Three-nucleon interactions and nuclear structure

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Strong interaction physics in the lab and cosmos

Matter at the extremes:
density \( \rho \sim 10^{11} \ldots 10^{15} \) g/cm\(^3\)
neutron-rich to proton-rich
\( Z/N \sim 0.05 \ldots 0.6 \)
temperatures \( T \sim \ldots 30 \) MeV

Interaction challenges:
QCD \( \Rightarrow \) chiral EFT \( \Rightarrow \) RG evolved
low-momentum interactions for all nuclei

Many-body challenges

Astrophysics challenges
Outline

Resolution dependence of nuclear forces

Low-momentum three-nucleon interactions

Impact on binding energies

Impact on density dependences

Impact on spin-orbit and spin dependences

Impact on isospin dependences
Resolution dependence of nuclear interactions

with high-energy probes: quarks+gluons

at low energies:
complex QCD vacuum

lowest energy excitations:
pions, nearly massless, $m_\pi=140$ MeV
“phonons” of QCD vacuum

$\Lambda_{\text{chiral}}$
momenta $Q \sim \lambda^{-1} \sim m_\pi$

$\Lambda_{\text{pionless}}$
$Q \ll m_\pi=140$ MeV
\[ \Lambda / \text{Resolution dependence of nuclear interactions} \]

with high-energy probes: quarks+gluons

**Effective theory for NN, many-N interactions, operators depend on resolution scale \( \Lambda \)**

\[ H(\Lambda) = T + V_{\text{NN}}(\Lambda) + V_{\text{3N}}(\Lambda) + V_{\text{4N}}(\Lambda) + \ldots \]

\( \Lambda_{\text{chiral}} \)

momenta \( Q \sim \lambda^{-1} \sim m_\pi \): chiral effective field theory

nucleons interacting via pion exchanges + contact interactions

typical Fermi momenta in nuclei \( \sim m_\pi \)

\( \Lambda_{\text{pionless}} \)

\( Q \ll m_\pi = 140 \text{ MeV} \): pion not resolved

pionless effective field theory

large scattering lengths + corrections

applicable to loosely-bound, dilute systems, reactions at astro energies

... halo nuclei
Resolution dependence of nuclear interactions with high-energy probes: quarks+gluons

\[ Q \sim \lambda^{-1} \sim m_{\pi}: \text{chiral effective field theory} \]

\[ Q \ll m_{\pi} = 140 \text{ MeV}: \text{pion not resolved} \]

\[ \Lambda_{\text{pionless}} \]

Loosely-bound, dilute systems, reactions at astro energies

Lattice QCD

Effective theory

Beane et al. (2006)

Edwards et al. (2006)
Chiral Effective Field Theory for nuclear forces

Separation of scales: low momenta $\frac{1}{\Lambda} = Q \ll \Lambda_b$ breakdown scale $\Lambda_b$

explains pheno hierarchy: $\text{NN} > 3\text{N} > 4\text{N} > \ldots$

$\text{NN-3N, } \pi\text{N, } \pi\pi, \text{ electro-weak, ...}$

consistency

$3\text{N,4N: 2 new couplings to } N^3\text{LO}$

resolution/$\Lambda$-dependent couplings

error estimates from truncation order, lower bound from $\Lambda$ variation

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner, Nogga, Machleidt, …
Low-momentum interactions from the Renormalization Group evolve to lower resolution/cutoffs by integrating out high-momenta, can be carried out exactly for NN interactions Bogner, Kuo, AS (2003)

implemented by RG equations or unitary transformation

find ≈ universal interaction for low momenta

evolution to $V_{\text{low } k}(\Lambda)$ decouples high momenta

method to vary resolution scale without loss of low-energy NN physics
Chiral EFT and RG

find $\approx$ universality from different $N^3LO$ potentials

weakens off-diag coupling
Advantages of low-momentum interactions for nuclei

high momenta/large cutoffs lead to slow convergence for nuclei

evolution of chiral EFT interactions to low-momentum beneficial

weakens off-diagonal coupling in HO states

lower cutoffs need smaller basis

Bogner et al. (2007)

$10^3$ states for $N_{\text{max}}=2$ vs.

$10^7$ states for $N_{\text{max}}=10$

direct convergence in structure calcs
Impact on binding energies

$V_{\text{low } k}(\Lambda)$ defines class of NN interactions with cutoff-independent low-energy NN observables

Cutoff variation estimates errors due to neglected parts in $H(\Lambda)$

Cutoff dependence explains “Tjon line”

Three-nucleon interactions required by renormalization, to break off line, for neutron-/proton-rich systems,…
NN-only results lead to Tjon lines in $^{16}\text{O}$±1 \textit{Hagen, Dean, AS, in prep.} \Rightarrow 3N truncations in oscillator shells N, different $\hbar\omega$ approx on same lines slopes agree with nuclear matter limit $A\pm1/A +$ surface+Coulomb corr.
Precision era in nuclear masses

First Penning-trap mass measurement in the milli-second half-life range: the two-neutron halo nucleus $^{11}$Li

Dilling et al. TITAN@TRIUMF (2008)

with relative uncertainty $10^{-8}$
Three-nucleon interactions: a frontier
from H.-O. Meyer @ TRIUMF workshop (2007)
p-deuteron scattering
a way to look at 880 data points...

coherent 3N effort needed with theoretical uncertainties
Low-momentum 3N interactions

from leading N$^2$LO chiral EFT $\sim (Q/\Lambda)^3$ van Kolck (1994), Epelbaum et al. (2002)

c_i from $\pi N$, consistent with NN Meissner (2007)

generally improves 3N scattering $pd @ 65$MeV

c_3, c_4 important for structure, large uncertainties at present

chiral EFT is complete basis $\rightarrow$ 3N up to truncation errors
D term could be fixed by tritium beta decay
Low-momentum 3N fits

fit D,E couplings to $A=3,4$ binding energies for range of cutoffs

linear dependences in fits to triton binding

3N interactions perturbative for $\Lambda \lesssim 2 \text{ fm}^{-1}$

Nogga, Bogner, AS (2004)

nonperturbative at larger cutoffs
cf. chiral EFT $\Lambda \approx 3 \text{ fm}^{-1}$

3N exp. values natural
$\sim (Q/\Lambda)^3 V_{\text{NN}} \sim 0.1 V_{\text{NN}}$

$A=3$…nuclear matter

Navratil et al. (2007)
Subleading chiral EFT 3N interactions

parameter-free $N^3LO \sim (Q/\Lambda)^4$ Status from Epelbaum @ TRIUMF 3N workshop (2007)

- $1/m$-corrections to 1 insertion from $L_{1/m}^{(2)} = \text{[diagram]} + \text{[diagram]} + \text{[diagram]} + \mathcal{O}(\pi^3)$
  - rich operator structure (includes spin-orbit interactions)

- 1-loop diagrams with all vertices from $L_{\text{eff}}^{(0)}$

  \[ 2\pi - \text{exchange} \]
  \[
  \begin{align*}
  \text{[diagram]} &= \text{[diagram]} + \text{[diagram]} + \text{[diagram]} + \text{[diagram]} + \text{[diagram]} + \ldots
  \end{align*}
  \]

  The calculated corrections simply shift the LECs $c_i$ as follows:

  \[
  \begin{align*}
  \delta c_1 &= \frac{g_A^2 M_\pi}{64 \pi F_\pi^2} \sim 0.13 \text{ GeV}^{-1} \\
  \delta c_3 &= \frac{3g_A^4 M_\pi}{16 \pi F_\pi^2} \sim 2.5 \text{ GeV}^{-1} \\
  \delta c_4 &= \frac{-g_A^4 M_\pi}{16 \pi F_\pi^2} \sim -0.85 \text{ GeV}^{-1}
  \end{align*}
  \]

  \[ 2\pi - 1\pi - \text{exchange} \]
  \[
  \begin{align*}
  \text{[diagram]} &= \text{[diagram]} + \text{[diagram]} + \text{[diagram]} + \text{[diagram]} + \text{[diagram]} + \ldots
  \end{align*}
  \]

  \[ \text{ring diagrams} \]
  \[
  \begin{align*}
  \text{[diagram]} &= \text{[diagram]} + \text{[diagram]} + \text{[diagram]} + \text{[diagram]} + \text{[diagram]} + \ldots
  \end{align*}
  \]

  \[ \text{contact-1}\pi - \text{exchange} \]
  \[
  \begin{align*}
  \text{[diagram]} &= \text{[diagram]} + \text{[diagram]} + \text{[diagram]} + \text{[diagram]} + \text{[diagram]} + \ldots
  \end{align*}
  \]

  \[ \text{contact-2}\pi - \text{exchange} \]
  \[
  \begin{align*}
  \text{[diagram]} &= \text{[diagram]} + \text{[diagram]} + \text{[diagram]} + \text{[diagram]} + \text{[diagram]} + \ldots
  \end{align*}
  \]
Theoretical uncertainties

Cutoff variation estimates errors due to neglected parts in $H(\Lambda)$

Radii of light nuclei approximately cutoff-independent, agree with exp.

Can provide lower limits on theoretical errors

goal: uncertainties of matrix elements needed in fundamental symmetry tests

isospin-symmetry breaking corrections $V_{ud} = 0.97416(13)\ (14/18)\text{theo.}$

neutrinoless double-beta decay

atomic EDMs ……

from A. Nogga
Impact on density dependences

Possibility of perturbative nuclear matter with RG-evolved interactions
nuclear matter converged at $\approx$ 2nd order
3N drives saturation

Nuclear matter with “bare” $N^3$LO

3N contributions not expected to be small for $N^3$LO interactions

nuclear matter from different $N^3$LO potentials
Neutron matter from NN and 3N

Uncertainties from $c_1$ overwhelm errors due to cutoff variation, mainly $c_3$ for neutron matter.

Combine with knowledge of basic nuclear properties important for dense matter in astrophysics.

Neutron star mergers $\rightarrow$ gravitational waves.

Different EOS models Oechslin, Janka (2007)
Three-nucleon interactions and nuclear structure

ab-initio calculations highlight the importance of 3N interactions

Navratil et al. (2007)

same $1^+$ vs. $3^+$ inversion in closed shell +3p+3n without 3N interactions

Nowacki @ Oslo 2008; Nowacki et al., in prep.

3N crucial for T=1 spin-orbit shell closures in $^{22}$O, $^{48}$Ca,…

see e.g., AS, Zuker (2006)
Location of the neutron drip line: Why so near in Oxygen?

Discovery of $^{40}$Mg and $^{42}$Al suggests neutron drip-line slant towards heavier isotopes

Neutron orbits in Oxygen isotopes

neutron $d_{3/2}$ - proton $d_{5/2}$ interaction pulls down $d_{3/2}$ neutrons in Fluorine

Why do $d_{5/2}$ neutrons not pull down $d_{3/2}$ in oxygen?
Monopole interaction and drip lines

Monopole part of nuclear forces \( \nu_{st}^T = \frac{\sum_J \nu_{stst}^J (2J + 1) [1 - (-)^{J+T} \delta_{st}]}{\sum_J (2J + 1) [1 - (-)^{J+T} \delta_{st}]} \)

determines interaction of s with t orbit \( \rightarrow \) change in \( d_{3/2} \) by \( N_{d5/2} \nu_m \)
\( \Rightarrow \) small changes in monopoles enhanced by number of neutrons

microscopic results based only on NN interactions require phenomenological repulsive contribution to \( T=1 \) monopoles

\( \rightarrow \) neutron \( d_{3/2} \) remains high, dripline at \( N=16 \) for Oxygen

first results indicate that \( \nu_{m,pheno} \) due to 3N interactions

Utsuno et al. (1999)
Towards 3N interactions in medium-mass nuclei
based on low-momentum $V_{\text{low } k}(\Lambda) + V_{3N}(\Lambda)$

Hagen et al. (2007) developed coupled-cluster theory with 3N interactions, first benchmark for $^4\text{He}$

Results show that 0-, 1- and 2-body parts of 3N interaction dominate

2-body part

occupied orbits

residual 3N interaction can be neglected
very promising and practical

extrapolated: $-28.23\text{MeV}$
exact, FY: $-28.20(5)\text{MeV}$
Monopole shifts and 3N interactions

0-, 1- and 2-body parts of 3N interaction dominate, supports that monopole shifts are due to 3N interactions cf. Zuker (2003)

shell model matrix elements for different cores probe 3N dependence monopoles from $V_{\text{low } k}(\Lambda) + 2\text{nd order (6hw)}$

cutoff independent in $T=1$

find repulsive contributions from 3N interactions

reproduces hierarchy for low orbits dominated by $c_i$ terms $\rightarrow \Delta$-hole contributions

Holt, Otsuka, AS, Suzuki, in prep.
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Summary

Exciting era with advances on many fronts

Exciting intersections with problems in many related areas

For the first time, approaches from light to heavy nuclei and for astrophysics based on the same interactions

Three-nucleon interactions play a central role