Partons and jets in gauge theories at strong coupling

Edmond Iancu
IPhT Saclay & CNRS

Collaboration with Yoshitaka Hatta and Al Mueller
(arXiv:0710.2148, 0710.5297, and 0803.2481)
Introduction (1)

- ‘Hard probes for strongly–coupled quark–gluon plasma’
  
  (‘hard’ = high energy, momentum, virtuality ... \( \gg T \))
  
  ◆ energy loss, momentum broadening, meson screening length, jet lifetime/stopping length ...

- QCD at high energy, finite temperature, ... or both
  
  ◆ confinement is not so important
  ◆ a conformal field theory might by qualitatively similar

- AdS/CFT methods come naturally into the game!

- The respective results can be compared to theoretical expectations in pQCD, and/or to experimental measurements
  
  ◆ particle multiplicities, momentum/rapidity distributions
  ◆ the structure of the ‘final state’
Main question: Is this regime of QCD mostly on the ‘strong coupling’ side, or on the ‘weak–coupling’ one?

The AdS/CFT calculations remain highly non–trivial, even in the strong coupling (‘supergravity’) limit:
- global, or ‘inclusive’, results, like total cross–sections
- difficult to identify the actual ‘final state’
- multi–particle \((n \geq 3)\) correlations

An ‘alternative’ (simpler but less rigorous) strategy:
combine the results of (simpler) AdS/CFT calculations with physical insight coming from general principles, in order to develop a physical picture for the final state

Follow the footsteps of QCD:
start with an electromagnetic current!
Introduction (3)

Electromagnetic current in (perturbative) QCD

- the simplest way to produce jets: $e^+ e^- \rightarrow \gamma^* \rightarrow \text{jets}$

- the most direct way to study the parton structure: DIS

- universal ‘parton distributions’ $\Rightarrow$ hadron–hadron collisions
Introduction (4)

- Particularly well suited for AdS/CFT purposes:
  - well-posed problem (conceptually unambiguous)
  - minimal formulation of the AdS/CFT correspondence
  - physical interpretation facilitated by ‘non-renormalization theorems’

- Results which admit a natural physical interpretation
  - ‘parton branching’ (in the vacuum, or in the plasma)
  - the partons of the underlying $\mathcal{N} = 4$ SYM theory
  - simple explanation for drag force, jet broadening & lifetime

- Some challenging results (at variance with QCD)
  - no jets (say, in $e^+e^-$, or hadron–hadron, collisions)
  - momentum broadening due to medium–stimulated parton branching (as opposed to thermal rescattering)
Outline

- Partons and jets in QCD at weak coupling
- AdS/CFT calculation: Methodology
- $\mathcal{N} = 4$ SYM theory in the strong coupling limit
  - $\mathcal{R}$–current in the vacuum
  - $\mathcal{R}$–current in the plasma
- Focus on time–like current (‘jets’)
- Related work on space–like current (‘DIS’)
  
  J. Polchinski and M. J. Strassler, [hep-th/0209211]
  
  
$e^+ e^- \text{ annihilation}$

- Lowest–order in perturbative QCD: $e^+ e^- \rightarrow \gamma^* \rightarrow q\bar{q}$

- A time–like current ($Q^2 = s > 0$) decaying into a $q\bar{q}$ pair

- Center of mass frame: a pair of back–to–back ‘jets’

- Bare partons cannot appear in the final state (confinement)

- The structure of the final state is determined by
  - parton branching
  - hadronisation
Parton branching at weak coupling

Gluon emission to lowest order in perturbative QCD:

\[ dP_{\text{Brem}} \sim \alpha_s(k_{\perp}^2) N_c \frac{d^2k_\perp}{k_{\perp}^2} \frac{dx}{x} \]

Phase–space enhancement for the emission of

- **collinear** \((k_\perp \rightarrow 0)\)
- **and/or soft** \((x \rightarrow 0)\) gluons

Generic for a theory with **dimensionless coupling** and **massless vector bosons**
‘Multi–jet event’ : large emission angle & \( x \sim \mathcal{O}(1) \)

\[ k_\perp \sim k \sim \sqrt{s} \implies \mathcal{P}_{\text{Brem}} \sim \alpha_s(s) \ll 1 \]

small probability for emitting an extra gluon jet !

‘Intra–jet activity’ : collinear and/or soft gluons

\[ \Lambda_{\text{QCD}} \ll k_\perp \ll k \ll \sqrt{s} \implies \mathcal{P}_{\text{Brem}} \sim \alpha_s \ln^2 \frac{\sqrt{s}}{\Lambda_{\text{QCD}}} \sim \mathcal{O}(1) \]

modifies particle multiplicity but not the number of jets
Final state

- Few, well collimated, jets
- $e^+e^- \text{ cross–section computable in perturbation theory}$

$$\sigma(s) = \sigma_{\text{QED}} \times \left( 3 \sum_f e_f^2 \right) \left( 1 + \frac{\alpha_s(s)}{\pi} + O(\alpha_s^2(s)) \right)$$

$\sigma_{\text{QED}}$ : cross–section for $e^+e^- \rightarrow \mu^+\mu^-$

- No logs: collinear and infrared singularities mutually cancel
3–jet event at OPAL (CERN)
Current–current correlator

- Total cross–section given by the **optical theorem**

\[
\sigma(e^+e^-) = \frac{1}{2s} \ell^{\mu\nu} \text{Im} \Pi_{\mu\nu}(q)
\]

\[
\Pi_{\mu\nu}(q) \equiv \int d^4x e^{-iq \cdot x} i\theta(x_0) \langle 0 | [J_\mu(x), J_\nu(0)] | 0 \rangle
\]

\[
J^\mu = \sum_f e_f \bar{q}_f \gamma^\mu q_f
\]

- Vacuum polarization tensor for time–like momenta \((q^\mu q_\mu > 0)\)
Current–current correlator

- Total cross–section given by the optical theorem

\[ \sigma(e^+ e^-) = \frac{1}{2s} \ell^{\mu\nu} \text{Im} \Pi_{\mu\nu}(q) \]

- Valid to leading order in \( \alpha_{em} \) but all orders in \( \alpha_s \)

\[ \Pi_{\mu\nu}(q) \equiv \int d^4x \, e^{-iq\cdot x} \, i\theta(x_0) \langle 0 \mid [J_\mu(x), J_\nu(0)] \mid 0 \rangle \]

\[ J^\mu = \sum_f e_f \, \bar{q}_f \, \gamma^\mu \, q_f \]
Current–current correlator

- Total cross-section given by the optical theorem

\[ \sigma(e^+e^-) = \frac{1}{2s} \ell^{\mu\nu} \text{Im} \Pi_{\mu\nu}(q) \]

- Inclusive calculation (a ‘black box’)

- No specific information about the structure of the final state
  (‘how many jets, how they are distributed’)

\[ \Pi_{\mu\nu}(q) \equiv \int d^4x \ e^{-i q \cdot x} i\theta(x_0) \langle 0 \mid [J_\mu(x), J_\nu(0)] \mid 0 \rangle \]
AdS/CFT: Methodology (I)

- Abelian $\mathcal{R}$–current: $J_{\mu}(q) \propto e^{-i\omega t + i q z}$ with $Q^2 = \omega^2 - q^2$

- Retarded polarization tensor (in plasma, or in the vacuum)

\[ \Pi_{\mu\nu}(q) \equiv \int \! d^4x \, e^{-iq \cdot x} \, i\theta(x_0) \left\langle [J_{\mu}(x), J_{\nu}(0)] \right\rangle_T \]

$\triangleright$ 'vacuum': $T = 0$

- Im $\Pi_{\mu\nu}$: dissipation, jet production (time–like: $Q^2 > 0$), structure functions (space–like: $Q^2 < 0$)

- AdS/CFT correspondence: ‘minimal’ formulation ($\mathcal{N} = 4$ SYM, no D-Branes, no IR cutoff)

- Large 't Hooft coupling limit: $\lambda \equiv g^2 N_c \rightarrow \infty$ for fixed $g \ll 1$

$\implies$ ‘supergravity approximation’ (classical limit)

- $\mathcal{R}$–current $J_{\mu}$ in 4D $\leftrightarrow$ Maxwell–like wave in $AdS_5$
AdS/CFT: Methodology (II)

- Maxwell wave in $AdS_5$ (BH): $A_\mu(t, x, \chi) = e^{-i\omega t + i q z} A_\mu(\chi)$

- Radial coordinate on $AdS_5$: $\chi \equiv r_0/r$, $r_0 = \pi R^2 T$
  - $0 \leq \chi \leq 1$ (boundary at $\chi = 0$, horizon at $\chi = 1$)
AdS/CFT: Methodology (III)

- Maxwell equation in curved space ($AdS_5$ BH)

$$\partial_m (\sqrt{-g} g^{mn} g^{pq} F_{nq}) = 0 \quad \text{where} \quad F_{mn} = \partial_m A_n - \partial_n A_m$$

- Boundary conditions

  - $A_\mu(t, \vec{x}, \chi = 0) = e^{-i\omega t + iqz} A_\mu^{(0)}$, $A_5(t, \vec{x}, \chi = 0) = 0$

  - No reflected wave at large $\chi$ (say, near $\chi = 1$)

- The (retarded) polarization tensor:

$$\Pi_{\mu\nu}(\omega, q) = \frac{\partial^2 S_{cl}}{\partial A_\mu^{(0)} \partial A_\nu^{(0)}}$$

AdS/CFT analog of computing Feynman graphs in momentum space

- Time-dependent perturbations: $A_\mu(t \to 0, \chi) \propto \delta(\chi)$

essential to develop a physical space–time picture
The vacuum polarization tensor

- **The AdS/CFT result** turns out to be quite simple:

\[
\Pi_{\mu\nu}(q) = \left( \eta_{\mu\nu} - \frac{q_\mu q_\nu}{Q^2} \right) \frac{N_c^2|Q^2|}{32\pi^2} \left( \ln \frac{|Q^2|}{\mu^2} - i\pi \Theta(Q^2) \text{sgn}(\omega) \right)
\]

- The same as the respective result at one–loop level!

- **Non–renormalization property due to SUSY**

- The physical picture at 1–loop level is well understood

  \[Q^2 > 0: \text{imaginary part due to decay into a pair massless fields}\]

- ... and can be used to ‘translate’ the AdS/CFT calculation!
‘Schrödinger equation’: vacuum

- Maxwell eq. for $A_0 \iff$ Schrödinger eq. for $\psi(\chi, t) \equiv \sqrt{\chi} A_0$

\[ i \frac{\partial \psi}{\partial t} = \left( -\frac{1}{2k} \frac{\partial^2}{\partial \chi^2} - \frac{1}{8k\chi^2} \pm \frac{K^2}{2k} \right) \psi \]

- Dimensionless variables: $k = \omega/T$ and $K^2 = |Q^2|/T^2$

- $V(\chi)$
  - $V_A$
  - $V_B$
  - $K^2/k$
  - $1/K$
  - space–like

- $V(\chi)$
  - $V_A$
  - $V_B$
  - $-K^2/k$
  - $1/K$
  - time–like

- $\psi(\chi, t = 0)$: a wave–packet localized near $\chi = 0$
Early times: diffusion

- Early times/small $\chi$: $t \lesssim k/K^2$ and $\chi \lesssim 1/K$

- Same dynamics for both space–like and time–like currents

- Quantum diffusion into $AdS_5$:

\[ \psi(\chi, t) \sim \exp\left\{ \frac{ik\chi^2}{2t} \right\} \quad \longleftrightarrow \quad \chi(t) \sim \sqrt{\frac{t}{k}} \]
Late times $t \gtrsim k/K^2$: free streaming

- **Space–like**: Potential barrier due to energy conservation
  - the wave gets stuck at $\chi \sim 1/K$

- **Time–like case**: Free motion towards larger values of $\chi$

$$\chi(t) \sim \frac{K}{k} t = \sqrt{1 - v_z^2} t \quad \text{with} \quad v_z \equiv \frac{q}{\omega}$$
This picture must be consistent with the one–loop dynamics in the gauge theory (at least, formally).
The one–loop space–time picture

- **Space–like current**: The transverse size $L$ of the $q\bar{q}$ fluctuation grows diffusively, up to a maximal size $L \sim 1/Q$

  \[ L(t) \simeq \sqrt{\frac{t}{\omega}}, \quad t \lesssim \frac{\omega}{Q^2} = t_{\text{form}} \]

  \[ q = (\omega, 0, 0, q) \sim t^{1/2} \quad L \sim 1/Q \]

- **Time–like current**: Free transverse expansion for $t \gtrsim t_{\text{form}}$

  \[ L(t) \simeq 2v_{\perp} t, \quad v_{\perp} \equiv \sqrt{1 - v_z^2} \]

  \[ q = (\omega, 0, 0, q) \sim t^{1/2} \quad \sim v_{\perp} t \quad \text{massless partons} \]
The $L - \chi$ duality

- The comparison suggests the following ‘duality’

\[ \chi \leftrightarrow TL \quad \text{or} \quad \frac{R^2}{r} \leftrightarrow L \]

- $\text{COM } (v_z = 0)$: this looks like a pair of back-to-back jets
The $L - \chi$ duality

- The comparison suggests the following ‘duality’

\[ \chi \leftrightarrow TL \quad \text{or} \quad \frac{R^2}{r} \leftrightarrow L \]

- Particular realization of the UV/IR correspondence

\[ (L. \ Susskind \ and \ E. \ Witten, \ 98; \ A. \ Peet \ and \ J. \ Polchinski, \ 98) \]

- Generally formulated as:
  
  “Radial distance in $AdS_5$ $\leftrightarrow$ Energy on the boundary”

- At high energy this becomes ambiguous: dissymmetry between longitudinal and transverse dynamics
  
  - energy $\leftrightarrow$ longitudinal dynamics
  - ... but the radial dimension $r \leftrightarrow$ transverse size!

- A similar correspondence noticed by Brodsky and de Teramond

  the $AdS_5$ radius $\leftrightarrow$ the transverse size of a hadron
Parton branching at strong coupling

- One–loop picture: ‘two–particle final state’

- But, of course, this is merely formal!
  - Recall: specific final states cannot be discriminated solely by the AdS/CFT calculation of $\text{Im} \Pi_{\mu\nu}$

- At strong coupling, one rather expects a very different physical picture
  - No reason why branching should stop at 2 parton level
  - No reason to favour special corners of phase–space

- Energy and momentum should be quasi–democratically divided among the daughter partons

- ‘Democratic branching’ yields the same result as the AdS/CFT calculation!
Quasi–democratic branching

\[
\omega_n \sim \frac{\omega_{n-1}}{2} \sim \frac{\omega}{2^n}
\]

\[
Q_n \sim \frac{Q_{n-1}}{2} \sim \frac{Q}{2^n}
\]

\[
\Delta t_n \sim \frac{\omega_n}{Q_n^2}
\]

(uncertainty principle)

\[
\frac{Q_n - Q_{n-1}}{\Delta t_n} \sim - \frac{Q}{\omega} Q_n^2 \quad \Rightarrow \quad \frac{dQ(t)}{dt} \sim - \frac{Q^2(t)}{\gamma}
\]

\[
L(t) \sim \frac{1}{Q(t)} \quad \Rightarrow \quad L(t) \sim \frac{t}{\gamma} = \sqrt{1 - v_z^2} t
\]
No jets in $e^+ e^-$ at strong coupling!

- IR cutoff $\Lambda \rightarrow$ splitting continues down to $Q \sim \Lambda$

- $2^N = Q/\Lambda$ final particles $\rightarrow$ multiplicity grows like $Q$

- Transverse momenta $p_\perp \sim \Lambda \rightarrow$ particles are isotropically distributed in transverse plane

- In the COM frame $\rightarrow$ spherical distribution!

*Similar conclusion by Hofman and Maldacena, arXiv:0803.1467 [hep-th]*
Time–like current in the plasma (1)

‘Hard probe’ : \( \frac{Q}{T} \equiv K \gg 1 \) & ‘High energy’ : \( \frac{\omega}{Q} \equiv \frac{k}{K} \gg 1 \)

\[ V(\chi) \]

- \( K^2/k \)
- \( 1/K \)
- \( 1/k^{1/3} \)
- \( \sqrt{K/k} \)

\( V_A \) \quad \( V_B \) \quad \( V_C \)

diffusion \quad free streaming \quad medium branching

- The potential becomes **attractive** at sufficiently large \( \chi \)
The physical situation so long as $\chi_{cr} \gg 1/K$

$$\omega \ll \frac{Q^3}{T^2} \quad \text{or} \quad Q^2 \gg Q_s^2(\omega, T)$$

The plasma saturation momentum: $Q_s \equiv (\omega T^2)^{1/3}$
Jets in the plasma: **early times**

\[ q = (\omega, 0, 0, q) \]

\[ \gamma = \frac{\omega}{Q} \]

Free streaming up to the point of no return:

\[ \chi_{cr} \sim \frac{1}{\sqrt{\gamma}} \implies L_{cr} \sim \frac{1}{\sqrt{\gamma} T} = \frac{(1 - v^2_z)^{1/4}}{T} \]
Meson screening length

The same as the meson screening length!

\[ q = (\omega, 0, 0, q) \]

\[ L \sim \frac{1}{Q} \]

\[ \gamma = \frac{\omega}{Q} \]

\[ \frac{1}{\gamma^{1/2}T} \]

The same as the meson screening length!

[Liu, Rajagopal, Wiedemann; Chernicoff et al; Caceres et al (2006)]

Here, an effective, dynamical, ‘meson’: globally colorless partonic system which expands in transverse space up to the critical size \( L_{cr} \) at which it starts feeling the plasma.
A quasi-stable ‘meson’ can be built with a space–like current

\[ q=(\omega,0,0,q) \approx t^{1/2}, \quad L \sim 1/Q \]

Tunneling probability (DIS structure function)

\[ P \sim \exp \left\{ -\frac{Q^{3/2}}{Q_s^{3/2}} \right\} \sim \exp \left\{ -\sqrt{\frac{Q}{\gamma T}} \right\} \]
Falling into the Black Hole

\[ q = (\omega, 0, 0, q) \]

\[ \gamma = \frac{\omega}{Q} \]

\[ L \sim \frac{1}{Q} \]

\[ \frac{1}{\gamma^{1/2} T} \]

- Accelerated fall in \( AdS_5 \): 
  \[ \frac{d\chi}{dt} \sim \chi^2 \implies \chi(t) \sim \frac{\chi_{cr}}{1 - \chi_{cr}(t - t_{cr})} \]

- ... constant decelerating force in the transverse direction 
  \[ \chi \sim TL \sim \frac{T}{p_{\perp}} \implies \frac{dp_{\perp}}{dt} \sim -T^2 \]
Falling into the Black Hole

- Partons disappear into the plasma when $\chi \sim 1$ or $L \sim 1/T$

- $t_3 \sim t_2 \sim \sqrt{\gamma}/T \gg t_1 \implies$ lifetime of the partonic system

$$\Delta t \sim t_1 + t_2 + t_3 \sim \frac{\sqrt{\gamma}}{T}$$

- What is the physical interpretation of the ‘fall into the BH’?
Medium induced parton branching

- Quasi–democratic branching in the presence of the uniform transverse force $\sim T^2$ exerted by the plasma

$$\omega_n \sim \frac{\omega_{n-1}}{2} \sim \frac{\omega}{2^n}$$

$$\Delta t_n \sim \frac{\omega_n}{Q_n^2}$$

(uncertainty principle)

$$\frac{\Delta Q_n}{\Delta t_n} \sim -T^2 \quad \Rightarrow \quad Q_n \sim \frac{\omega_n}{Q_n^2} T^2 \sim (\omega_n T^2)^{1/3}$$

- The saturation momentum at step $n$ : $Q_s \equiv (\omega T^2)^{1/3}$
Energy loss: dynamical ‘drag force’

\[- \frac{d\omega(t)}{dt} \simeq Q_s^2(t) \equiv \gamma(t) T^2 \quad \text{with} \quad \gamma \equiv \frac{\omega}{Q_s} = \left(\frac{\omega}{T}\right)^{1/3}\]

- Time–dependent generalization of the ‘drag force’
  (here, the velocity is not constant anymore !)

[Herzog, Karch, Kovtun, Kozcaz, Yaffe; Gubser, 2006]

Universality of the dissipation mechanism
Enveloping curve: the trailing string

- The ‘string’ pulled by a heavy quark moving through the plasma at constant speed: \( z(\chi, t) = v_z t - \Delta z(\chi) \)

- After replacing \( \chi \rightarrow TL \) 

  the enveloping curve \( \Delta z(L) \) of the partonic system produced via medium–induced branching
The lifetime of a (colored) parton

\[ \frac{d\omega(t)}{dt} \sim - \left( \frac{\omega(t)}{T} \right)^{1/3} T^2 \quad \Rightarrow \quad \Delta t \sim \frac{1}{T} \left( \frac{\omega_0}{T} \right)^{1/3} \]

- The same as the ‘gluon penetration length’ \( \Delta z = \Delta t \)
  

- Recall: parton formation time \( t_{\text{form}} \sim \omega_0/Q^2 \)

- If ‘lifetime’ is shorter than ‘formation time’, the parton never becomes on–shell: jets cannot form!

\[ t_{\text{form}} \sim \Delta t \quad \Rightarrow \quad Q \sim (\omega T^2)^{1/3} \equiv Q_s \]

- Partons with momenta \( Q \lesssim Q_s \) are freed into the plasma

- The mechanism for both energy loss and momentum broadening in the strongly–coupled plasma!
  
Instead of Conclusions

- The AdS/CFT calculation is not a black box!

- Thank you!