Tuning the heavy-light guitar: Notes on holographic $\mathcal{N} = 2$ mesons

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INT, April 10, 2008

Outline

Heavy quarks in QCD
  Preliminaries
  Heavy quark limit of QCD

Meson spectroscopy in strongly coupled $\mathcal{N} = 2$ SYM
  Basics of AdS/CFT
  Meson spectra: Preliminaries
  Heavy-light puzzle

Holographic heavy-light mesons
  Fluctuations: Preliminaries
  Fluctuation spectra
  Rotating strings: Preliminaries
  Rotating strings: Results

Conclusions
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**Conclusions**
Hadron spectrum of QCD

- Fundamental challenge in QCD: Understand and predict mass spectrum of bound states (hadrons and glueballs)
  - Lattice QCD provides accurate numerical results — excellent agreement with experiments
  - Analytic results difficult to obtain due to non-perturbative nature of problem (1M$ for derivation of mass gap)
Heavy quarks in QCD

- Simplifications for heavy quark systems ($m_h \gg \Lambda_{QCD}$)
  - Asymptotic freedom $\Rightarrow$ Heavy quark physics perturbative
  - Separate conservation of heavy and light quark angular momenta $\Rightarrow$ Interactions of heavy and light dof’s independent of heavy quark spin and flavor
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- Heavy-heavy meson: Non-relativistic, perturbative positronium-like system
  - Balancing between potential and kinetic energies of two point-like particles
  - Some sensitivity to details of confinement: Interpolating "Cornell" potential
Heavy quarks in QCD

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- Heavy-light meson: Small object of size $1/m_h$ surrounded by brown muck of size $1/\Lambda_{QCD}$
  - Heavy quark almost at rest; motion suppressed by $\Lambda_{QCD}/m_h$
  - Surrounded by light (virtual) quarks and gluons, blind to heavy quark spin and flavor
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- Formal approach Heavy Quark Effective Theory (HQET)
  - Effective theory based on $1/m_h$ expansion
  - Predictions: $m_h$ dependence of hadron spectrum and weak decay amplitudes, e.g.
    \[
    m_{HL} = m_Q + \bar{\Lambda} + \frac{\Delta m^2}{2m_Q}
    \]
  - Fine splitting from light quark quantum numbers, hyperfine from heavy quark spin
  - Experimentally $m_{B_s} - m_B \approx m_{D_s} - m_D \approx 100$ MeV
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- Physical motivation for heavy quark physics:
  - Flavor changing weak decays: CKM matrix elements
  - Heavy quark masses and meson decay constants
Setting up goals

- Ultimate dream: First principles analytic calculation of masses and lifetimes of different hadrons in QCD
  - Implies solving confinement
- More realistic goal: Increase qualitative understanding of confinement and other non-perturbative physics
- Novel approach: Use AdS/QCD and answer questions through dual gravitational model
- One step back: Address the problem in a theory with known gravity dual, $\mathcal{N}=4$ SYM with $N_f$ massive $\mathcal{N}=2$ hypermultiplets
  - Direct access to strongly coupled non-Abelian gauge theory
  - Important difference: $\mathcal{N}=2$ essentially conformal, $\Lambda_{QCD}$ replaced by lightest $m_q$
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$\mathcal{N} = 4$ SYM and gauge/gravity duality

- Special feature of 4d $\mathcal{N} = 4$ SYM: Known string dual
  - $\mathcal{N} = 4$ SYM $\Leftrightarrow$ IIB string theory on $AdS_5 \times S_5$
  
  \[ ds^2 = L^2 \left[ u^2 \eta_{\mu\nu} dx^\mu dx^\nu + \frac{\delta_{ij} dy^i dy^j}{u^2} \right] \]
  
  - $L$ = (curvature) radius of $S_5$ and $AdS_5$, $u^2 \equiv \sum_{i=1}^{6} (y^i)^2$
  - Parameters string coupling $g_s$ and length scale $\ell_s = \sqrt{\alpha'}$

- AdS/CFT dictionary:
  - Radial coordinate in AdS $\sim$ energy scale in CFT
  - $(L/\ell_s)^4 = \lambda$, $g_s = \lambda/(4\pi N_c)$

- Beautiful aspect:
  - Classical sugra limit: string coupling $g_s \ll 1$ and $L/\ell_s \gg 1$
  - Gauge theory at $\lambda \gg 1$, $N_c \to \infty \Leftrightarrow$ Classical GR!
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Fundamental flavors in AdS/CFT

- Karch, Katz: Fundamental $\mathcal{N} = 2$ hypermultiplet in field theory equivalent to adding a D7 brane on gravity side
  - Quarks $\leftrightarrow$ Strings hanging from D7 brane
  - Mesons $\leftrightarrow$ Strings with both endpoints on D7 branes
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- Dynamics of D7 brane governed by DBI action:
  \[
  S_{DBI} = -\tau_7 L^8 \int d^4\xi \sqrt{\det \left( \frac{\partial y}{\partial \xi^a} \cdot \frac{\partial y}{\partial \xi^b} \right)}
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  - D7 spans $\mathbb{R}^{3,1}$ and four $y^i$ directions
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  - Usual choice $y^5 = c_i$, $y^6 = 0$
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  - Large $N_c$ counting: SUGRA $\gg$ DBI $\gg$ Nambu-Goto
Meson mass spectra

- Meson masses dual to fluctuation and rotation energies of strings hanging from D7 branes
Meson mass spectra

- Meson masses dual to fluctuation and rotation energies of strings hanging from D7 branes
- String may fluctuate in directions parallel and perpendicular to the D7 brane

\[
ds^2 = L^2 \left[ (\rho^2 + y_5^2 + y_6^2) \left\{ -dt^2 + dr^2 + r^2 d\phi^2 + dx^2 \right\} \\
+ \frac{d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2 \theta d\Omega_2^2 + dy_5^2 + dy_6^2}{\rho^2 + y_5^2 + y_6^2} \right]
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Meson mass spectra

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- String may rotate in real (= 4d spin) and internal (= finite R charge) space

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Meson mass spectra

- Meson masses dual to fluctuation and rotation energies of strings hanging from D7 branes
- Equal quark masses, small spin: Open string excitations of D7 brane (Kruczenski et al. hep-th/0304032)
  - Analytic spectrum $M_M \sim \frac{m_Q}{\sqrt{\lambda}}$
  - Spectrum depends on R charge and radial ($\rho$) excitation quantum numbers
Meson mass spectra

- Meson masses dual to fluctuation and rotation energies of strings hanging from D7 branes

- Equal quark masses, large 4d spin $J$: Classical solutions to string (Nambu-Goto) equations of motion for strings rotating in real space

$$S = -\frac{L^2}{2\pi\alpha'} \int dt \, d\sigma \frac{1}{z^2} \sqrt{(1 - \Omega^2 r^2)((z')^2 + (r')^2)}$$

- In the limit of large $J$, solutions U-shaped
- Energy in terms of $J$ interpolates between Regge behavior at small $J$ and the energy of two non-relativistic particles in Coulomb potential
Meson mass spectra

\[ \omega \rightarrow 0 \]

- \( z = 0 \)
- \( \tilde{z} = 0 \)
- Boundary
- \( z = 1 \)
- \( \tilde{z} = \infty \)
- D7 brane

- \( \rho_0 \sim \omega^{-2/3} \)
- \( \tilde{\rho}_0 \sim \omega^{1/3} \)
- \( \tilde{z}_0 \sim \omega^{-2/3} \)
- \( \tilde{\omega}_0 \sim \omega^{1/3} \)
Meson mass spectra
What about non-equal masses?

- To explore heavy quark physics in strongly coupled $\mathcal{N} = 2$, want spectrum of heavy-light mesons, $m_h \gg m_l$
  - Preserving $\mathcal{N} = 2$ SUSY, ground state meson mass must satisfy $m_h - m_l < m_{HL} < m_h + m_l$
- Question: Is it possible to find fine and hyperfine structure from fluctuation spectra?
- Apparent answer: No (Erdmenger et al. hep-th/0605241)
  - Energy splitting proportional to $m_h$!
  - Could this be due to rigid string approximation?
- Rest of the talk: Finding the complete HL mass spectrum
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Conclusions
String fluctuations

- Starting point: Choose $\sigma = y_5 \equiv y$ and consider
  
  $$m_l = \frac{L^2}{2\pi\alpha'} y_l \ll \frac{L^2}{2\pi\alpha'} y_h = m_h$$

- Solve for fluctuations in $x$, $\rho$ and $y_6$ around $x = y_6 = 0$, $\rho = \rho_0$, using quadratic Nambu-Goto action

- Integrate $\pi^0_t$ over $\sigma$ to obtain
  
  $$E = m_h - m_l + \sum_{w,n} N_{w}^{n} \omega_n^w$$

- Valid for $N_w^n \ll \sqrt{\lambda}$
String fluctuations

- Starting point: Choose $\sigma = y_5 \equiv y$ and consider
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Aleksi Vuorinen, CERN  Tuning the heavy-light guitar
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**x and y\textsubscript{6} fluctuations**

- Defining \( q \equiv \rho_0 L^2 / (2\pi \alpha') \) and using \( L^2 / \alpha' = \sqrt{\lambda} \), we obtain analytically

\[
E_n^{x} = E_n^{y} = m_h - m_l + \frac{2\pi q}{\sqrt{\lambda}} \sqrt{\frac{n^2 \pi^2}{(\arctan[q/m_l] - \arctan[q/m_h])^2}} - 1
\]

- For \( x \) fluctuations, in addition a zero mode

- Above function monotonously increasing in \( q \Rightarrow \) For non-spinning, fluctuating string, set \( q = 0 \)

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E_n^{x} = E_n^{y} = m_h - m_l + \frac{m_h m_l}{m_h - m_l} \frac{2\pi^2 n}{\sqrt{\lambda}}
\]

- For \( m_h \gg m_l \), factor \( 1/(1 - m_l/m_h) \) leads to heavy mass independence & fine structure in energy splittings!

- Very natural: Compare to guitar string
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\( \delta \rho \) fluctuations

- Equation of motion not analytically solvable \( \Rightarrow \) Must resort to numerics
\( \delta \rho \) fluctuations

- Equation of motion not analytically solvable \( \Rightarrow \) Must resort to numerics
- Expansion in \( m_l/m_h \) gives

\[
\frac{E_n^\rho - m_h + m_l}{m_l} = \frac{2\pi}{\sqrt{\lambda}} \left\{ \omega_{n,0}(q/m_l) + \frac{m_l^3}{m_h^3} \omega_{n,3}(q/m_l) + \mathcal{O}(m_l^4/m_h^4) \right\}
\]

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Tuning the heavy-light guitar
Spinning strings

- Want to model mesons with spin and electric charge
  - Mesons w/ 4d spin $J$ dual to strings rotating in real space
  - Mesons w/ R charge $Q$ dual to strings rotating in int. space
- Assume uniform rotation in time and $t$ indep. string profile
  - $\phi = \Omega t$, $\theta = \Omega t$
  - $r = r(z) = r(1/u)$, $\rho = \rho(y)$
- (Numerically) solve full equations of motion for $r$ and $\rho$, integrate $\pi_A$ to obtain $E(\Omega)$ and $J(\Omega)/Q(\Omega)$, and invert for $E(J/Q)$
- Immediate observations (for int. space, $J \rightarrow Q$):
  - Small $\Omega \Leftrightarrow$ large $J$, large $\Omega \Leftrightarrow$ small $J$ (one exception)
  - $E = m_h - m_l + m_l \times f(2\pi J/\sqrt{\lambda})$
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Small $J / Q$ limit accessible from fluctuation calculation

- $E_{r\phi} = m_h - m_l + m_l \omega_n \frac{2 \pi J}{y_1 \sqrt{\lambda}}$ and $E_{\rho\theta} = m_h - m_l + m_l \omega_n z_1 \frac{2 \pi Q}{\sqrt{\lambda}}$
- $n$ counts extrema in string profile

Increasing $J / Q$, find continuous family of solutions indexed by $n$

Critical $J / Q$ values at $E \approx m_h$ and $E \approx m_h + m_l$

- $E \approx m_h$: Light endpoint of string reaches speed of light $\Rightarrow$ Transition to long strings
- $E \approx m_h + m_l$: Solutions break down at finite $J / Q$, as $\Omega \rightarrow 0$ (exception $n = 1$ branch of real space case)

Curiosities

- For real space $n = 1$ branch, first critical point at $E = m_h - m_l^2 / (2m_h) + \ldots$
- In internal space case, additional $n = 0$ branch from uplifting of zero mode
Small $J/Q$ limit accessible from fluctuation calculation

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Strings spinning in real space
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\[ \frac{E - m_h + m_l}{m_l} \]

\[ \frac{2 \pi J}{\sqrt{\lambda}} \]

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Strings spinning in real space

For $n = 1$ branch, large $J$ limit from classical system of two particles in Coulomb potential

$$E = m_h + m_l - 2 \left( \frac{\Gamma(3/4)}{\Gamma(1/4)} \right)^4 \frac{m_l m_h}{m_h + m_l} \frac{\lambda}{J^2}$$
Strings spinning in internal space
Strings spinning in internal space

\[ \frac{E - m_h + m_i}{m_l} \]

\[ \frac{2 \pi Q}{\sqrt{\lambda}} \]
Internal space $n = 0$ branch

- $n = 0$ branch appears from uplift of zero fluctuation mode
  - Small $\Omega \iff$ Small $Q$
  - Small $Q$ limit analytically obtainable and non-Regge-like

\[ E \approx m_h - m_l + \frac{m_l}{2\gamma} \left( \frac{2\pi Q}{\sqrt{\lambda}} \right)^2 \]

- As $\Omega, Q \to 0$, string profile approaches constant
  - $\rho = \rho_0 = 0$ or $1.825y_l$!

- No long strings: Solution break down when $E \approx m_h$
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Outline

Heavy quarks in QCD
  Preliminaries
  Heavy quark limit of QCD

Meson spectroscopy in strongly coupled $\mathcal{N} = 2$ SYM
  Basics of AdS/CFT
  Meson spectra: Preliminaries
  Heavy-light puzzle

Holographic heavy-light mesons
  Fluctuations: Preliminaries
  Fluctuation spectra
  Rotating strings: Preliminaries
  Rotating strings: Results

Conclusions

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Summary and conclusions

- AdS/CFT duality offers a novel way of looking into physics of bound hadronic states in $\mathcal{N} = 2$ SYM
  - Interesting possibilities for comparison with perturbative HQET predictions
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  - Heavy mass independence of mass splittings, *i.e.* fine structure, found both from fluctuating and rotating strings
  - Hyper-fine splitting not immediately visible; perhaps less SUSY could help
  - Smallness of observed EM splittings in accordance with small-$Q$ limit of $n = 0$ branch energy
- Still to do: Intersecting branes (less SUSY), different ground states, hybrid mesons,...
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