Magnetic Component of Strongly Coupled Quark-Gluon Plasma & QCD Phase Diagram from E-M Duality Perspective

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OUTLINE

- E-M Duality for sQGP
- When Magnetic Scenario Meets Heavy Ion Data
- Pin Down the Parameters of Magnetic Component
- Discussions of QCD Phase Diagram
E-M Duality in General

What E-M duality says:

a) D+1 local field theory E, allowing D-dim. Topological Excitations M
   → convenient at E-coupling < 1 (M mass 1/E-coupling)

b) an eff. local field theory based on M with E non-local
   → convenient at E-coupling > 1

c) further more the M-coupling ~ 1/E-coupling !!!

working examples? YES!

- 2-D Ising model:
  \[ Z[\beta, \sigma] = Z[\beta', \mu] \]
  \[ \sinh(2\beta') = \frac{1}{\sinh(2\beta)} \]

- 2-D sine-Gordon --- Thirring Model
  \[ \frac{\beta^2}{4\pi} = \frac{1}{1 + g/\pi} \]
  \[ S_{SG} = \int d^2x \left( \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{\alpha}{\beta^2} (\cos \beta \phi - 1) \right) \]
  \[ S_T = \int d^2x \left( \bar{\psi} \gamma_\mu \phi \psi + m \bar{\psi} \psi - \frac{g}{2} \bar{\psi} \gamma^\mu \psi \bar{\psi} \gamma_\mu \psi \right) \]

More nontrivial: \text{\cal N=2 SYM Seiberg-Witten:}

Minimal Lesson:
D.o.F. in per. Spectrum may NOT be the D.o.F. at strong coupling!
New Quest for D.o.F

hadrons: \textbf{E-Composite}

interaction so strong $\rightarrow$ quarks/gluons confined!
however: hadrons are merely excitations of some more complicated stuff $\rightarrow$

\textbf{QCD vacuum: Magnetic}

dual superconductor monopole condensation
\textit{('t Hooft -- Mandelstam)}
(lattice: Giacomo/Suzuki/Greensite/...)

\begin{itemize}
  \item $T_c \approx 4T_c$
  \item \textbf{Very High} $T$
\end{itemize}

quarks, gluons: \textbf{EQP}

dominating thermodynamics
pQCD calculable

non-pert. scale: $e^2T$

\textbf{M-sector setting in}

dim. reduction $\rightarrow$ MQCD
heavy, strongly correlated monopoles
\textit{(Kolthas-Altes, Meyer, ...)}

\begin{itemize}
  \item Hadronic World
  \item \textbf{S-QGP}
  \item \textbf{wQGP}
\end{itemize}
Magnetic D.o.F: Lessons from S-W

Summarizing the **E-component** in sQGP

- quarks/gluons from high-T down to Tc become heavy and rare, gradually ceasing already about 1.5Tc
- hadrons from low-T do NOT necessarily melt right at Tc, instead they survive to about 1.5Tc and then dissolve into quarks and gluons at higher T

Q: what about **M-component**?

- N=2 SYM: solved by Seiberg-Witten
- Deg. vacua: complex higgs U
  - energy scale set by |U| (our analog: T)
- confining point: monopole becomes massless and forms condensate
- on way to that: gluons more and more heavy and strongly coupled
- monopoles the OPPOSITE!!

From: Lerche, hep-th/9611190
Magnetic Scenario for sQGP

JL & Shuryak, PRC75:054907, 2007

hadrons: E-Composite

1.5 Tc: e=g ?!

quarks, gluons: EQP

E-S-C

M-W-C

M-S-C

QCD vacuum: Magnetic

dual superconductor, monopole ondensate

Magnetic Plasma

made of light and abundant magnetic monopoles

non-pert. scale: e^2 T

M-sector setting in

~ 4 Tc

Very High T

S-QGP

wQGP

JL & Shuryak, PRC75:054907, 2007

\( \frac{eg}{4\pi \hbar c} = 1 \)

\( \tilde{\beta}^E(\alpha) + \tilde{\beta}^M(\alpha) = 0 \)
sQGP: a strongly-coupled plasma with both Electric and Magnetic charges

What would be the transport properties of such a mixture plasma, e.g. viscosity, diffusion,…

Strong constraints from heavy ion data!

Also, is the magnetic component important for jet quenching?
Warmup: Single Monopole Motion I

Charge-Monopole

- studied in great details by many people, both classical and quantum
- Poincare cone: focusing & bouncing

K. A. Milton, hep-ex/0002040
Warmup: Single Monopole Motion II

eDipole-Monopole

- Again: focusing & bouncing
- Now: even trapping
- Important for transport!

E-M PING-PONG
Warmup: Single Monopole Motion III

A grain of salt:

- Very complicated trajectories
- Cone-like structure near the corners
- Multi-bouncing before eventually escaping
Warmup: Single Monopole Motion III

A Cage for Monopole:

Lorentz Trapping Effect

- Enhance very much the collision rate
- Monopole can be trapped locally for a time scale much longer than micro. time scale

Absent for E-charge in the same setting

\[ C = \frac{v \tau_{\text{esc.}}}{L} \]
\[ \Gamma = \frac{PE}{KE} \sim \frac{1}{v^2} \]
\[ C \sim \Gamma^{0.47} \]
\[ MFP \sim \frac{1}{C} \]
\[ D \sim \frac{1}{\Gamma^{0.47}} \]
MD for E-M Mixture Plasma

**MD is a powerful tool:**
- QED Coulomb plasma extensively studied
- Non-Abelian Coulomb plasma studied by Gelman, Shuryak, and Zahed
- Extremely useful for calculating transport properties

**New Window -- 1st MD for E-M mixture**
- Lorentz force between E-M
- ~1000 particles in a “cup”
- Varying E/M ratio: M00, M25, M50
- Varying Gamma as well
- Measure viscosity, diffusion
- Mapping to sQGP parameters

\[ \Gamma = \left| \frac{\langle U \rangle}{\langle E_k \rangle} \right| \approx \frac{e^2/a}{k_BT} \]

- \( \Gamma < 1 \) weakly-coupled (gas)
- \( \Gamma > 1 \) strongly-coupled (liquid)
  - \( \sim 1 \)-few 10; solid \( \sim 100 \)

For sQGP: \( \Gamma \approx 3 - 10 \)

Transport properties are sensitive to \( \Gamma \) regime

\[ M \sim 3T \]
\[ r_0 \text{ core size} \sim 1/3T \]
\[ \tau = \sqrt{mr_0^3/e^2} \sim 1/4.2T \]

\[ \rightarrow \text{Observable}(\Gamma)[UNIT] = #MD(\Gamma) \times UNIT_{QGP} \]
Transport: Diffusion

\[ D(\tau) = \frac{1}{3N} \left< \sum_{i=1}^{N} v_i(\tau) \cdot v_i(0) \right> \]

\[ D = \int_{0}^{\infty} D(\tau) \, d\tau \]

- More mixing \( \rightarrow \) less diffusion
- power law
- the power is close to 0.47 (cube-analysis) and the 0.5 (AdS/CFT)
Transport: Viscosity

\[ \eta(\tau) = \frac{1}{3VT} < \sum_{l<k}^{1,2,3} T_{lk}(\tau) T_{lk}(0) > \]

\[ \eta = \int_0^\infty \eta(\tau) d\tau \]

\[ T_{lk} = \sum_{i=1}^{N} m(v_i)_l (v_i)_k + \frac{1}{2} \sum_{i \neq j} (r_{ij})_l (F_{ij})_k \]

\[ = \sum_{i=1}^{N} m(v_i)_l (v_i)_k + \sum_{i=1}^{N} m(r_i)_l (a_i)_k \]

- More mixing \( \rightarrow \) less viscosity
- rapid rising for Gamma < 1
- tendency to rise again for Gamma ~ 10
Transport Summary

RHIC Results:
viscous hydro → \( \eta/s \)
heavy flavor → diffusion

Weakly coupled limit:
both proportional to M.F.P.

AdS/CFT predictions:
(Kovtun, Son, Strainet; Casselderrey & Teaney)

\[
\frac{\eta}{s} = \frac{1}{4\pi} \left( 1 + \frac{135 \zeta(3) 2^{-9/2}}{\lambda^{3/2}} \right) + \ldots
\]

\[
D(2\pi T) = \frac{4}{\sqrt{\lambda}}
\]
Jet Quenching at RHIC

Nuclear Modification Factor:

\[ R_{AA}(p_T) = \frac{1}{N_{AA}^{\text{evt}}} \frac{d^2N_{AA}^{\pi^0}}{dp_T \, dy} \frac{T_{AA}}{\langle T_{AA} \rangle} \times \frac{d^2\sigma_{pp}^{\pi^0}}{dp_T \, dy}, \]

\[ R_{aa} = 1 : \text{NO medium effect, just bunch of pp} \]
Raa & V2 from Phenix

Large Azimuthal Asymmetry!
Jet Energy Loss

\[
\frac{dE}{dx} = \mathcal{F} \left[ J \mid M \mid M + J \right] \rightarrow \Delta E = \int_i^f \mathcal{F} \cdot dx
\]

**Probe dependence:**

\[
\mathcal{F} \rightarrow \langle \mathcal{F} \rangle \left|_{\text{species}} \propto E \rightarrow E_f = E_i \times e^{-\int_i^f \tilde{\mathcal{F}} \cdot dx}
\]

**Medium dependence:**

\[
\tilde{\mathcal{F}} \propto \kappa[s(x)] \cdot s(x)
\]

**Entangled dependence:**

\[
\tilde{\mathcal{F}} \ dx \rightarrow \tilde{\mathcal{F}} \ x \ dx
\]

**All together:**

\[
E_f = E_i \times e^{-\int_i^f \kappa[s(x)] \cdot s(x) \cdot x \ dx}
\]

Note:
mid rapidity
high Pt > 5GeV
\[\rightarrow\ E \sim Pt\]
Raa AND V2 of High Pt Hadron

Raa : well described by most models

\[ E_f = E_i \times f \quad f = e^{-\int_i^f \kappa(s(x)) \cdot s(x) \cdot x \cdot dx} \]

\[ R_{AA}(P_T) = f^{(n-2)} \quad n \approx 8.10 \]

V2 in non-central collisions : surprisingly large !
no satisfactory model ; what’s the geometric limit ?

Shuryak, PRC66:027902,2002
• hard sphere nuclei with sharp edges
• constant \( \kappa \)
→ even black limit can NOT produce enough \( v_2 \)

Drees & Feng & Jia, PRC71:034909,2005
• Woods-Saxon + medium dilution + quadratic length
• constant \( \kappa \)
→ only bring down the \( v_2 \) (except cylindrical geo. up \( \sim 10\% \))

Possible improvement : CGC like initial → 10% increase of the final \( v_2 \) !
Rethinking about the \kappa


The dependence could be very non-trivial!

From "almond" to "ONION"

Jet absorption retarded by a "latent time" of about 2-3 fm

\kappa [s(x)]

The dependence could be very non-trivial!
Scanning the RHIC Fireball with Jet

Key idea of our geometric model:

\[ \kappa[s] = \kappa_c \times \theta[s - s_{\text{min}}] \times \theta[s_{\text{max}} - s] \rightarrow f = e^{-\kappa_c \int_{s=s_{\text{min}}}^{s=s_{\text{max}}} s(x) \cdot dx} \]

Varying \( s_{\text{min}} \) & \( s_{\text{max}} \) to scan layer by layer: really a kind of jet tomography!

The jet:
- ignited in \( x-y \) plane according to density of binary collision
- randomly escaping near \( \phi=0, \pi/2, \pi, 3\pi/2 \) directions

The medium:
- initial profile \( \rightarrow \) Woods-Saxon with impact \( b \) & \( N_{\text{part.}} \) scaling
- evolution \( \rightarrow \) Bjorken dilution \( \sim 1/\tau \)

\[ \bar{R}_{AA}(P_T) = \frac{R_{AA}(P_T, \phi = 0) + R_{AA}(P_T, \phi = 90)}{2} \]

\[ v_2(P_T) = \frac{[R_{AA}(P_T, \phi = 0) - R_{AA}(P_T, \phi = 90)]/2}{R_{AA}(P_T, \phi = 0) + R_{AA}(P_T, \phi = 90)} \]
Black Limit Raa & V2

\[ b=7, \quad N_{\text{part}} \sim 260, \quad \tau_0 = 1 \text{ fm} \]
**V2 with constrained Raa**

*R_aa is constrained by measurements:*

\[ b = 7 \text{ fm} \rightarrow R_{aa} \sim 0.33 \]  
*(PHENIX 20-30%)*

*For each layer scan:*  

→ first adjust \( \kappa \) to fit the \( R_{aa} \)  
→ then calculate the \( V_2 \)

*PHENIX V2 20-40%*  
5-15 GeV: \( \sim 0.10-0.14 \)

\[ b = 7, \; N_{part} \sim 260, \; \tau_0 = 1 \text{ fm} \]
**V2 with constrained Raa @ another centrality**

*R_aa is constrained by measurements:

\( b = 10 \text{fm} \rightarrow R_{aa} \sim 0.52 \)  
(PHENIX 40-50%)

For each layer scan:

→ first adjust \( \kappa \) to fit the \( R_{aa} \)
→ then calculate the \( V_2 \)

**PHENIX V2 40-60%**

5-15 GeV: \(~ 0.11-0.15\)

\( b = 10, \ N_{\text{part}} \sim 120, \ \tau_0 = 1 \text{ fm} \)
The Near Tc Region

No wonder the Near Tc Region shall be special on general ground. However:

**Our study of azimuthal asymmetry in high Pt hadron yield suggest in particular that -----**

Jet quenching happens mostly in this Near Tc Region!

Is there any independent evidence for such a conclusion?

Yes → the conical flow created from interaction of away-side jet with the medium

---

From: Roy Lacey at ICHP08

In light of $\eta/s$

From: Csernai, Kapusta, McLerran

Minimal viscosity around $T_c$
→ Strong dissipation/relaxation concentrated locally

From: Lacey & Collaborators

Thanks to Larry for pointing out this to me.
Magnetic Quenching of Electric Jet

Why more quenching in the Magnetic Plasma near $T_c$ rather than in the QGP at higher $T$?

- **Time argument**:
  \[
  \int \frac{d\tau}{\tau} \to \ln \left( \frac{T_f}{T_i} \right) \to \ln[2]_{MP} : \ln[5]_{QGP} \approx 0.4 : 1
  \]

- **Mass argument**:
  \[
  \frac{\alpha_e \cdot \alpha_m \cdot v_m^2 \cdot \Delta t^2}{M_m} / \frac{\alpha_e^2 \cdot \Delta t^2}{M_e} \sim M_e : M_m \approx 3 : 1
  \]

- **Density argument**:
  \[
  \eta_m : \eta_e > 1
  \]

- **Transport cross-section argument**:
  \[
  \sigma_m : \sigma_e[\theta \to \pi] \sim 2
  \]
Lattice Gauge Theory (LGT) is another important way of studying sQGP.

LGT provides important support for the dual superconductivity scenario of color confinement.

Are there evidence for the magnetic component of sQGP from LGT?

Further: can the parameters of magnetic component be measured or inferred?
Magnetic Monopoles on the Lattice

\[ \rho(T) \approx T_c^3 \quad (T_c < T < 2T_c) \]

\[ \zeta(3)/\pi^2 \approx 0.12 \]

Monopole Density & Correlation

M-anti-M, M-M equal-time spatial correlation functions

\[ G_{ab}(r) \equiv \frac{< \sum_{i=1,N_a} \sum_{j=1,N_b} \delta(|r^a_i - r^b_j| - r)>}{N_a N_b 4\pi r^2/V} \]
The Correlation Teaches us something.

From: Gelman, Shuryak, & Zahed, nucl-th/0601029
From Correlation to M-Coupling

$G_{MM}(r) \sim \exp \left[ \frac{\alpha_M e^{-r/R_d}}{rT} \right]$
Plasma coupling of M-component:

- a magnetic component of sQGP made of dense monopoles;
- the magnetic coupling runs oppositely to the electric, crossing around 1.5Tc;
- the magnetic component should be a good liquid:

all these are supported by the lattice calculations need more
More E-M Crossing

Screening mass


M. Baker, PRD78(2008)014009

Dual QCD : vacuum & finite T<3T
Electric component of sQGP 1-2Tc

- Susceptibilities $\rightarrow$ quarks/gluons: heavy below 1.5Tc
- Potential model Q.M. $\rightarrow$ hadrons: survive above 1.5Tc

They are NOT the only players, nor the dominant!

E-M duality $\rightarrow$

Magnetic component of sQGP 1-2Tc

- A good liquid made of dense monopoles
- Condense at lower T while become dilute and stronger coupled at higher T, oppositely to the electric
- Using MD we showed a mixture plasma explains the transport properties of the RHIC measurements
- It helps us understand the geometry & physics of jet quenching

Magnetic Scenario (with time)

$\rightarrow$ a deeper theoretical understanding of sQGP
New Phase Diagram from E-M Duality
The Order of Transition?
The Order-Disorder Transition?

Is dual superconductivity the correct picture for QCD confinement?

Symmetric Phase ---- deconfined
\[ <\text{Magnetic } U(1)> = 0 \]

Asymmetric Phase ---- confined
\[ <\text{Magnetic } U(1)> \neq 0 \]

NO CROSSOVER

Pisa group (Di Giacomo and collaborators) \textit{arXiv:0710.1174 \& refs therein}

\[ L(\vec{x}) = Tr [ Pe^{i \int_0^T A_0(\vec{x}, t) dt} ] \]

CROSSOVER ??
[ real QCD vacuum is confining, for sure. ]

\[ \mu(\vec{y}, t) = \exp \left[ \frac{1}{e} \int d^3 x \; \vec{E}(\vec{x}, t) \vec{b}(\vec{x} - \vec{y}) \right] \]

\[ \vec{b}(\vec{x} - \vec{y}) = \frac{q}{2} \frac{\vec{r} \wedge \vec{n}_3}{r(r - \vec{r} \cdot \vec{n}_3)} \]

\[ N_f = 2 \; \text{QCD} \]

\[ C_V - C_0 = L_s^{\frac{2}{3}} \Phi_c(\tau L_s^{\frac{1}{3}}, mL_s^{y_h}) \]

\[ \chi - \chi_0 = L_s^{\frac{1}{3}} \Phi(\tau L_s^{\frac{1}{3}}, mL_s^{y_h}) \]

\textit{DISCLAIMER: I am not to be blamed for the results ☺}
Backup: QCD in General

\[ \mathcal{L} = -\frac{1}{4} \sum_{A=1}^{8} F_{\mu\nu}^{A} F_{\mu\nu}^{A} + \sum_{j=1}^{n_f} \bar{q}_j (iD_j - m_j)q_j \]

- QCD & QCD-like theories
  simple Lagrangian, rich phenomena
- \( N_c \): # of color
  \( N_f \): # of flavor
- Asymptotic freedom (UV)
  \( \rightarrow \) IR : strongly coupled
- **Color confinement**
- Chiral Sys. Breaking

\textbf{D.o.F. in per. Spectrum is NOT the D.o.F. in QCD vacuum!}
What is the M/T for q,g in sQGP? Abundant or NOT?

Direct Info. from lattice:

<table>
<thead>
<tr>
<th>T</th>
<th>M_q / T</th>
<th>M_g / T</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 T_c</td>
<td>3.9(2)</td>
<td>3.4(3)</td>
</tr>
<tr>
<td>3.0 T_c</td>
<td>1.7(1)</td>
<td>1.2(1)</td>
</tr>
</tbody>
</table>

Already at 1.5Tc, EQPs are very heavy!
They may become even heavier further down to Tc!

Indirect probe: thermodynamic variables
e.g. **baryonic & isospin susceptibilities**

\[
d_n(T) = \frac{\partial^n(p/T^4)}{\partial(\mu/T)^n} \bigg|_{\mu=0} = n! c_n(T)
\]

\[
d_n^c(T) = \frac{\partial^n(p/T^4)}{\partial(\mu/T)^n} \bigg|_{\mu=\mu_f=0} = n! c_n^c(T)
\]

1. Quarks dominate close to 2Tc
2. Baryons dominate close to Tc
In sQGP 1-2 Tc:
- **EQPs are heavy**
- **Binary interaction is strong**

→ **forming bound state!**
hadrons surviving above Tc
even more exotic states
(Shuryak & Zahed,
PRC 70(2004)021901
PRD 70(2004)054507)

**First studied and found robust multi-body bound states:**
- baryons (qqq) and glueballs (ggg)
- polymer chains Q-g-g----g-antiQ
Backup: RHIC Results


Backup: Masses from Lattice

- Large masses found at lattice for quasi-quarks/glucions

<table>
<thead>
<tr>
<th></th>
<th>$M_q / T$</th>
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<td>$1.5 \ T_c$</td>
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<td>1.7(1)</td>
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</tr>
</tbody>
</table>

Backup: Baryonic Susceptibilities

## Backup: Bound States

<table>
<thead>
<tr>
<th>Channel</th>
<th>Representation</th>
<th>Charge factor</th>
<th>No. of states</th>
<th>Structure</th>
<th>-body</th>
<th>$C/C_{\bar{q}q}$</th>
<th>$E_b/T_c$ at $T_c$</th>
<th>$T_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{q}q$</td>
<td>1</td>
<td>9/4</td>
<td>$9_s$</td>
<td>$\bar{q}q$</td>
<td>2</td>
<td>1</td>
<td>-1.45</td>
<td>2.1</td>
</tr>
<tr>
<td>$\bar{q}g \cdots gq$</td>
<td>(polymer chain)</td>
<td>N</td>
<td>1</td>
<td>$\bar{q}g \cdots gq$</td>
<td>N</td>
<td>1</td>
<td>-1.45*(N-1)</td>
<td>2.1</td>
</tr>
<tr>
<td>$qqg$</td>
<td>(closed chain)</td>
<td>3</td>
<td>1</td>
<td>$qqg$</td>
<td>3</td>
<td>1</td>
<td>-7.64</td>
<td>2.6</td>
</tr>
<tr>
<td>$\bar{q}q \ / , \bar{q}q$</td>
<td>2</td>
<td>1/2</td>
<td>-0.13</td>
<td>1.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$qqg \ / , \bar{q}gq$</td>
<td>(closed chain)</td>
<td>3</td>
<td>1/2</td>
<td>$qqg \ / , \bar{q}gq$</td>
<td>3</td>
<td>1/2</td>
<td>-1.10</td>
<td>1.6</td>
</tr>
</tbody>
</table>

From: Shuryak & Zahed; Liao & Shuryak
Backup: Bound States


\[ \phi(r) = \exp \left[ -Ar - \frac{C^2r^2}{(C^2r^2 + 1)} (B - A)r - \frac{1}{2} \log(C^2r^2 + 1) \right]. \]

\[ \psi(\vec{x}_1, \vec{x}_2, \vec{x}_3) = \phi(r_{12}) \phi(r_{23}) \phi(r_{31}) \]

\[ V(T, r) = -\left( 4 \frac{T}{3r} + \frac{8T}{3} \right) \frac{e^{-2Tr}}{\log(1 + 3T)} - \frac{4T}{r(1 + 3T)} \frac{e^{-2Tr}}{(\log(1 + 3T))^2}. \]

From: Shuryak & Zahed; Liao & Shuryak
Backup: Polymer Chains

From: Liao & Shuryak.
Backup: Dirac Monopoles

\[ \nabla \cdot D = 4\pi \rho_e \]

\[ -\nabla \times E = \frac{1}{c} \frac{\partial B}{\partial t} + \frac{4\pi}{c} J_m \]

\[ \nabla \cdot B = 4\pi \rho_m \]

\[ -\nabla \times E = \frac{1}{c} \frac{\partial B}{\partial t} + \frac{4\pi}{c} J_m \]

\[ \vec{F} = e(\vec{E} + \frac{\vec{u}}{c} \times \vec{B}) + g(\vec{B} - \frac{\vec{u}}{c} \times \vec{E}) \]

\[ \frac{ge}{\hbar c} = \frac{n}{2}, \quad n = 0, \pm 1, \pm 2, \ldots \]

K. A. Milton, hep-ex/0002040
Backup: eDipole-Monopole

\[ H = \frac{(\vec{p} + g\vec{A}_e/c)^2}{2M} \]

\[ V_{eff} = \frac{\hbar^2}{2Ma^2} \left[ \frac{1}{r/a} \left( \frac{ge}{\hbar c} \frac{rA_e}{e} + f \right) \right]^2 \]

\[ \vec{A}_e(r, \phi, z) = -\frac{e}{r} \left[ 2 + \frac{z-a}{\sqrt{r^2 + (z-a)^2}} - \frac{z+a}{r^2 + (z+a)^2} \right] \phi \]

“Magnetic Bottle “
(Budker)
in controlled thermal nuclear fusion
Backup: eDipole-Monopole
Backup: Lorentz Trapping
Backup: MD setting

\[
\frac{d^2 r_i}{dt^2} = \sum_{j \neq i} \left[ \frac{C}{r_{ij}^{K+1}} \hat{r}_{ji} + \frac{e^2 e_{ij}}{4\pi r_{ij}^2} \hat{r}_{ji} + \frac{g^2 g_{ij}}{4\pi r_{ij}^2} \hat{r}_{ji} + \frac{ge_\kappa_{ij}}{4\pi r_{ij}^2 c dt} \times \hat{r}_{ji} \right]
\]

\[
K = 9 + \frac{g^2 g_{ij}}{4\pi r_{ij}^2} \hat{r}_{ji} + \frac{ge_\kappa_{ij}}{4\pi r_{ij}^2 c dt} \times \hat{r}_{ji}
\]

\[
v_{1,2,3}^i(t = 0) = V \ast (\text{RANDOM\#})
\]

\[
V = [B \ast (r - R_{cut})]^L \ast \theta(r - R_{cut})
\]

\[
B = 5 \text{ and } L = 11
\]

\[
\vec{r} = r/r_0 \quad \text{with} \quad r_0 = (C/e^2)^\frac{1}{K-1}
\]

\[
\tilde{t} = t/\tau \quad \text{with} \quad \tau = (mr_0^3/e^2)^\frac{1}{2}
\]
Backup: Transport from MD

$$D(\tau) = \frac{1}{3N} < \sum_{i=1}^{N} v_i(\tau) \cdot v_i(0) >$$

$$\eta(\tau) = \frac{1}{3VT} < \sum_{l<k}^{1,2,3} T_{lk}(\tau) T_{lk}(0) >$$
\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{2} D^\mu \Phi^a D_\mu \Phi^a - V(\Phi) \]

\[ V(\phi) = \frac{\lambda}{4} (\phi^a \phi^a - v^2)^2 \]

\[ \Phi^a = \frac{\hat{\Phi}^a}{er} H(\text{ver}) \]

\[ A_i^a = -\epsilon^a_{ij} \frac{\hat{\Phi}^j}{er} (1 - K(\text{ver})) \]

\[ g = \int_{S_2} \vec{B} \cdot d\vec{S} = \frac{1}{2ev^3} \int_{S_2} \epsilon^{ijk} \epsilon^{abc} \Phi^a \partial_j \Phi^b \partial_k \Phi^c = \frac{4\pi N}{e} \]

\[ M_M \geq vg \]
Maximal Abelian Projection: 

SU(2) example

\[ \int d^4 x \left\{ [A_\mu^1(x)]^2 + [A_\mu^2(x)]^2 \right\} \]

\[ A_\mu(x) \equiv t^a A_\mu^a(x) \to t^3 A_\mu^3(x) \]

Monopole density counting

From wrapped trajectories

\[ \int \frac{1}{V_{3d}} \langle |s| \rangle \]

More about the way to monopole:

Abelian Dominance & Monopole Dominance

for IR properties in the QCD vacuum are well established.

Chernodub & Zakharov, arXiv:0806.2874
Backup: E-M Equilibrium Point

\[ \langle g^2 A^2 \rangle = [1.64(15) \text{ GeV}]^2 \]

\[ \langle g^2 A^2 \rangle = \langle g^2 A_E^2 \rangle + \langle g^2 A_M^2 \rangle \]


Approaching \( T_c \) from above:
- E-screening mass decrease
  \( \rightarrow \) less E-charge
  \( \rightarrow \) E-charge getting heavier
- M-screening mass increase
  \( \rightarrow \) more M-charge
  \( \rightarrow \) M-charge getting lighter

Chernodub & Ilgenfritz, arXiv: 0805.3714

\[ T_c = 0.694(18) \sigma^{1/2} = 305(8) \text{ MeV} \]

symmetric point, \( T_0 = 2.21(5) T_c \)

\[ \frac{\Delta A^2}{T^2} \]

\[ T/T_c \]

\( 16^3 \times 4 \)

\( 24^3 \times 6 \)

\( 32^3 \times 8 \)

fit \( (T/T_c = 1.5 \ldots 6.2) \)