined from a maximum of the spectral function for $p=10$ MeV (dynamic chiral susceptibility).

Although the finite quark mass tend to suppress the 3-peak structure, 3-peak structure survive in the NJL model with the finite $m$ for the van Hove singularity caused by the change of the meson dispersion relation.

Can be observed?
Dilepton production as a probe

- Clean from final-state interaction
- Recently experimental results at RHIC are available
Dilepton production rate and quark quasi-particle in HTL approx.

E. Braaten, R. D. Pisarski and T. C. Yuan ('90)

\[
\frac{d\Gamma}{dq_0 d^3 q} = \frac{\alpha}{12\pi^4} \frac{1}{Q^2} \frac{1}{e^{\beta q_0} - 1} \text{Im} \Pi^\mu_\mu(q_0 + i\eta, \bar{q})
\]

\[\propto \quad P_{++}(q_0) + P_{+-}(q_0) + P_{--}(q_0)\]

\[P_{ij}(q_0) = \int_0^\infty p^2 dp \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} d\omega' \delta(q_0 - \omega - \omega') \Lambda_{ij}(\omega, \omega', p) \rho_i(\omega, p) \rho_j(\omega', p)\]

\[\Pi^{\mu\nu}(Q) = \]

\[\rho = \rho_+ \Lambda_+ + \rho_- \Lambda_-\]

\(\Lambda: \text{quark number projection}\)
For example

\[ P_{- -}(q_0) \sim \int_0^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty p^2 \, dp \, d\omega \, d\omega' \, \Lambda_{- -}(\omega, \omega', p) \delta(\omega - \omega_-(p)) \delta(\omega' - \omega_-(p)) \]

\[ = \int_0^\infty dp \, p^2 \, \Lambda(\omega_-(p), \omega_-(p), p) \delta(q_0 - 2\omega_-(p)) \]

\[ = \int_0^\infty dp \, p^2 \, \Lambda(\omega_-(p), \omega_-(p), p) \frac{1}{2 |\partial_p \omega_-(p)|} \delta(p - \tilde{p}(q_0)) \]

Divergence of the DoS

\[ D(\omega) \propto \frac{dp}{d\omega} \]

\[ q_0 - 2\omega_-(\tilde{p}) = 0 \]
Dilepton Production rate in a Yukawa model

In collaboration with M. Kitazawa (Osaka) and T. Kunihiro (Kyoto)

\( mf = 0 \), \( mb \neq 0 \), fermion is coupled with scalar and photon field

\[
\Pi^{\mu\nu}(q_0, q=0) = \begin{pmatrix}
\omega
\end{pmatrix}^{-1} \begin{pmatrix}
\omega
\end{pmatrix}^{-1}
\]

\[
\frac{d\Gamma}{dq_0 d^3 q} = \frac{\alpha}{12\pi^4} \frac{1}{Q^2} \frac{1}{e^{\beta q_0} - 1} \text{Im}\Pi^\mu_\mu(q_0 + i\eta, \vec{q})
\]

• though we must use effective vertex to keep gauge invariance, we adopt bare one for numerical simplicity here.
Approx. for the spectral function

Numerical calculation is heavy if calculating naively

Breit-Wigner type approximation

\[
\rho_{\text{approx.}}(\omega, p) = \sum_{i=1,2,3} \frac{1}{\pi} \frac{Z_i(p) \Gamma_i(p)}{(\omega - E_i(p))^2 + \Gamma_i^2}
\]

- For each \( p \) fit the parameters \( (E_i, G_i, Z_i) \) to \( \rho(\omega,p) \)
- Assume an analytic form for each function \( (E_i(p) \) etc), and fit that to the result got in step 1
Approx. spectral function \((T/m_b = 1.13)\)

Approximated form of the spectral function well reproduce the qualitative features of the original spectral function.
Result (preliminary)

\[ \frac{d\Gamma}{dq_0 d^3 q} \]

Peak structure!
Physical interpretation

There is no minimum, so it is not a van Hove singularity.

A possibility: resonance.
Summary

• 3-peak structure of quark may cause a `peak` in the photon decay rate.
  – Which indicate that fermion-scalar interaction cause a resonance of fermion-anti-fermion pair

• Result is generic : taking the density fluctuation as the soft mode, the same discussion can be near CP.

Future Work

• Physical interpretation

• Vertex correction
Back Up
meson by HTL quark pair

- the mesonic modes can exist even if \((p^\mu)^2 < (2m_T)^2\)
Model And Approximation

\[ \mathcal{L} = \bar{\psi} (i \gamma \cdot \partial - m_f) \psi + \frac{1}{2} [ (\partial_\mu \phi)^2 - m_b^2 \phi^2 ] - g \phi \bar{\psi} \psi \]

• Self Energy (1-loop)

\[ \Sigma(\omega, \vec{p}) = \]

- \[ p = 0 \text{ only} \]
- \[ \text{to concentrate on the } T\text{-dependence} \]
- \[ \text{plot on the } w-T\text{ plane} \]

Yukawa coupling of scalar filed and fermion

Quark Green function

\[ G^R(\omega) = \left[ (\omega + i \eta) - m_f - \Sigma^R(\omega) \right]^{-1} \]

\( g = 1 \): qualitative behavior is the same irrespective of \( g \)
The spectral function

\[ \mathcal{A}(p_0, \vec{0}) = -\frac{1}{\pi} \text{Im} G^R(p_0, \vec{0}) \]

projection op. \( \Lambda_\pm(\vec{0}) = \frac{1 \pm \gamma_0}{2} \)

\[ = [\rho_+ \Lambda_+(\vec{0}) + \rho_- \Lambda_- (\vec{0})] \gamma_0 \]

Deal with only positive number component
cf: parity property \( \rho_+(\omega, p) = \rho_-(-\omega, p) \)

\[ \rho_+(\omega) = -\frac{1}{\pi} \frac{\text{Im} \Sigma^R_+ (\omega)}{(\omega - m_f - \text{Re} \Sigma^R_+ (\omega))^2 + \text{Im} \Sigma^R_+ (\omega)^2} \]

\[ \Sigma = [\Sigma_+ \Lambda_+(\vec{0}) + \Sigma_- \Lambda_- (\vec{0})] \gamma_0 \]

\[ \text{Re} \Sigma_+ = \omega - m_f \]

\[ \text{Im} \Sigma_+ (\omega) \]
Poles of the quark propagator

Poles are found by solving

\[ z - m_f - \sum_{+}^{R}(z) = 0 \]

The residue at pole which indicate the strength of the excitation

\[ Z = \left[ 1 - \frac{\partial \Sigma_{+}^{R}(z)}{\partial z} \right] \bigg|_{z=z_{+}} \]

Pole approximation of the spectral function

\[ \rho_{\text{pole}}^{\omega}(\omega) = \sum_{i=1}^{3} \left( -\frac{1}{\pi} \right) \text{Im} \left[ \frac{Z_i}{\omega - z_i} \right] \]

A sum rule for the fermion spectral function

\[ \int_{-\infty}^{\infty} d\omega \rho_{+}(\omega) = 1 \quad \text{If pole approximation is good} \]

\[ \sum_{i} \text{Re}Z_i \approx 1 \]
Pole Structure Of Propagators \( \square (m_f = 0) \)

- Poles of the retarded functions in the lower half plane

\[ z : \text{complex energy variable} \]

\[ z = \text{complex energy variable} \]
How the poles move

- pole (A) stay at the origin irrespective of $T$
- Imag. parts of the poles (B) and (C) become smaller as $T$ is raised
Breit-Wigner approximations

\[ T/m_b = 0.5 \]

\[ T/m_b = 1.0 \]

\[ T/m_b = 1.5 \]

The pole approximation describes the spectral function except near the origin: the residues at many poles near the origin have negative real parts.

\[ \rho_{\text{pole}}(\omega) = \sum_{i=1}^{3} \left( -\frac{1}{\pi} \right) \text{Im} \left[ \frac{Z_i}{\omega - z_i} \right] \]
The residues

- Residues have similar values at $T \sim m_b$ : consistent with 3-peak structure!
How the poles move \((m_f/m_b=0.1)\)

- Pole (A) moves toward the origin as \(T\) is raised
- Pole (C) have larger imaginary part than pole (B)
Breit-Wigner approximations

- The spectral function is well described by the three poles we picked up!
The residues and a sum rule

- Residue at pole (A) decreases at high $T$
- Residue at pole (B) is larger than that for pole (C) at $T \sim m_b$
- Sum rule is approximately satisfied
Structural change in the pole behavior

$\frac{m_f}{m_b} = 0.2$

$\frac{m_f}{m_b} = 0.3$

- The $T$-dependence of the poles qualitatively change
Residues
- It seems that behavior of poles A and B is exchanged.

\[ \frac{m_f}{m_b} = 0.2 \]

\[ \frac{m_f}{m_b} = 0.3 \]

\[ \text{critical mass is } \frac{m_f}{m_b} \sim 0.21 \]
The behavior at high temperature

- \( \text{Im } z / \text{Re } z \) is small at high \( T \)
- \( m_T \) of HTL well approximate the real part at high \( T \)

\( m_T = gT/4 \)

\( \omega_Q = m_b \)

Phase space for decays vanish
The Level Mixing

The Physical Origin
Of Multi Peak Structures
Level Mixing

~ massless fermions coupled with a massless boson ~

\[ \rho + (\omega, k) \]

\[ \rho - (\omega, k) \]

H.A. Weldon, PRD40(1989)2410
Level Mixing
~ massless fermions coupled with a massive boson ~

Energies of the mixed levels

In the discussions above, the momentum of the boson is set to zero. Here I consider general values of the absorbed/emitted boson.

\[ 0 < \omega < |m_b - m_f| \]

\[ (E_f, k) \]

\[ 0 > \omega > - |m_b - m_f| \]

\[ (E_b, k) \]

The energy of the state which mixed with the original (free) state

\[ \omega^> = E_b - E_f \ (>0) \]

\[ \omega^< = E_f - E_b \ (<0) \]

\[ E_b = \sqrt{m_b^2 + k^2} \]

\[ E_f = \sqrt{m_f^2 + k^2} \]
Im $\Sigma$ prop. to the decay rate

This suggest that the mixing processes occur most often at two energy value $\rightarrow$ effectively the mixing b/w three states $\rightarrow$ three peak structure

suppression of the peak

with negative energy

competition b/w

1) phase volume $\propto k^2$

2) dist. func. $n(k)$, $f(k)$
the structural change from the aspect of level mixing

Intermediated states are thermally excited
- at low $T$ effect of level mixing is weak
- graphs are schematic picture at enough high $T$ so that level mixing become effective and pole (A) start to move

original level is pushed down in energy at high $T$

$m_f < m^*$

original level is pushed up in energy at high $T$

$m_f > m^*$

$m_f = 0, 0.1, 0.2$

Effective mixed level

$m_f = 0.3$
Subtracted Dispersion Relation

Kramers-Kroenig Relation for $f(x)$

$$\text{Re} f(z) = \frac{1}{\pi} \mathcal{P} \int \frac{dz'}{z - z'} \text{Im} f(z)$$

Finiteness of $\text{Re} \Sigma$ require $|f(z)|$ to converge as $z \to \pm \infty$. Else one should use

$$\text{Re} f(z) = f(a) + \frac{(z - a)}{\pi} \mathcal{P} \int \frac{dz'}{(z' - z)(z - a)} \text{Im} f(z')$$

$$\text{Re} f(x) = f(a) + (x - a) f'(a) + \frac{(x - a)^2}{\pi} \mathcal{P} \int \frac{dx'}{(x' - a)^2(x' - x)} \text{Im} f(x')$$

Those are called once “subtracted” and twice “subtracted” dispersion relation respectively.
Explicit expression of $T = 0$ part

$$\Sigma = p_0 \Sigma_0 - \hat{p} \cdot \vec{\gamma} \Sigma_V + \Sigma_2$$

$$\text{Im} \Sigma_0(p_0, \vec{p}) = -\frac{g^2}{32\pi} \frac{(p_\mu^2 - \tilde{m}_I \cdot M)}{p_\mu^2} \frac{\sqrt{(p_\mu^2 - \tilde{m}_I^2)(p_\mu^2 - M^2)}}{p_\mu^2} p_0 \epsilon(p_0) \theta(p_\mu^2 - M^2)$$

$$\text{Im} \Sigma_V(p_0, \vec{p}) = -\frac{(p_\mu^2 - \tilde{m}_I \cdot M)}{p_\mu^2} \frac{\sqrt{(p_\mu^2 - M^2)(p_\mu^2 - \tilde{m}_I^2)}}{p_\mu^2} p_\epsilon(p_0) \theta(p_\mu^2 - M^2)$$

$$\text{Im} \Sigma_0 \to p_0 \quad \text{as} \quad p_0 \to \infty$$

$$\text{Re} \Sigma(p_0, p) = \frac{1}{\pi} \mathcal{P} \int \frac{dz}{z - p_0} \text{Im} \Sigma(z, p) \quad \text{Diverge}!$$
Renormalization Of $T = 0$ Part

- We use twice subtracted disp. rel. for regularization of integral
  
  General complex function $f(x)$ obey to the following relation:
  
  $\text{Re} f(z) = \frac{1}{\pi} \mathcal{P} \int \frac{dz'}{z - z'} \text{Im} f(z)$ □ Dispersion Relation □

  Even if above expression diverge, following expression sometimes converge.
  
  $\text{Re} f(x) = f(a) + (x - a)f'(a) + \frac{(x - a)^2}{\pi} \mathcal{P} \int \frac{dx'}{(x' - a)^2(x' - x)} \text{Im} f(x')$

  This expression is called “twice-subtracted” dispersion relation

  $\text{Re} \Sigma(p_0, p) = \Sigma(\pm \tilde{E}_p, p) + (p_0 \mp \tilde{E}_p) \Sigma'(\pm \tilde{E}_p) + \frac{(p_0 \mp \tilde{E}_p)^2}{\pi} \mathcal{P} \int \frac{dx'}{(x' \mp \tilde{E}_p)^2(x' - p_0)} \text{Im} \Sigma(x')$
Renormalization Of $T = 0$ Part

$$\text{Re}\Sigma(p_0, p)_{T=0} = \Sigma_{T=0}(\pm \tilde{E}_p, p) + (p_0 \mp \tilde{E}_p)\Sigma'(\pm \tilde{E}_p, p)_{T=0} + \frac{(p_0 \mp \tilde{E}_p)^2}{\pi} \mathcal{P} \int \frac{dx'}{(x' \mp \tilde{E}_p)^2(x' - p_0)} \text{Im}\Sigma(x')_{T=0}$$

- dbl sign: for $p_0 > 0$ upper, for $p_0 < 0$ lower

Mass Shell Renormalization

- Mass Renorm.
  $$\delta m(p^2 = m_R^2) = 0$$

  $$\delta Z_2(p^2 = m_R^2) = 0$$

In terms of $\Sigma$

$$\Sigma(p^2 = m_R^2) = 0$$
$$\Sigma'(p^2 = m_R^2) = 0$$

$$\text{Re}\Sigma(p_0, p) = \mathcal{P} \int \frac{dx'}{(x' \mp \tilde{E}_p)^2(x' - p_0)} \text{Im}\Sigma(x')$$

- dbl sign: for $p_0 > 0$ upper, for $p_0 < 0$ lower

- Finite temperature part converge $\rightarrow$ disp. rel. without subtraction.
Decomposition of the integral

\[ \text{Im} \Pi^\mu_\mu(q_0 + i\eta) \]

\[ \alpha \quad \sum \quad \int \frac{d^3p}{(2\pi)^3} \int d\omega \int d\omega' f(\omega) f(\omega') \rho_\pm(\omega, p) \rho_\pm(\omega', p) \delta(q_0 - \omega - \omega') \]

decomposition of spectral function

\[ \rho_\pm(\omega, p) = \sum_j Z_{p,j}^{\pm} \delta(\omega - E_p^{j}) + \rho_{\text{reg}}^\pm(\omega, p) \]

The integral is decomposed into three blocks

- \[ \square \square \] : pole-pole term
- \[ \rho_{\text{reg}}^\pm \square \] : pole-regular term
- \[ \rho_{\text{reg}}^\pm \rho_{\text{reg}}^\pm \] : regular-regular term