Three-Nucleon Interactions in Light Nuclei and Possible Approximations for Coupled Cluster Theory

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CANADA'S NATIONAL LABORATORY FOR PARTICLE AND NUCLEAR PHYSICS

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Outline

• Nuclear Observables Sensitive to 3NF in Light Nuclei
  ★ Electroweak transitions to the continuum
  ★ The Lorentz Integral Transform Method → Talk by Nir Barnea last week
  ★ Photo-absorption, Neutrino and Electron Scattering

• Possible 3NF approximations for Coupled Cluster Theory

• Conclusions and Outlook

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Winfried Leidemann
Giuseppina Orlandini Trento University, Italy
Achim Schwenk TRIUMF Canada
Three-Nucleon Forces in Light Nuclei

From meson exchange theory + phenomenological short range
2NF AV18
3NF UIX, IL2

Nuclear low energy spectra

Hadronic Reactions
n\(^{4}\)He \rightarrow n^{4}\)He

Pieper et al. (2002)

Nollett et al. (2007)
Three-Nucleon Forces in Light Nuclei

Nuclear low energy spectra

From chiral EFT $\rightarrow$ 3NF arise naturally

2NF $N^3$LO

3NF $N^2$LO

Navratil et al. (2007)

Quaglioni et al. (2008)
Electroweak Reactions on Light Nuclei

As a theoretical laboratory to test 3NF effects!!

Why electroweak reactions?

- The coupling constant $\ll 1$  
  \[ \sigma \propto \left| \langle \Psi_f | J^\mu | \Psi_0 \rangle \right|^2 \]

  Excellent tool to investigate properties of nuclei

Why light nuclei?

- In few-body physics one can perform exact calculations both for bound and scattering states

  For $A \leq 4$ the explicit evaluation of the FSWF is possible within the F-Yakubovski, only up to a three-body disintegration channel

  For higher energy one needs a method capable to treat the "far continuum"

  The Lorentz Integral Transform
The Lorentz Integral Transform

\[ R(\omega) = \sum_{f} \left| \langle \psi_f | \hat{O} | \psi_0 \rangle \right|^2 \delta(E_f - E_0 - \omega) \]

Efros, Leidemann, Orlandini, PLB 338 (1994) 130
The Lorentz Integral Transform

\[ R(\omega) = \sum_{f} \left| \langle \psi_f | \hat{O} | \psi_0 \rangle \right|^2 \delta(E_f - E_0 - \omega) \]

\[ L(\sigma, \Gamma) = \int d\omega \frac{R(\omega)}{(\omega - \sigma)^2 + \Gamma^2} \]
The Lorentz Integral Transform

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\[ = \langle \psi_0 | \hat{O} \frac{1}{H - E_0 - \sigma - i\Gamma} \frac{1}{H - E_0 - \sigma + i\Gamma} | \psi_0 \rangle = \langle \tilde{\psi} | \tilde{\psi} \rangle \]

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\[ = \left\langle \psi_0 | \hat{O} \frac{1}{H - E_0 - \sigma - i\Gamma} \frac{1}{H - E_0 - \sigma + i\Gamma} \hat{O} | \psi_0 \right\rangle = \left\langle \tilde{\psi} | \tilde{\psi} \right\rangle \]

\[ (H - E_0 - \sigma + i\Gamma) | \tilde{\psi} \rangle = \hat{O} | \psi_0 \rangle \]

- Due to imaginary part \( \Gamma \) the solution \( | \tilde{\psi} \rangle \) is unique
- If the r.h.s. is finite \( | \tilde{\psi} \rangle \) has bound state asymptotic behavior

Efros, Leidemann, Orlandini, PLB 338 (1994) 130
The Lorentz Integral Transform

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- Due to imaginary part \( \Gamma \) the solution \( \left| \tilde{\psi} \right\rangle \) is unique
- If the r.h.s. is finite \( \left| \tilde{\psi} \right\rangle \) has bound state asymptotic behavior

You can use any good bound state method to solve the LIT equation
The Lorentz Integral Transform

\[ R(\omega) = \sum_f f \left| \langle \psi_f | \hat{O} | \psi_0 \rangle \right|^2 \delta(E_f - E_0 - \omega) \]

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\[ = \left< \psi_0 | \hat{O} \frac{1}{H - E_0 - \sigma - i\Gamma} \frac{1}{H - E_0 - \sigma + i\Gamma} \hat{O} | \psi_0 \right> = \left< \tilde{\psi} | \tilde{\psi} \right> \]

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- Due to imaginary part \( \Gamma \) the solution \( | \tilde{\psi} \rangle \) is unique
- If the r.h.s. is finite \( | \tilde{\psi} \rangle \) has bound state asymptotic behavior

**CC-LIT equation**

\[ [\bar{H}, R(z)]|\phi_0 \rangle = (z - E_0)R(z)|\phi_0 \rangle + \bar{O}|\phi_0 \rangle \]

\[ z = E_0 + \sigma + i\Gamma \]
How to solve the LIT equation

What we do:

\[(H - E_0 - \sigma + i\Gamma) |\tilde{\psi}\rangle = \hat{O} |\psi_0\rangle\]

- Bound-state method to expand \( |\psi_0\rangle, |\tilde{\psi}\rangle \) in terms of a complete set of basis state

Hyper-spherical Harmonics

\[\rho, \Omega = (\theta, \phi, \vartheta_1, \varphi_1, \vartheta_2, \varphi_2, \vartheta_3, \varphi_3)\]

\[\rho^2 = \sum_i r_i^2\]

\[\Psi = \sum_{[K], \nu}^{K_{\text{max}}, \nu_{\text{max}}} c_{\nu}^{[K]} e^{-\rho/2} \rho^{n/2} L_{\nu}^{n}(\rho) [\mathcal{Y}_\nu^{\mu}(\Omega)] \chi_{ST}^{\mu} a_{JT}\]

\[\Psi \sim e^{-\alpha \rho} \quad \rho \to \infty\]
How to solve the LIT equation

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\[\Psi \sim e^{-a\rho} \quad \rho \to \infty\]

- For fixed \(\Gamma\) and different \(\sigma\) solve the generalized eigenvalue problem to get the solution for \(\langle \tilde{\psi} | \tilde{\psi} \rangle\)
How to solve the LIT equation

What we do: \[(H - E_0 - \sigma + i\Gamma) | \tilde{\psi} \rangle = \hat{O} | \psi_0 \rangle\]

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Hyper-spherical Harmonics

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\rho, \Omega = (\theta, \phi, \vartheta_1, \varphi_1, \vartheta_2, \varphi_2, \vartheta_3, \varphi_3)
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\rho^2 = \sum_i r_i^2
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- For fixed \(\Gamma\) and different \(\sigma\) solve the generalized eigenvalue problem to get the solution for \(\langle \tilde{\psi} | \tilde{\psi} \rangle\)

- Numerical inversion of the transform \(L(\sigma) \leftrightarrow R(\omega)\)
  - Best fit method for \(c_n\)

\[
L(\sigma) = \sum_n c_n \tilde{\chi}_n^\alpha(\sigma) \quad \rightarrow \quad R(\omega) = \sum_n c_n \chi_n^\alpha(\omega)
\]
How to solve the LIT equation

What we do: \[(H - E_0 - \sigma + i\Gamma) \mid \tilde{\psi} \rangle = \hat{O} \mid \psi_0 \rangle\]

- Bound-state method to expand \(\mid \psi_0 \rangle, \mid \tilde{\psi} \rangle\) in terms of a complete set of basis state

\[\rho, \Omega = (\theta, \phi, \vartheta_1, \varphi_1, \theta_2, \varphi_2, \theta_3, \varphi_3)\]
\[\rho^2 = \sum_i r_i^2\]

Hyper-spherical Harmonics

\[\Psi = \sum_{[K, \nu]} c^{[K]}_{\nu} e^{-\rho/2} \rho^{n/2} L_{n}^{m}(\rho) [Y_{K}^{\mu}(\Omega) \chi_{ST}^{\alpha}]_{JT}\]
\[\Psi \sim e^{-\alpha \rho} \quad \rho \to \infty\]

- For fixed \(\Gamma\) and different \(\sigma\) solve the generalized eigenvalue problem to get the solution for \(\langle \tilde{\psi} \mid \tilde{\psi} \rangle\)

- Numerical inversion of the transform \(L(\sigma) \leftrightarrow R(\omega)\)
  - Best fit method for \(c_n\)
  \[L(\sigma) = \sum_n c_n \tilde{\chi}_n^{\alpha}(\sigma) \quad \quad R(\omega) = \sum_n c_n \chi_n^{\alpha}(\omega)\]

- Redo for different \(\Gamma\)
How to solve the LIT equation

What we do: \[ (H - E_0 - \sigma + i\Gamma) | \tilde{\psi} \rangle = \hat{O} | \psi_0 \rangle \]

- Bound-state method to expand \(|\psi_0\rangle, |\tilde{\psi}\rangle\) in terms of a complete set of basis state

Hyper-spherical Harmonics

\[ \Psi = \sum_{[K,\nu]}^{K_{\text{max}},\nu_{\text{max}}} c_{\nu}^{[K]} e^{-\rho/2} \rho^{n/2} L_{\nu}^n(\rho) [\mathcal{Y}_r^{\mu}(\Omega)] \tilde{\chi}_{ST}^{\alpha} \]

\[ \Psi \sim e^{-a\rho} \quad \rho \rightarrow \infty \]

- For fixed \(\Gamma\) and different \(\sigma\) solve the generalized eigenvalue problem to get the solution for \(\langle \tilde{\psi} | \tilde{\psi} \rangle\)

- Numerical inversion of the transform \(L(\sigma) \longleftrightarrow R(\omega)\)
  - Best fit method for \(c_n\)

\[ L(\sigma) = \sum_n c_n \tilde{\chi}_n^{\alpha}(\sigma) \quad R(\omega) = \sum_n c_n \chi_n^{\alpha}(\omega) \]

- Redo for different \(\Gamma\) Obtain a response function independent on \(\Gamma\) which includes the full final state interaction
Photo-absorption reaction

Inclusive cross section $\gamma A \rightarrow X$ \hspace{1cm} $\sigma_\gamma(\omega) = 4\pi^2 \alpha \omega R^{E_1}(\omega)$

\[ \begin{align*}
\sigma_\gamma(\omega) &= 4\pi^2 \alpha \omega R^{E_1}(\omega) \\
\text{AV18+UIX} &\quad \text{Gazit, S.B. et al. (2006)}
\end{align*} \]
Photo-absorption reaction

Inclusive cross section $\gamma A \rightarrow X$  

$$\sigma_\gamma(\omega) = 4\pi^2\alpha\omega R^{E1}(\omega)$$

$EFT$ NN+NNN

AV18+UIX  Gazit, S.B. et al. (2006)
AV18      Quaglioni and Navratil (2007)
Photo-absorption reaction

Inclusive cross section \( \gamma A \rightarrow X \quad \sigma_\gamma(\omega) = 4\pi^2 \alpha \omega R^{E1}(\omega) \)

\[
\begin{align*}
\text{Tagged photons} & \\
\text{shaded area} & \\
\text{Bermann ('80) + Feldman ('90)} & \\
\text{box} & \text{Wells et al. ('92)} \\
\text{Nilsson et al. ('05)} & \\
\text{Shima et al. ('05)} & \\
\end{align*}
\]

\( 4\text{He} \)

AV18+UIX Gazit, S.B. et al. (2006)
Chiral EFT Quaglioni and Navratil (2007)
Photo-absorption reaction

Inclusive cross section \( \gamma A \rightarrow X \)

\[
\sigma_\gamma(\omega) = 4\pi^2\alpha\omega R^{E1}(\omega)
\]

\( \omega \) [MeV]

\( \alpha_\gamma(\omega) \) [mb]

Soft-dipole Resonance

Giant Dipole Resonance

neutron halo \( \alpha \)-core

neutrons protons

AV4' potential

S.B. et al. (2004)

\( ^6\text{He} \)
Photo-absorption reaction

Inclusive cross section \( \gamma A \rightarrow X \) 
\[ \sigma_\gamma(\omega) = 4\pi^2 \alpha \omega R^{E1}(\omega) \]

AV4' potential

\( \alpha(\omega) \) [mb]

\( \omega \) [MeV]

\( ^6\text{He} \)

Soft-dipole Resonance

Giant Dipole Resonance

neutron halo \( \rightarrow \) \( \alpha \)-core

neutrons \( \rightarrow \) protons

S.B. et al. (2004)
Photo-absorption reaction

Inclusive cross section $\gamma A \rightarrow X$

$$\sigma_\gamma(\omega) = 4\pi^2 \alpha \omega R^{E_1}(\omega)$$

Diagram showing the AV4' potential with peaks labeled as Soft-dipole Resonance and Giant Dipole Resonance.

The graph shows the cross section $\sigma_\gamma(\omega)$ as a function of energy $\omega$ for the 6He nucleus. The peaks correspond to neutron halo, $\alpha$-core, and neutrons and protons configurations.

S.B. et al. (2004)
Photo-absorption reaction

Inclusive cross section $\gamma A \rightarrow X$

$\sigma_\gamma(\omega) = 4\pi^2\alpha\omega R^{E1}(\omega)$

$\omega$ [MeV]

$\alpha_\gamma(\omega)$ [mb]

6He

Soft-dipole Resonance

Giant Dipole Resonance

What happens if one uses realistic 2NF and 3NF?
Neutrino scattering reaction

\[ ^4He(\nu, \nu')X \] important in a SN environment

A microscopic calculation of the cross section, based on modern Hamiltonian is possible with the LIT!
Neutrino scattering reaction

$^4\text{He}(\nu, \nu')X$ important in a SN environment

A microscopic calculation of the cross section, based on modern Hamiltonian is possible with the LIT!

Sensitivity to the nuclear Hamiltonian

\[
\frac{1}{q^2} R_{E2}(\omega)
\]

Gazit, Barnea (2007)
Neutrino scattering reaction

$^4\text{He}(\nu, \nu')X$ important in a SN environment

A microscopic calculation of the cross section, based on modern Hamiltonian is possible with the LIT!

![Diagram](image)

Temperature averaged $\langle \sigma \rangle_T$ neutral cross section

<table>
<thead>
<tr>
<th>$T$ [MeV]</th>
<th>AV18</th>
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<tbody>
<tr>
<td>4</td>
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Sensitivity to the nuclear Hamiltonian

$\frac{1}{q^2} R_{E2}(\omega)$

Gazit, Barnea (2007)
Neutrino scattering reaction

$^4\text{He}(\nu, \nu')X$ important in a SN environment

A microscopic calculation of the cross section, based on modern Hamiltonian is possible with the LIT!

![Diagram of neutrino scattering](image)

Effect of 3NF shows up in neutrino scattering!

**Temperature averaged $\langle \sigma \rangle_T$ neutral cross section**

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Sensitivity to the nuclear Hamiltonian

$\frac{1}{q^2} R_{E2}(\omega)$

Gazit, Barnea (2007)
Electron scattering reaction

Virtual Photon

\[(\omega, \mathbf{q})\]

can vary independently

Inclusive cross section \(A(e,e')X\)

\[
\frac{d^2 \sigma}{d\Omega d\omega} = \sigma_M \left[ \frac{Q^4}{q^4} R_L(\omega, \mathbf{q}) + \left( \frac{Q^2}{2q^2} + \tan^2 \frac{\theta}{2} \right) R_T(\omega, \mathbf{q}) \right]
\]

with \(Q^2 = -q^2 = q^2 - \omega^2\) and \(\theta\) scattering angle

and \(\sigma_M\) Mott cross section
**Electron scattering reaction**

![Diagram of electron scattering reaction](image)

**Virtual Photon**

$$(\omega, \mathbf{q})$$

can vary independently

**Inclusive cross section** \(A(e,e')X\)

$$\frac{d^2 \sigma}{d\Omega d\omega} = \sigma_M \left[ \frac{Q^4}{q^4} R_L(\omega, \mathbf{q}) + \left( \frac{Q^2}{2q^2} + \tan^2 \frac{\theta}{2} \right) R_T(\omega, \mathbf{q}) \right]$$

with \(Q^2 = -q^2 = q^2 - \omega^2\) and \(\theta\) scattering angle

and \(\sigma_M\) Mott cross section
Comparison with experiment

Calculation of $R_L(\omega, q)$ with the LIT/EIHH method

Medium-$q$ kinematics

- The comparison with experiment improves with addition of 3NF
- 3NF mainly in quasi-elastic peak region
Calculation of $R_L(\omega, q)$ with the LIT/EIHH method

Low-q kinematics

- This observable is very sensitive to 3NF
- Quest for new precise measurements! Proposal in MAMI@Mainz
Nuclear Structure

We want to investigate nuclei with $A>4$ with $HH$

1. Use smooth cutoff low-momentum interactions $V_{\text{lowk}}$
   NN evolved from chiral N3LO

2. Find out some approximation to include 3NF effects in an easier way to reduce the computational cost

$\text{3}\rightarrow\text{2N}$ relative coordinate and coupled scheme
Nuclear Structure

We want to investigate nuclei with $A>4$ with $HH$

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   $3\rightarrow2N$ relative coordinate and coupled scheme

Could be used in spherical CC!
Nuclear Structure

We want to investigate nuclei with $A>4$ with $\text{HH}$

1. Use smooth cutoff low-momentum interactions $V_{\text{lowk}}$
   NN evolved from chiral N3LO

2. Find out some approximation to include 3NF effects in an easier way to reduce the computational cost

3$\rightarrow$2N relative coordinate and coupled scheme

Could be used in spherical CC!

Idea triggered by CC results of normal ordered Hamiltonian in $m$-scheme

Hagen et al. (2007)
Nuclear Structure

1. Use $V_{\text{lowk}}$ evolved from chiral N3LO

$$\hat{V} = \sum_{n, n'}^{n_{\text{max}}, \ell_{\text{max}}} |n(\ell s)j\rangle v^{j, h\Omega}_{n n' \ell \ell'} \langle n'(\ell' s')j| \text{ with } v^{j, h\Omega}_{n n' \ell \ell'} = \langle n(\ell s)j|\hat{V}|n'(\ell' s')j\rangle$$
Nuclear Structure

1. Use $V_{\text{lowk}}$ evolved from chiral N3LO

$$\hat{V} = \sum_{nn', \ell\ell'} |n(\ell s)j\rangle v_{nn'\ell\ell'}^{j,\hbar\Omega} \langle n'(\ell's')j|$$

with $v_{nn'\ell\ell'}^{j,\hbar\Omega} = \langle n(\ell s)j|\hat{V}|n'(\ell's')j\rangle$

- Expansion of Hilbert space size $\rightarrow K_{\text{max}}$
- Expansion of the potential $\rightarrow n_{\text{max}}, \ell_{\text{max}} \rightarrow \hbar\Omega$
Nuclear Structure

1. Use $V_{\text{low } k}$ evolved from chiral N3LO

\[ \hat{V} = \sum_{n_{\text{max}}, \ell_{\text{max}}} |n(\ell s)j\rangle v^{i,h\Omega}_{n n' \ell \ell'} \langle n'(\ell' s')j| \text{ with } v^{i,h\Omega}_{n n' \ell \ell'} = \langle n(\ell s)j|\hat{V}|n'(\ell' s')j\rangle \]

- Expansion of Hilbert space size $\rightarrow K_{\text{max}}$
- Expansion of the potential $\rightarrow n_{\text{max}}, \ell_{\text{max}} \rightarrow h\Omega$

\[ E_0 \text{[MeV]} \]

\begin{tabular}{l|ccc}
\hline
 & F-FY* & HH & CC$^\dagger$ \\
\hline
$^3$H & -8.40(1) & -8.41(2) & - \\
\hline
\end{tabular}

* from A. Nogga
† form G. Hagen

Perfect agreement!
Nuclear Structure

1. Use \( V_{\text{low}k} \) evolved from chiral N3LO

\[
\hat{V} = \sum_{n_{\text{max}}, \ell_{\text{max}}} |n(\ell s)j\rangle v_{n_{\text{max}}, \ell_{\text{max}}}^{j, h\Omega} \langle n'(\ell' s')j| \quad \text{with} \quad v_{n_{\text{max}}, \ell_{\text{max}}}^{j, h\Omega} = \langle n(\ell s)j| \hat{V} |n'(\ell' s')j\rangle
\]

- Expansion of Hilbert space size \( \rightarrow K_{\text{max}} \)
- Expansion of the potential \( \rightarrow n_{\text{max}}, \ell_{\text{max}} \rightarrow \hbar\Omega \)

Perfect convergence: no \( \hbar\Omega \) dependence!
Nuclear Structure

2. Find out some approximation to include 3NF effects in an easier way $3 \rightarrow 2N$
Nuclear Structure

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2. Find out some approximation to include 3NF effects in an easier way \(3\rightarrow2N\)

\[
|\alpha\rangle = n_{\eta_1} n_{\eta_2} \left( (\ell_{\eta_1} s) j_{\eta_1} \left( \ell_{\eta_2} \frac{1}{2} \right) j_{\eta_2} \right) JM \leftrightarrow |(12)_{\eta_1 q_{\eta_2}}\rangle = n_{\eta_1} (\ell_{\eta_1} s) j_{\eta_1} m_{\eta_1} \bigg| n_{\eta_2} \left( \ell_{\eta_2} \frac{1}{2} \right) j_{\eta_2} m_{\eta_2} \bigg\rangle
\]
Nuclear Structure

2. Find out some approximation to include 3NF effects in an easier way $3 \rightarrow 2N$

$$|\alpha\rangle = n_{\eta_1} n_{\eta_2} \left( (\ell_{\eta_1} s_{\eta_1} \frac{1}{2}) j_{\eta_1} (\ell_{\eta_2} \frac{1}{2}) j_{\eta_2} \right) JM \leftrightarrow |(12)_{\eta_1 q_{\eta_2}}\rangle = n_{\eta_1} (\ell_{\eta_1} s_{\eta_1} j_{\eta_1} m_{\eta_1}) n_{\eta_2} (\ell_{\eta_2} \frac{1}{2}) j_{\eta_2} m_{\eta_2}$$

$$\langle \alpha | V^{3N} | \beta \rangle \leftrightarrow \langle (12)_{\eta_1 q_{\eta_2}} | V^{3N} | (12)'_{\eta_1 q'_{\eta_2}} \rangle$$
2. Find out some approximation to include 3NF effects in an easier way \(3 \rightarrow 2N\)

\[
|\alpha\rangle = \left| n_{\eta_1} n_{\eta_2} \left( (\ell_{\eta_1} s) j_{\eta_1} \left( \ell_{\eta_2} \frac{1}{2} \right) j_{\eta_2} \right) J M \right\rangle \quad \rightarrow \quad |(12)_{\eta_1} q_{\eta_2}\rangle = \left| n_{\eta_1} (\ell_{\eta_1} s) j_{\eta_1} m_{\eta_1}\right\rangle \left| n_{\eta_2} \left( \ell_{\eta_2} \frac{1}{2} \right) j_{\eta_2} m_{\eta_2}\right\rangle
\]

\[
\langle \alpha | V^{3N} | \beta \rangle \quad \rightarrow \quad \langle (12)_{\eta_1} q_{\eta_2} | V^{3N} | (12)'_{\eta_1} q'_{\eta_2} \rangle
\]

\[
\sum_{q_{\eta_2}} \langle (12)_{\eta_1} q_{\eta_2} | V^{3N} | (12)'_{\eta_1} q_{\eta_2} \rangle \propto \langle (12)_{\eta_1} | V^{3-2N} | (12)'_{\eta_1} \rangle
\]

\[
\langle n_{\eta_1} (\ell_{\eta_1} s) j_{\eta_1} | V^{3-2N} | n'_{\eta_1} (\ell'_{\eta_1} s) j_{\eta_1} \rangle \quad \text{effective two-body force}
\]
2. Find out some approximation to include 3NF effects in an easier way \(3 \rightarrow 2N\)

\[
|\alpha\rangle = n_{\eta_1} n_{\eta_2} \left( (\ell_{\eta_1} s) j_{\eta_1} \left( \ell_{\eta_2} \frac{1}{2} \right) j_{\eta_2} \right) JM \longleftrightarrow |(12)_{\eta_1} q_{\eta_2}\rangle = n_{\eta_1} (\ell_{\eta_1} s) j_{\eta_1} m_{\eta_1}\rangle \left| n_{\eta_2} \left( \ell_{\eta_2} \frac{1}{2} \right) j_{\eta_2} m_{\eta_2}\right\rangle
\]

\[
\langle \alpha | V^{3N} | \beta \rangle \longleftrightarrow \langle (12)_{\eta_1} q_{\eta_2} | V^{3N} | (12)_{\eta_1} q_{\eta_2}\rangle
\]

\[
\sum_{q_{\eta_2}} \langle (12)_{\eta_1} q_{\eta_2} | V^{3N} | (12)'_{\eta_1} q_{\eta_2}\rangle \propto \langle (12)_{\eta_1} | V^{3->N} | (12)_{\eta_1}' \rangle
\]

\[
\langle n_{\eta_1} (\ell_{\eta_1} s) j_{\eta_1} V^{3->2N} | n'_{\eta_1} (\ell'_{\eta_1} s) j_{\eta_1} \rangle \text{ effective two-body force}
\]

\[
\sum_{q_{\eta_2}} \langle (12)_{\eta_1} q_{\eta_2} | V^{3N} | (12)'_{\eta_1} q_{\eta_2}\rangle
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\[
\sum_{q_{\eta_2}} \langle (12)_{\eta_1} | V^{3->N} | (12)_{\eta_1}' \rangle
\]

\[
\langle n_{\eta_1} (\ell_{\eta_1} s) j_{\eta_1} V^{3->2N} | n'_{\eta_1} (\ell'_{\eta_1} s) j_{\eta_1} \rangle
\]
Nuclear Structure

Test the $3\to2N$ on $A=4$

→ NN: $V_{\text{lowk}}$ evolved from 2N chiral N3LO

3N: chiral 3N N2LO with $c_D$, $c_E$ fitted to $A=3,4$

→ $3\to2NF$:

\[ V_{\text{lowk}} + 3\to2N \ J=1/2^+ \ T=1/2 \]

\[ h\Omega = 14 \text{ MeV} \]

\[ \begin{array}{|c|c|c|c|}
 \hline
 \text{[MeV]} & \text{F-FY*} & \text{HH} & \text{CC†} \\
 \hline
 \hline
 \end{array} \]

* from A. Nogga
† from G. Hagen

Good, but not perfect agreement!
Nuclear Structure

Test the $3\rightarrow2N$ on $A=4$

→ NN: $V_{\text{lowk}}$ evolved from 2N chiral N3LO

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→ $3\rightarrow2NF$:

\[\begin{align*}
V_{\text{low k}} & \quad \Lambda=2.0 \text{ fm}^{-1} \\
& + \\
3\rightarrow2N J=1/2^+ T=1/2
\end{align*}\]

\[\begin{align*}
\hbar\Omega \text{ dependence!}
\end{align*}\]
Nuclear Structure

Test the $3\rightarrow2N$ on $A=4$

$\rightarrow$ NN: $V_{\text{low k}}$ evolved from 2N chiral N3LO

3N: chiral 3N N2LO with $c_D$ $c_E$ fitted to $A=3,4$

$\rightarrow$ 3$\rightarrow$2NF:

$V_{\text{low k}}$ $\Lambda=2.0$ fm$^{-1}$

$3\rightarrow2N$ $J=1/2^+$ $T=1/2$

$h\Omega$ dependence!

- $3\rightarrow2N$ is too repulsive for large $h\Omega$
  when the HO density is larger

\begin{itemize}
  \item $h\Omega$ dependence!
  \item $3\rightarrow2N$ is too repulsive for large $h\Omega$
  when the HO density is larger
\end{itemize}
Nuclear Structure

Test the $3\rightarrow2N$ on $A=4$

- **NN:** $V_{\text{low } k}$ evolved from 2N chiral N3LO
- **3N:** chiral 3N N2LO with $c_D$, $c_E$ fitted to $A=3,4$

- **3\rightarrow2NF:**

\[ V_{\text{low } k} \Lambda=2.0 \text{ fm}^{-1} \]
\[ 3\rightarrow2N J=1/2^+ T=1/2 \]

$\hbar\Omega$ dependence!

- $3\rightarrow2N$ is too repulsive for large $\hbar\Omega$ when the HO density is larger
- For small $\hbar\Omega$, $3\rightarrow2N$ coincides with 2N only

$4\text{He}$
Test the $3\to2N$ on $A=4$

$\rightarrow$ NN: $V_{\text{lowk}}$ evolved from 2N chiral N3LO

$3N$: chiral 3N N2LO with $c_D$ $c_E$ fitted to $A=3,4$

$\rightarrow$ $3\to2\text{NF}$:

Understand the dependence from 3NF scaling

With a simple gaussian 3NF force $\langle V^{3N} \rangle \sim (\hbar \Omega)^3$
Test the 3→2N on \( A=4 \)

- **NN:** \( V_{\text{lowk}} \) evolved from 2N chiral N3LO
- **3N:** chiral 3N N2LO with \( c_D, c_E \) fitted to \( A=3,4 \)
- **3→2NF:**

Understand the dependence from 3NF scaling

With a simple gaussian 3NF force \( \langle V^{3N} \rangle \sim (\hbar \Omega)^3 \)

Need to improve on the \( \hbar \Omega \)-dependence to get a flattening over a physically motivated range of \( \hbar \Omega \)
Nuclear Structure

Test the 3→2N on A=4

→ NN: \( V_{\text{lowk}} \) evolved from 2N chiral N3LO

3N: chiral 3N N2LO with \( c_d \) \( c_E \) fitted to A=3,4

→ 3→2NF:

Understand the dependence from 3NF scaling
With a simple gaussian 3NF force \( \langle V^{3N} \rangle \sim (\hbar \Omega)^3 \)

\[
\begin{align*}
\text{linear fit: } y &= ax^3 + bx^2 - E_0(2NF) \\
\text{non linear fit: } y &= ax^b + c \\
& \quad \text{b~3.2}
\end{align*}
\]

Need to improve on the \( \hbar \Omega \)-dependence to get a flattening over a physically motivated range of \( \hbar \Omega \)
Structure of halo nuclei

Lightest halo system: $^{6}\text{He}$

Investigate it with realistic forces + 3NF with a w.f. with “good” fall off at large $r$

$e^{-\alpha \rho} \quad \rho \rightarrow \infty$

Hyper-spherical Harmonics

$^{6}\text{He}\quad$ PRELIMINARY

$V_{\text{low } k}$

$V_{\text{low } k} + 3 \rightarrow 2\text{N}$

$J=1/2^+ \quad T=1/2$

$^{4}\text{He-core} \quad \hbar \Omega = 14 \text{ MeV}$

EXP

$E_0 \text{ [MeV]}$

$K_{\text{max}}$

$0 \quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \quad 12 \quad 14$
Structure of halo nuclei

Lightest halo system: $^6$He

Investigate it with realistic forces + 3NF with a w.f. with "good" fall off at large $r$

$e^{-\alpha \rho}$ \quad \rho \rightarrow \infty \quad \text{Hyper-spherical Harmonics}$

$^6$He

$V_{\text{low } k}$ $^3$->$^2$N

$J=1/2^+ \quad T=1/2$

$^4$He-core $\hbar \Omega = 14 \text{ MeV}$

Still unsatisfactory convergence

Add other 3NF partial waves

Investigate $\hbar \Omega$ dependence
**Conclusion and Outlook**

- Three-body forces are fundamental to describe nuclear physics.

- The LIT is a very powerful method to an exact study of perturbation induced reactions of few-body systems, and can possibly be extended to heavier systems if used in conjunction with the CC theory.

- Need a systematic improvement of the 3→2N approximation for CC theory of medium-mass nuclei.

**Future:** CC + LIT + 3→2NF

- Extend the ab initio treatment of inelastic reactions to heavier systems.
- Shed more light on role of 3NF.
- Have new intersection theory-experiment.