Jets in N=4 SYM

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based on work with Paul Chesler and Kristan Jensen
Outline

- Introduction and Review
- Jets at zero temperature
  * basic construction
  * universal late time behavior
- Jets at finite temperature
- Baryon Number Densities
Introduction and Review
Jets at hadron colliders:

Long distance physics = complicated (all orders showering of colored objects, nonperturbative hadronization = organization into color singlets)

Measure this in the detector

Short distance physics = simple (perturbative)

Want to talk about this

Correlated by Underlying Event (UE) color correlations

Stuck with this, small?

(Graphics courtesy of S. Ellis)

More long distance physics, but measured in pdfs
Focus on showering:

Need to evolve single parton into spray of partons (which then can hadronize once the typical energy per parton is of order \( \_ \)). Perturbative as long as all involved momenta and momentum differences are large.

Or treat in SCET. But not completely perturbative.
Importance of jets at zero T

LHC is a jet machine.

If we could find a qualitative guide that allows us to use the substructure of the jet to distinguish pure QCD jets from tops and new physics, this would be extremely helpful.
Jets at finite $T$ -- jet quenching:

See one of two back-to-back created particles.

The other one got “stuck” in the fireball.

Jet quenching is a direct indication of large drag.
Jet Quenching – quantitative:

Nuclear modification factor characterizes medium-effects:

\[ R_{AA}(p_T) = \frac{dN^{AA}/dp_T}{n_{coll} \ dN^{NN}/dp_T} \]

- \( R_{AA}(p_T) = 1.0 \) no suppression
- \( R_{AA}(p_T) = 0.2 \) factor 5 suppression
Jet Quenching, experimental.

$R_{AA}$

$\pi^0$ 0-5% Central
PHENIX preliminary
Toy model needed, since:

Strong Coupling prevents perturbation theory from being applicable.

Real Time dynamics challenging for the lattice
Gauge/String duality or AdS/CFT

Solvable Toy Model(s) of non-equilibrium strong coupling dynamics.

“Finite temperature field theory = Gravity with Black Hole”
Review: Heavy Quark Energy Loss

- is understood in AdS/CFT
- gives nice qualitative picture of energy loss in strongly coupled gauge theory
- quantitative guidance … but need to pick value for coupling (unlike, say, viscosity to entropy density ratio)
Heavy quark at finite $T$:

(HKKKY, Gubser)

Constant $E$-field

Momentum flows at constant rate.

Gluon tail

IR "Divergence"

Loss rate: $\frac{dP}{dt} = -\frac{\sqrt{\lambda}}{2\pi} \frac{v}{\sqrt{1 - v^2}} (\pi T)^2$

Rate at which the external field does work.
Energy and Momentum Density

\( \frac{|x| \Delta \varepsilon(x)}{T^3 \sqrt{\lambda}} \)

\( \frac{|x| \Delta S(x)}{T^3 \sqrt{\lambda}} \)

(Chesler and Yaffe)
Energy Loss Mechanism

What is the dominant effect for energy loss? Single gluon Bremsstrahlung (weak coupling)? Glueball Emission (certainly not at large $N$)?

At strong coupling: Coherent gluon emission.

Strong coupling dynamics dominated by very many very soft gluons!
Questions addressed in this talk:

Massless quark jets at finite T.

Jets at zero temperature.

Baryon Number Densities.
Jets at zero temperature.
Holographic Image (zero T):

Two “blobs” of energy density / charge density rushing apart and expanding.
Universal endpoint behavior:

Endpoints = Straight lines = Geodesics.

$x = vt$, $u = wt$

Slight deviation from geodesic at early times.

Initial condition = Pointlike String
Analytic Late Time behavior

\[ x = \ell(t, \varphi) \cos \varphi \quad \text{1 is “radius” in } u/x \text{ plane} \]

\[ u = \ell(t, \varphi) \sin \varphi. \]

Late time expansion:

\[ \ell(\varphi, t) = \chi(\varphi) t + \sum_{n=0}^{\infty} \ell_n(\varphi) t^{-n}. \]

Result: \[ \chi(\varphi) = 1. \quad \text{Lightlike, circular arc.} \]
World Sheet Inflation.

Just as in inflation, short wavelength modes inflate away and become long wavelength:

\[ \ell' / \ell \rightarrow 0 \text{ as } t \rightarrow \infty. \]
Endpoint Motion:

Geodesic (n=0) + small corrections.

$$\varphi_s(t) = \sum_{n=0}^{\infty} \varphi_n^s t^{-n}.$$ 

Boundary Conditions:

$$\varphi^s_1 = \ell'_0(\varphi^s_0),$$

$$\ell_1(\varphi^s_0) = -\frac{1}{2} (\ell'_0(\varphi^s_0))^2.$$
Expansion around lightlike string.

- Late time configuration is a lightlike arc.
- Lightlike configuration has infinite energy, but serves as a good starting point for perturbation analysis.
- Energy density and flux depend crucially on two free functions worth of fluctuations, mapping to two free functions in initial conditions.
- Leading order endpoint motion is geodesic.
Evolution of Asymmetric String.
Validity of late time expansion.
Jets at finite temperature.
Jets = Falling Strings (finite T)

\[ \dot{x} \sim e^{-t(4\pi T)} \]
Generic Jet at Finite $T$

- For initial energies of order a few $(1/2)T$ string again quickly settles on geodesic motion.
- At finite temperature geodesics only travel finite distance before hitting the horizon = finite reach of jet. Jet stuck in plasma.
- Asymptotic time scales agree with non-hydro Quasinormal modes of Kovtun and Starinets.
- Quasi-Particles from non-generic strings.
Quasi-Particle excitations.

Zero T Jets
Quasiparticle in Plasma
Final Diffusion
Energy scales at finite T.

At early times: like $T = 0$ jets as long as $E >> T$

At late times: diffusion + damping of non-hydro modes. Observed late time behavior agrees with time scales from quasinormal modes!
Analytic Solution for Quasiparticle

\[ x(t, u) = x_{\text{steady}}(t, u) + \epsilon x_1(t, u) + \mathcal{O}(\epsilon^2) \]

where

\[ x_{\text{steady}}(t, u) = \xi t + x_0(u), \]

Instead of late time expansion we do an expansion around the dragging string of HKKKY. Generic “Quasi-Particle-String” (small initial u, velocity in slice direction) well described by this expansion.
Comparison Analytics/Numerics
Summary: String Configurations

- Both at zero and finite temperature endpoints quickly settle onto geodesic motion.
- We have analytic control as a systematic expansion around a lightlike configuration at late times (zero T) or large initial energy (finite T).
- Numerical Solutions are in perfect agreement with our analytic solutions.
Baryon Number Densities
Mapping out field theory charges.

\[ \langle T^{\mu\nu} \rangle \quad \langle j^\mu \rangle \]

\[ h_{MN} \]

\[ A_M \]

radial coordinate

\[ u_h \]
Baryon Number versus Energy.

Energy density depends on the whole string profile. Sensitive to the deviations from the analytic solution (lightlike solution itself has infinite energy!). Non-universal. Crucial initial condition dependence.

Baryon number density only depends on endpoint motion! Very constrained, e.g. at zero T, late times uniquely determined in terms of one number!
Universal Observables (zero T):

Define moments of Baryon number density that only depend on late time behavior of the bulk source (= the string).

\[
\frac{dB_{s}}{d\Omega} \equiv \int_{0}^{\infty} r^2 dr \rho_{s}(t, \mathbf{x})
\]

\[
\bar{x}_{s}(t) \equiv \frac{\int d^{3}x \mathbf{x} \rho_{s}(t, \mathbf{x})}{\int d^{3}x \rho_{s}(t, \mathbf{x})}
\]

Angular Distribution of Baryon Number in Jet. Center of charge position/motion.

(Can then be calculated in terms of geodesic source)
Analytic Expression at zero T.

\[
\frac{dB_s}{d\Omega} = \frac{(-1)^{s+1} 1 - V_s^2}{4\pi (1 - V_s \cos \theta)^2}.
\]

+1/-1 for quark/antiquark respectively.

\[v_s = \frac{1}{V_s} + \left(1 - \frac{1}{V_s^2}\right) \tanh^{-1} V_s\]
x-velocity of string endpoint; the one parameter that determines the late time bulk solution.
Baryon Density, zero T

\[ t_3 = 10t_1 \]

\[ t_2 = 5t_1 \]

\[ x_{\perp} \]

\[ x_{\parallel} \]
Baryon Density at finite $T$

- No good late time expansion, but instead expansion around “quasiparticle solution”
- Clear evidence of 3 important time scales:
  - $t \sim \text{initial height}$: 2 clearly separated lumps.
    - Zero temperature jets
  - $t \sim T$: quasi-particle forms
    - Almost entire distance travelled here!
  - $t \sim T^2/\text{initial height}$:
    - Quasi particle quickly diffuses
Jet in N=4 SYM:
Baryon Density, finite T.
Discussion, part I

Gubser et al:

**Stopping distance** of gluon is calculated based on geodesic analysis alone.

**Map of initial energy to geodesic non-trivial.** Small changes in geodesic lead to large difference in final distance traveled.

Our numerical solutions violate “bounds” by about a factor of 1.5
Discussion, part II

Strassler, Hofman and Maldacena:

\[ \text{N}=4 \text{ states created by } \text{N}=4 \text{ currents are round.} \]

Generic state created by N=4 current has already many soft gluons and quarks. Subsequent showering does just spread the round gluon/quark cloud.

No jet like structure!
We study special initial state (mimicking what a hard QCD scattering event would do).

This is certainly not the generic state created by N=4 current, but it is a much better model of how a QCD jet looks like.

We have complete control over the soft emission from the hard jet in the strongly coupled N=4 setting.
Conclusions:

Gauge/gravity duality is a great toy model to study non-equilibrium phenomena at strong coupling. In particular, showering of jets can both be followed at zero and finite temperature. **Baryon number universal.**

**Question:** Is there a universal observable for energies? Stopping distance? We can trace the shower back in time, can one build a N=4 inspired model that can do this in QCD, too?