Spectral functions near the QCD critical point in chiral models

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HF, PRD67, 094018
HF-M.Ohtani, PRD70, 014016
Introduction

- LQCD + Models suggested the QCD-CP in $T-\mu$
  - cross-over along $T$ axis
  - $1^{st}$ order along $\mu$ axis
Introduction

- LQCD + Models suggested the QCD-CP in $T-\mu$
  - cross-over along T axis
  - 1st order along mu axis
- Difficult to conclude in LQCD+Models today
  - about the location
  - about the existence
- Needs for exp'tal signatures / confirmation
  - What is the characteristics of the QCD-CP?
    - fluctuations, focusing, particle ratios, ..., etc.
How to use chiral models

- Difficult to locate the QCD-CP in $T_\mu$
- But, generic features of the CP can be demonstrated in the (simplest) models

- What are the spectral properties of QCD-CP?
model

- **NJL** (and other chiral models)

\[ \mathcal{L} = \bar{q}(i\slashed{D} - m)q + g\left[ (\bar{q}q)^2 + (\bar{q}i\gamma_5 \tau^a q)^2 \right] \]

- simplest quark dynamics
- chiral symmetry breaking by $q^\text{bar}$-$q$ attraction
- no dynamic gluons, no confinement (nucleons)
- $\sigma$ and $\pi$ as fluct of $<q^\text{bar}q>$
Chiral transition

- order parameter $\langle q^\text{bar} q \rangle \sim \langle \sigma \rangle$
  - $=0$ in Wigner, $\neq 0$ in NG phase
- flat potential at CP
- chiral susceptibility
  - $\chi_{mm} \sim \int d^3 r \frac{e^{-Mr}}{r} \sim \frac{1}{M^2} \rightarrow \infty$
  - long-range fluctuation
  - $M$: screening mass
- $\chi_{\mu\mu}$, $C$ finite
CP with nonzero $m$

- Across co-existence surface (Clapeyron-Clausius)
  \[
  \frac{dT}{d\mu} = -\frac{\Delta n}{\Delta s}, \quad \frac{dT}{dm} = -\frac{\Delta \sigma}{\Delta s}
  \]

- Order parameter = deviations from equilibrium $\sigma$, $n$, or $s$
  - linear mixing due to finite $m$

- $\chi_{mm}$, $\chi_{\mu\mu}$, and $C$ diverge
χ's & spectral change

• strong $q^{\text{bar}} - q$ attraction generates the CP
  – Which mode softens?
\( \chi_{mm} \) & dropping mass

- Spectral fn near O(4) CP
  - \[ \chi_{mm}(q) = \int \frac{d\omega}{2\pi} \frac{\rho_{mm}(\omega, q)}{\omega} \]
  - dynamic \( \sigma \)-mode as time-like excitation
  - space-like (p-h) mode characteristic in a medium

\[ \rho_{mm}/A^2 \]

\( T > T_c \)
\( \chi_{mm} \) & dropping mass

- Spectral fn near O(4) CP
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\( T>T_c \) & \( T<T_c \)
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\[ R = \text{sp-mode / total} \]

Sp-mode gives finite fraction of divergence

\[ T<T_c \]
\[ \chi_{mm} \& \text{dropping mass} \]

- **Toward the TCP**
  - space-like mode enhances

\[ R = \text{sp-mode / total} \]

Sp-mode saturates the sum at TCP

\[ T < T_c \]
$\chi_{mm}$ & dropping mass

- Toward the TCP
  - quark number $\chi_{\mu\mu}$ diverges

\[
\begin{array}{c}
\text{Graph: } \\
\chi_{\mu\mu}/\Lambda^2 \\
\end{array}
\]

\[
\begin{array}{c}
\text{Diagram: } \\
T<T_c
\end{array}
\]
**CP with nonzero m**

- mass of sigma?
  - Scavenius et al. posed a [Q]:

![Graph showing meson masses as functions of temperature for different chemical potentials](image)

**FIG. 6.** The sigma mass (solid line) and pion mass (dashed line) in the sigma model (left) and NJL model (right) as functions of temperature for $\mu = 0$ (right pair) and for $\mu = \mu_c$ (left pair).
CP with nonzero $m$

- mass of sigma?
  - Scavenius et al. posed a [Q]:
  - comparing different things
    - screening vs pole masses

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![Graphs showing meson masses as functions of temperature for different values of $\mu$.](image)

FIG. 6. The sigma mass (solid line) and pion mass (dashed line) in the sigma model (left) and NJL model (right) as functions of temperature for $\mu = 0$ (right pair) and for $\mu = \mu_c$ (left pair).
**CP with nonzero m**

- Divergent $\chi$'s due to p-h mode
  - no dropping mass!

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(a) scalar

(b) quark number
Why is p-h mode enhanced?

- $\chi_{\mu\mu} =$ fluctuation of conserved density
  - cannot fluctuate at $q=0$ (total charge)
    - $\omega \sim D q^2$
    - $\lim_{q \to 0} \rho_{\mu\mu}(\omega, q) \propto \omega \delta(\omega)$
  - strength diverges as approaching QCD-CP
Why is $p$–$h$ mode enhanced?

- Schematic sketch

\[ \lim_{q \to 0} \sigma \propto \omega \]

\[ \omega \quad \text{w/ exact chiral symmetry} \]
\[ \omega \quad \text{w/ non-zero quark mass} \]

\[ \omega \quad \text{drop} \]
\[ \sigma \quad \text{hydro-modes} \]
\[ \omega/\Lambda \quad \text{mix & repel} \]
\[ \omega \quad \text{hydro-modes} \]
Implications to HIC

- Critical softening in p-h mode
  - decoupling of $\sigma$ meson ($\omega$ meson as well)
  - no direct access to dileptons, $\pi\pi$
  - particle scattering may produce $\gamma^*$
  - low pT dist. of scattered particles
    - standard ...
Dynamic Universality

- Static universality
  - 3D Z(2) Ising

- Dynamics constrained by conservation laws
  - possible slow modes near QCD-CP
    - $\sigma$, $\pi$ massive due to $m_q$ = 0
    - $T^0$, n ... 5 densities
  - decoupling of $\sigma$ due to finite m
  - slow modes = sound (2), shear (2), heat (1) = liq.gas
Hints to phenomenology

- within critical region of gas-liquid CP
  - mode-coupling between hydro-modes important
  - shear viscosity diverges, but only very weakly
    - \( \eta \sim \xi^{(1/19)} \epsilon \),
  - thermal conductivity diverges
    - \( \lambda \sim \xi^{(18/19)} \epsilon \), \( \omega \sim \xi^{-3} \)

- But, finite size and time in HIC set the limit to these critical effects?
  Berdnikov-Rajagopal
Summary

- Model demonstrates:
  - sigma softens near chiral CP, and p-h mode contributes too if approached from broken phase
  - near QCD-CP, p-h mode saturates the divergence
- Critical mode has p-h type spectrum
- Dynamic Universality is the same as liq.-gas
  - $\eta$ & $\lambda$ diverge
  - finite size/time may limit the critical behavior
Region of spectrum

- vacuum
Region of spectrum

- mass modification
Region of spectrum

- medium fluctuation