What can we learn from model studies on the chiral critical end-point?

– Old and New Perspectives –

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Discovery in Old Days

- Color Confinement – Deconfinement
  - Center Symmetry
  - 2-Color – Second-Order 3-Color – First-Order
  - High Baryon Density?

- Chiral Effective Model
  - Chiral Symmetry
  - 2-Flavor – Second-Order 3-Flavor – First-Order
  - Mesons, Baryons **sigma-model**
  - Quarks **NJL-model**
**Thermodynamic Potential**

Thermodynamic Potential in the NJL Model

- **Condensation Energy**

\[
\Omega_{\text{cond}} = g_S (\langle \bar{u}u \rangle^2 + \langle \bar{d}d \rangle^2 + \langle \bar{s}s \rangle^2) + 4g_D \langle \bar{u}u \rangle \langle \bar{d}d \rangle \langle \bar{s}s \rangle
\]

- **Zero-Point Energy** (responsible for chiral symmetry breaking)

\[
\Omega_{\text{zero}} = -2N_c \sum_i \int \frac{d^3p}{(2\pi)^3} \varepsilon_i(p)
\]

- **Thermal Energy**

\[
\Omega_{\text{quark}} = -2T \sum_i \int \frac{d^3p}{(2\pi)^3} \left\{ \ln \det \left[ 1 + L e^{-\left(\varepsilon_i(p) - \mu\right)/T} \right] \right. \\
+ \left. \ln \det \left[ 1 + L^\dagger e^{-\left(\varepsilon_i(p) + \mu\right)/T} \right] \right\}
\]
Why First-Order?

Some Numerics

\[ M_{ud} = 336 \text{ MeV}, \; \mu_c = 345 \text{ MeV}, \; T = 0 \]

Increasing \( \mu \)

- \( \mu < 336 \text{ MeV} \) \( n_B = 0 \) nothing changed

- \( 336 \text{ MeV} < \mu < 345 \text{ MeV} \) \( n_B \neq 0 \)

\[ p \propto (\mu^2 - M^2)^2, \quad f \propto 2 \mu^2 M^2 - M^4 \]

on top of \( f = -c_2 M^2 + c_4 M^4 + c_6 M^6 \)

- \( \mu > 345 \text{ MeV} \)

\[ n_B \neq 0, \quad p \propto (\mu^2 - m^2)^2 \]

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Some People Claim

Density-Density Interaction Changes

\[ \mathcal{L}_V = -g_V (\bar{\psi} \gamma_\mu \psi)^2 \rightarrow f \sim g_V n_q^2 \propto \mu^2 (\mu^2 - M^2)^2 \]

NJL Results  Kitazawa-Koide-Kunihiro-Nemoto (2002)
Some More from NJL

NJL Results Sasaki-Frimon-Redlich (2006)

So far all the discussions are limited to two-flavor quark matter in the NJL model.

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**Degrees of Freedom**

- **Quark Degrees of Freedom**
  - Two-Flavor Case
    \[(7/8) \times 3 \times 2 \times 4 = 21\]
  - Three-Flavor Case
    \[(7/8) \times 3 \times 3 \times 4 = 31.5\]

- **Gluon Degrees of Freedom**
  \[8 \times 2 = 16\]

The NJL model description is not bad, but the gluon contribution is comparable also.

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Quick History (only limited milestones)

- **Fukushima 2003**
  Simultaneous crossover of deconfinement and chiral restoration

- **Ratti-Thaler-Weise 2005**
  Fitting the thermodynamic quantities on the lattice

- **Fu-Zhang-Liu 2007**
  Ciminale-Gatto-Ippolito-Nardulli-Ruggieri 2007
  Fukushima 2008
  2+1 flavor with 't Hooft interaction

It works much better than I thought...
Neat Properties

- Degrees of freedom are different!
  - 16 gluons at high $T$ should be transverse.
  - Polyakov loop is $A_0$ that is longitudinal.

**Polyakov loop saturates the thermodynamics near $T_c$.**

- No clear way to fix the Polyakov-loop potential?
  - Pressure, entropy, internal energy fitting.
  - Simple parametrization with $T_c$ fixing.

**Little sensitivity to the choice of the potential.**
Model Ingredients

Thermal Energy

\[
\Omega_{\text{quark}} = -2T \sum_i \int \frac{d^3p}{(2\pi)^3} \left\{ \ln \det \left[ 1 + L e^{-\left(\varepsilon_i(p) - \mu\right)/T} \right] \\
+ \ln \det \left[ 1 + L^\dagger e^{-\left(\varepsilon_i(p) + \mu\right)/T} \right] \right\}.
\]

Polyakov-Loop Potential

\[
\Omega_{\text{Polyakov}} = -b \cdot T \left\{ 54 e^{-a/T} \ell \bar{\ell} \\
+ \ln \left[ 1 - 6 \ell \bar{\ell} - 3(\ell \bar{\ell})^2 + 4(\ell^3 + \bar{\ell}^3) \right] \right\}
\]

\(a\) determines the pure-gluonic \(T_c = 270\text{MeV}\)

\(b\) determines the simultaneous crossover \(T_c = 200\text{MeV}\)

No information on bulk thermodynamic quantities

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Thermodynamics at Zero Density

Interaction Measure

2+1 flavor PNJL

Lattice Data
Cheng et al.

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More on Thermodynamics

(A part of) Sound Velocity

2+1 flavor PNJL

Lattice Data
Cheng.et al.


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More and More

$4^{\text{th}}$ order Cumulant

![Graph showing the susceptibility ratio $\chi_q(4)/\chi_q$ as a function of temperature $T$ in MeV. The graph displays a sharp decrease around $T = 200$ MeV, illustrating the behavior of the susceptibility ratio at higher temperatures.]
Strong correlation is evident, but!
the Polyakov loop stays small at low temperature.
Susceptibility

(Light-Quark) Chiral and Quark Number Susceptibility

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Prediction from the PNJL

Zero Polyakov Loop at Zero Temperature

\[
\langle \text{det} [1 + L e^{-(\varepsilon - \mu)/T}] \rangle = 1 + e^{-3(\varepsilon - \mu)/T} + 3 l e^{-(\varepsilon - \mu)/T} + 3 \bar{l} e^{-2(\varepsilon - \mu)/T}
\]

\[
\langle \text{det} [1 + L^{\dagger} e^{-(\varepsilon + \mu)/T}] \rangle = 1 + e^{-3(\varepsilon + \mu)/T} + 3 \bar{l} e^{-(\varepsilon + \mu)/T} + 3 l e^{-2(\varepsilon + \mu)/T}
\]

dominant!

Degenerate quark matter with all colors equally occupied does not break global center symmetry.

No Deconfinement at High Density

Quark matter with \( \langle \bar{\psi} \psi \rangle \approx 0, \ l \approx 0, \ n \neq 0 \)

Quarkyonic Matter
McLerran-Pisarski (2007)

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Evidence?

Talk by Simon Hands at XQCD (2-color simulation)

**Order Parameters**

Superfluid condensate approaches BCS scaling as $\mu$ increases.

Polyakov loop $\approx 0$ throughout – no deconfinement!

Due to poor signal:noise? Or has it disappeared?
Phase Diagram

Without Color Superconductivity

With Color Superconductivity

Z. Zhang will surprise you with three critical-end-points.

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Do not ask me a question...

Zhang-Fukushima-Kunihiro (coming soon)

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Model Assumptions

Polyakov Loop treated as a Mean Field.
- Cannot describe the spatial correlations correctly.
- Sign Problem not completely resolved...

**Bulk thermodynamics is OK.**

Parameters fixed at $T=\mu=0$
- No new interaction induced?
- No coupling running with $T$ and $\mu$.

**Four-coupling as a result of dressed (heavy) gluons is OK unless at high $T$ or $\mu$.**
**There are two dangerous parts...**
**Dangerous Parts**

Interaction Directly Coupled to Density

\[ \mathcal{L}_V = -g_V (\bar{\psi} \gamma_\mu \psi)^2 \]

Fierz trans generates various interaction channels. Wave-function may have a larger component at finite density.

Interaction Induced by Anomaly (Instanton)

\[ \mathcal{L}_A = g_D \left[ \text{det} \bar{\psi} (1 - \gamma_5) \psi + \text{h.c.} \right] \]

Instanton excitation should be suppressed at finite \( T \) and \( \mu \). Effective \( U_A(1) \) symmetry restoration.
Columbia Plots

First-Order Regions
Their Effects

Shift of the Critical Point

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What Model Studies Can Say

- There IS a critical end-point.
  - Some reasonable model parameters lead to existence of a critical end-point on the phase diagram.

- There IS NOT a critical end-point.
  - Some other model parameters within reasonable range lead to non-existence of a critical point.

- There ARE more critical end-points.
  - Some other dynamics at high density lead to more than one (up to three!) critical end-points.

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Still Useful Way to Go

Meaningful Approach (Attitude)
- Let us assume one of them.
- We can see what happens in the model.
- Model prediction based on a scenario.

One Critical End-Point (standard picture)
- Let us accept “reasonable model parameters.”
- We can see the isentropic trajectories on the phase diagram.
- Model prediction for the “hydrodynamical” evolution.
Isentropic Trajectories

$s/n = \text{constant lines}$

Critical region is very small and neglected. Then, nothing is special around the end-point.
Strangeness-free Isentropic Trajectories

$n_s$ must be zero in the heavy-ion collision

Positive $\mu_s$ is necessary, why so?
PNJL model can answer!

Lattice Data
MILC 2008

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Qualitative Comparison II

Heavy Quark Sector

Graphs showing the relationship between temperature and quark chemical potential, with data points for different values of $\mu_h$ at specified $T$ values.
Polyakov Loop Feedback

Particle-rich state favors anti-Polyakov-loop.

\[ \ell = \frac{1}{N_c} \langle \text{tr} L \rangle, \quad \bar{\ell} = \frac{1}{N_c} \langle \text{tr} L^\dagger \rangle \quad \bar{l} > l \text{ for } \mu > 0 \]

Anti-Polyakov-loop acts as a negative chem. pot.

\[
\Omega_{\text{quark}} = -2T \sum_i \int \frac{d^3p}{(2\pi)^3} \left\{ \ln \det \left[ 1 + L e^{-(\bar{\epsilon}_i(p)-\mu)/T} \right] \\
+ \ln \det \left[ 1 + L^\dagger e^{-(\bar{\epsilon}_i(p)+\mu)/T} \right] \right\}.
\]

Positive chem. pot. is necessary.

\[ \log(\bar{l}/l) \approx 2\mu_s/T \]

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Polyakov Loop Difference

Difference and Ratio

\[ \bar{l} - l \quad \log(\bar{l}/l) \]
Summary

- PNJL model can reproduce bulk thermodynamic properties at zero density.
- PNJL model can give predictions for finite density.
- Two model parameters, the coupling strength of the vector-channel interaction and the magnitude of 't Hooft term, are not under theoretical control.
- Under assumption of one critical end-point, PNJL model can draw isentropic trajectories.
- Strangeness-free isentropic trajectories are close to ones with no such constraint.
- Anti-Polyakov-loop induces strange chem. pot.

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