QCD Critical Point and
Its Effect on Physical Observables

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QCD Phase Diagram

160-190 MeV
100 MeV ~ 10^{12} K

Hadron Phase
- chiral symmetry breaking
- confinement

QGP (quark-gluon plasma)

RHIC

LHC

CEP (critical end point)
crossover
1st order

CSC (color superconductivity)

5-10 \rho_0

\mu_B

M. Asakawa (Osaka University)
Nuclear Physics A504 (1989) 668-684
North-Holland, Amsterdam

CHIRAL RESTORATION AT FINITE DENSITY AND TEMPERATURE

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Fig. 8. The phase transition line in the cases (I) and (II).
Where is CEP, if any?
**CEP = 2nd order phase transition, but...**

- **CEP = 2nd Order Phase Transition Point**
- **Divergence of Fluctuation**
- **Correlation Length**
- **Specific Heat ?**

If expansion is adiabatic, even if the system goes right through the critical end point...

There is no conservation law that slows down the change of those quantities!

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Subject to Final State Interactions

*M. Asakawa (Osaka University)*
Furthermore...

Furthermore, critical slowing down limits the size of fluctuation, correlation length!

Time Evolution along given isentropic trajectories ($n_B/s : fixed$)

\[ \frac{d}{dt} m_\sigma(\tau) = -\Gamma[m_\sigma(\tau)] \left( m_\sigma(\tau) - \frac{1}{\xi_{eq}(\tau)} \right) \]

\[ \Gamma[m_\sigma(\tau)] = \frac{A}{\xi_0} \left( m_\sigma(\tau) \xi_0 \right) \]

\[ z \approx 3 \quad \text{Model H (Hohenberg and Halperin RMP49(77)435)} \]
**Principles to Look for Other Observables**

- We are in need of observables that are not subject to final state interactions.

After Freezeout, no effect of final state interactions.

### Chemical Freezeout

- Usually assumed momentum independent.
- But this is not right.

Chemical freezeout time: $p_T$ (or $y_T$) dependent.

- Larger $p_T$ (or $y_T$), earlier ch. freezeout.

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**Principle I**

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Emission Time Distribution

Au+Au, $E_{\text{lab}}=40$ GeV/A

Emission Time

- Larger $\beta_T$, earlier emission
- No CEP effect (UrQMD)

![Graph showing emission time distribution with different emission time bins and labels for protons and anti-protons with specific $\beta_T$ values.]
**Principle II**

**Universality:**

QCD CEP belongs to the same universality class as 3d Ising Model

Lattice QCD at finite density: still in its infancy

For critical behavior: need to carry out $V \rightarrow \infty$ limit

\[ T \text{ and } \mu_B \leftrightarrow (T, \mu_B) \leftrightarrow (r, h) \]

\[ r = \frac{T - T_C}{T_C} \]

\[ h : \text{external magnetic field} \]
What is not universal

Further Assumptions

- Size of Critical Region
  - No general universality
  - Lattice calculation: not yet $V \to \infty$ limit
  - Renormalization group analysis in Effective Models?

Mapping

$\vec{r} \perp \vec{h}$  
$\vec{r} \parallel$ 1st order PT line is not an assumption

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EOS on Ising Side

Critical Behavior on Ising Side

- parametric representation

\[ M = M_0 R^\beta \theta \]
\[ r = R(1 - \theta^2) \]
\[ h = h_0 R^{\beta \delta} h(\theta) = h_0 R^{\beta \delta} (\theta - 0.76201 \theta^3 + 0.00804 \theta^5) \]

\((R \geq 0, -1.154 \leq \theta \leq 1.154)\)

Condition for \(M_0\) and \(h_0\)

\[ M(r, h) = 1 \text{ at } (r, h) = (0, 1) \]
\[ M(r, h) = 1 \text{ at } (r, h) = (-1, 0) \]

\[ r = \frac{T - T_C}{T_C} \]
\[ h : \text{external magnetic field} \]
\[ \beta = 0.326 \]
\[ \delta = 4.8 \] (Critical Exponents)

R. Guida and J. Zinn-Justin, NPB486 (1997) 626
**Singular Part + Non-singular Part**

- Matching between Hadronic and QGP EOS
  
  - Entropy Density consists of *Singular and Non-Singular Parts*
  
  - **Only Singular Part** shows universal behavior

- Requirement:
  
  reproduce both the singular behavior and known asymptotic limits

- Matched Entropy Density

\[
s_{\text{real}} = (T, \mu_B) = \frac{1}{2} \left\{ 1 - \tanh \left[ S_c(T, \mu_B) \right] \right\} s_H(T, \mu_B) + \frac{1}{2} \left\{ 1 + \tanh \left[ S_c(T, \mu_B) \right] \right\} s_Q(T, \mu_B)
\]

- Dimensionless Quantity: \( S_c \)

\[
S_c(T, \mu_B) = s_c(T, \mu_B) \sqrt{\left( \Delta T_{\text{crit}} \right)^2 + \left( \Delta \mu_{\text{crit}} \right)^2} \times D
\]

\( D \): related to extent of critical region

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**Isentropic Trajectories**

- In each volume element, Entropy (S) and Baryon Number (N_B) are conserved, as long as entropy production can be ignored (= when viscosities are small)

**Isentropic Trajectories (n_B/s = const.)**

**An Example**

Near CEP s and n_B change rapidly

*isentropic trajectories show non-trivial behavior*

Bag Model EOS case
With Large Critical Region

Focusing of Isentropic Trajectories

with CEP

without CEP (EOS in usual hydro calculation)

Excluded Volume Approximation + Bag Model EOS

used in most hydro calculations

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Consequence

For a given chemical freezeout point, prepare three isentropic trajectories: w/ and w/o CEP

Along isentropic trajectory:

- FO, CO $\frac{\mu_B}{T}$
- QCP $\frac{\mu_B}{T}$

As a function of $p_T(y_T)$:

- FO, CO $\frac{\mu_B}{T}$
- QCP $\frac{\mu_B}{T}$

$\overline{p}/p$ ratio: near CEP steeper

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Evolution along Isentropic Trajectory

\[ \frac{\bar{p}}{p} \sim \exp \left( -\frac{2\mu_B}{T} \right) \]

with CEP steeper \( \bar{p} \) spectra at high \( P_T \)

\[ M. \text{ Asakawa (Osaka University)} \]
Effect on Spectra?

steeper $\bar{p}$ spectra at high $P_T$

NA49, PRC73, 044910(2006)

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Result of One Temperature Fit

- Only one experimental result for $\bar{p}$ slope
- Still error bar is large

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Summary

- Two Principles:
  i) Chemical Freezeout is $p_T(\beta_T)$ dependent
  ii) Isentropic Trajectory behaves non-trivially near CEP (focusing)

  $\bar{p}/p$ ratio behaves non-monotonously near CEP

  Information on the QCD critical point:
  such as location, size of critical region, existence...

- We then made a data search
  - turned out NA49 $\bar{p}$ data shows non-trivial behavior around 40 GeV/A
  - still error bar is large, finer energy scans at SPS, FAIR, RHIC: desirable

- Effect on Flow?
  $c_s$ changes differently from the case with EOS used in usual hydro cal.
  (3D hydro cal. with CEP + UrQMD: C. Nonaka in progress)