Dissipation in strongly coupled $\mathcal{N} = 4$ plasma

I. Amado, C.H., K. Landsteiner, S. Montero

0805.2570


JHEP 0709 (2007) 057 [0706.2750]

Institute for Nuclear Theory, Seattle WA

June 4, 2008
Outline

- Short motivation
- Dissipation in field theory and holography
- Results for hydrodynamic channels
- Limits of the hydrodynamic approximation and related issues
- Screening masses and causality
- Vague ideas about large $N$ and temperature
Finite temperature

It is an interesting topic for

- Theoretical reasons: both for gauge theories and the correspondence itself
- Applications to cosmology, astrophysics, nuclear physics

Many things can be done in field theory at weak coupling (perturbation theory, HTL, semiclassical analysis, ...)

Strong coupling?

- Lattice simulations... hard time with real time
- AdS/CFT?
$\mathcal{N} = 4$ computations for real world

Some pros and cons

:( We look to a SCFT, quite different to QCD (real world)

:) In some interval of temperatures $\text{RHIC} \lesssim T \lesssim \text{LHC}$ e.o.s. maybe not so different $p \sim \varepsilon/3$ [e.g. 0710.0354]

:( Different parameter values: large $N, \lambda \gg 1$ vs $N = 3, \lambda \sim 1$

:) Transport coefficients similar: $\zeta \simeq 0$, $\eta/s \ll 1$

What is generic and what depends on large $N$, strong coupling, susy, etc?
Thermal AdS/CFT

- Finite temperature $T > 0$ in CFT $\Rightarrow$ Same temperature $T$ measured at the boundary
- Deconfined phase $\Rightarrow$ Black hole in AdS ($T =$ Hawking temperature)
- Screening $\Rightarrow$ breaking of strings/branes that fall into the horizon
- Asymptotically AdS: usual field-operator correspondence
- Dissipation $\Rightarrow$ Small fluctuations fall into the black hole

$$ds^2 = \frac{r^2}{L^2} \left( -f(r) dt^2 + dx^2 \right) + \frac{L^2}{r^2} \frac{dr^2}{f(r)}, \quad f(r) = 1 - \frac{r_0^4}{r^4} \quad T = \frac{r_0}{\pi L^2}$$
Linear response

Small perturbation: energy negligible compared to the whole system. Properties at equilibrium determine the non-equilibrium values. Source $j(t, x)$ for gauge-invariant operator $\mathcal{O}$

$$\langle \mathcal{O}(t, x) \rangle = - \int dt' d^3x' G_R(t - t', x - x') j(t', x') ,$$

where $G_R$ is the retarded Green function

$$G_R(t - t', x - x') = -i \Theta(t - t') \langle [\mathcal{O}(t, x), \mathcal{O}(t', x')] \rangle$$

Fourier transform

$$\langle \mathcal{O}(t, x) \rangle = - \int \frac{d\omega}{2\pi} \frac{d^3q}{(2\pi)^3} e^{-i\omega t + iqx} \tilde{G}_R(\omega, q) \tilde{j}(\omega, q) .$$
Dissipation

Assume the only singularities of the Green function in the complex $\omega$ plane are poles

$$\tilde{G}_R(\omega, q) \simeq \sum_{\text{poles}} \frac{R_n(\omega, q)}{\omega - \omega_n(q)}, \quad \omega_n(q) = \Omega_n(q) - i\Gamma_n(q)$$

Retarded $t > 0$: Cauchy theorem with closed contour on $\text{Im} \omega < 0$:

$$\langle O(t, q) \rangle \simeq i\theta(t) \sum_n R_n(\omega_n(q), q) \tilde{j}(\omega_n(q), q) e^{-i\Omega_n(q)t - \Gamma_n(q)t}. $$

- $\Gamma_n > 0$ exponential decay of perturbations $\Rightarrow$ dissipation
- $\Gamma_n < 0$ exponential growth $\Rightarrow$ instability
Retarded Green function in AdS/CFT

Gauge-invariant operator ↔ Classical field
(e.g. $\text{Tr} \ F^2 \leftrightarrow \phi$, $J_\mu \leftrightarrow A_\mu$, $T_{\mu\nu} \leftrightarrow g_{\mu\nu}$)

- Linear approximation: 2nd order ODEs in the bulk
  $$\phi(t, x, r) = e^{-i\omega t + iq x} \phi(r)$$

- Infalling boundary conditions at the horizon
  $$\phi_{\text{in}}(r) \sim (r - r_0)^{-i\omega/2}$$

- At the AdS boundary:
  $$\phi_{\text{in}}(r) = A(\omega, q) \phi_{NN}(r) + B(\omega, q) \phi_N(r)$$

$$\tilde{G}_R(\omega, q) \sim \lim_{r \to \infty} \phi_{\text{in}}(r) \partial_r \phi_{\text{in}}(r) \propto \frac{B(\omega, q)}{A(\omega, q)}$$

[Son, Starinets]
Quasinormal modes and residues

- Poles of the Green function correspond to normalizable solutions (QNMs) \( A(\omega, q) = 0 \)

- QNMs describe the classical decay of a localized perturbation in the presence of a horizon

- QNMs with \( \lim_{q \to 0} \omega(q) = 0 \) describe hydrodynamics (diffusion, shear, sound) [Policastro, Son, Starinets]

How to compute the residues?

\[
R_n^{-1} = \left. \frac{d}{d\omega} \tilde{G}_R^{-1} \right|_{\omega=\omega_n}
\]

\[
\begin{align*}
\phi_{in} &= A\phi_{NN} + B\phi_N \\
\phi_{in}' &= A\phi_{NN}' + B\phi_N'
\end{align*}
\]

\[
\Rightarrow \quad \frac{B}{A} = \frac{\phi_{in}'\phi_{NN} - \phi_{in}\phi_{NN}'}{\phi_{in}\phi_N - \phi_{in}'\phi_N}
\]
EOMs

Metric (energy-momentum tensor) EOMs

\begin{align}
Z''_1 + \frac{(w^2 - q^2 f(u)) f(u) - u w^2 f'(u)}{u f(u)(q^2 f(u) - w^2)} Z'_1 + \frac{w^2 - q^2 f(u)}{u f(u)^2} Z_1 &= 0 , \\
Z''_2 - \frac{3 w^2 (1 + u^2) + q^2 (2u^2 - 3u^4 - 3)}{u f(u)(3w^2 + q^2(u^2 - 3))} Z'_2 + \\
&+ \frac{3w^4 + q^4 (3 - 4u^2 + u^4) + q^2 (4u^5 - 4u^3 + 4w^2u^2 - 6w^2)}{u f(u)^2(3w^2 + q^2(u^2 - 3))} Z_2 &= 0 , \\
Z''_3 + \frac{1 + u^2}{u f(u)} Z'_3 + \frac{w - q^2 f(u)}{u f(u)^2} Z_3 &= 0 .
\end{align}

Gauge field (R-current) EOMs

\begin{align}
E''_T + \frac{f'(u)}{f(u)} E'_T + \frac{w^2 - f(u)q^2}{u f(u)^2} E_T &= 0 , \\
E''_L + \frac{w^2 f'(u)}{f(u)(w^2 - f(u)q^2)} E'_L + \frac{w^2 - f(u)q^2}{u f(u)^2} E_L &= 0 .
\end{align}

(u = r_0^2/r^2, f(u) = 1 - u^2, w = \omega/(2\pi T), q = q/(2\pi T))
Location of the poles

There is an exact \textit{analytic result} in $AdS_5$! (for $q = 0$)

\begin{equation}
\rho_J(\omega) = -2 \text{Im} \tilde{G}_R(\omega, q = 0) = \frac{N_c^2 \omega^2}{16\pi} \frac{\sinh(\beta \omega/2)}{\cosh(\beta \omega/2) - \cos(\beta \omega/2)}
\end{equation}

\[\omega_n = 2\pi T(1 \pm i)n, \; n \in \mathbb{Z}\]

In $AdS_3$ $\omega_n = \pm q - i4\pi T(n + h)$ \cite{Birmingham, Sachs, Solodukhin}

There are many numerical/semiclassical results

\cite{Horowitz, Hubeny; Cardoso, Konoplya, Lemos; Cardoso, Natario, Schiapappa; Nunez, Starinets; Fidkowsky, Hubeny, Kleban, Shenker;...]

\[\omega_n \sim T(1 - i)n + \omega_0, \; n \in \mathbb{N}, \; \text{etc}\]

\begin{itemize}
  \item Our computation agrees with previous results
  \item Can we say anything from the field theory?
\end{itemize}
Quasinormal modes for R-current

The longitudinal channel of R-current has a diffusive mode

$$\omega = -iDq^2, \quad \omega, q \ll T$$

Qualitative change of QNM behavior after crossing the diffusion mode
Residues for R-current

\[ G_{tt}(\omega, q) \approx \frac{N^2 T}{16\pi} \sum_n \frac{q^2 R_n(q)}{\omega - \omega_n(q)}, \]

The diffusive mode decouples at high momentum
Energy-momentum tensor modes

\[ G_{\mu\nu,\alpha\beta}(x) = -i\theta(t) \langle [T_{\mu\nu}(x), T_{\alpha\beta}(0)] \rangle \]

Holographic dual described by metric fluctuations \( g_{\mu\nu} \)

- **Vector channel** → **Shear mode** \( \omega_H(q) \simeq -i \frac{n}{sT} q^2 \)

\[ \tilde{G}_{tx,tx}(\omega, q) \simeq \frac{N^2\pi T^3}{2} \frac{q^2 R_H(q)}{\omega - \omega_H(q)} + \frac{N^2 T}{8\pi} \sum_n \frac{q^2(\omega^2 - q^2) R_n(q)}{\omega - \omega_n(q)} \]

- **Scalar channel** → **Sound mode** \( \omega_H(q) \simeq v_s q - i \Gamma_s q^2 \)

\[ \tilde{G}_{tt,tt}(\omega, q) \simeq \frac{2}{3} \frac{N^2 T}{8\pi} \frac{q^4 R_H(q)}{\omega - \omega_H(q)} + \frac{2}{3} \frac{N^2 T}{8\pi} \sum_n \frac{q^4 R_n(q)}{\omega - \omega_n(q)} \]
Shear mode
Comparison with analytic results

\[ \omega_{\text{shear}} = -i \frac{\eta}{sT} q^2 - i \left( \frac{\eta}{sT} \right)^2 \tau_\Pi q^4 , \]

\[ \omega_{\text{sound}} = v_s q - i \Gamma_s q^2 + \frac{\Gamma_s}{v_s} \left( v_s^2 \tau_\Pi - \frac{\Gamma_s}{2} \right) q^3 . \]

AdS: \[ \tau_\Pi = \frac{2 - \log 2}{2\pi T} , \]

Hydro (shear): \[ \tau_\Pi = \frac{1 - \log 2}{2\pi T} . \]

[Baier, Romatschke, Son, Starinets, Stephanov]
Reconstructing the spectral function

- **First approximation:** \( \omega, q \to 0 \) limit, hydrodynamic mode
- The shear and diffusion spectral functions change sign with the residue
- The sound spectral function changes sign when the sound mode changes behavior
  - \( \Rightarrow \) Failure of the hydrodynamic approximation
  - \( \Rightarrow \) Higher-derivative expansion in effective theory breaks down
  
  Solution: add more modes

- The spectral function becomes positive \( \rho(\omega) \geq 0 \)
- The full retarded correlator can have analytic terms
Example: R-charge diffusive channel

The sum over poles can give divergent contributions:

\[
\text{Im } \tilde{G}_R \sim \omega \sum_n \frac{\text{Im} \left( \omega_n^2 R_n \right)}{(\omega^2 - \text{Re} \omega_n^2)^2 + (\text{Im} \omega_n^2)^2} \sim q^2 \omega \sum_n \frac{1}{n^2} < \infty
\]

\[
\text{Re } \tilde{G}_R \sim \sum_n \frac{|\omega_n|^2 \text{Re} \left( \omega_n^* R_n \right)}{(\omega^2 - \text{Re} \omega_n^2)^2 + (\text{Im} \omega_n^2)^2} \sim q^2 \sum_n \frac{1}{n} \rightarrow \infty
\]

The divergence can be cured by adding an analytic term

\[
\tilde{G}_R(\omega, q) \sim \frac{q^2 R_H(q)}{\omega - w_H(q)} + q^2 \sum_{n=1}^4 \left( \frac{R_n(q)}{\omega - w_n(q)} + \frac{-R_n^*(q)}{\omega + w_n^*(q)} \right) - C_4 q^2,
\]

Numerically, we find that \( C_4 \approx 3.9 \)
Retarded correlator
Energy-momentum tensor modes

Compare the exact retarded correlator with the sum over poles approximation and try to find the analytic contributions

- **Shear mode:**
  \[ \tilde{G}_{tx,tx} \simeq (\text{sum over poles}) + (\text{analytic term } A) \]
  \[ A \sim -\frac{i}{2} 0.9q^2\omega - 0.16q^2 + 1.89q^4 - 1.77q^2\omega^2 \]

- **Sound mode:**
  \[ \tilde{G}_{tt,tt} \simeq (\text{sum over poles}) + (\text{real constant } C') \]
  \[ C \sim 0.374 \]

⇒ The spectral function receives analytic corrections only in the shear channel

⇒ Kramers-Kronig relation:
  \[ \text{Re } \tilde{G}_R(\omega, q) = -\frac{1}{\pi} \mathcal{P} \int d\omega' \frac{\rho(\omega', q)}{\omega - \omega'} \]
Retarded correlators
Complex momentum modes

\[ \tilde{G}_R(\omega, q) \simeq \sum_n \frac{R'_n(\omega, q)}{q - q_n(\omega)}, \quad q_n = \pm k \pm i\Gamma \]

(a) Left-moving wave \((x < 0)\)
\[ e^{-i\omega t + iq x} \sim e^{-i(\omega t + k x)} e^{\Gamma x} \]

(b) Right-moving wave \((x > 0)\)
\[ e^{-i\omega t + iq x} \sim e^{-i(\omega t - k x)} e^{-\Gamma x} \]
Complex momentum modes

Why? Different physical information

\[ \langle O \rangle \sim \int d\omega \ e^{-i\omega t+iq_n(\omega)x} \tilde{j}(\omega, q_n(\omega)) \]

- \( \text{Im } q_n(\omega = 0) = m_n = 1/\ell_n \sim T \) screening mass/absorption length [Bak, Karch, Yaffe]

- Front velocity: assume \( x > 0 \), close the contour \( \text{Im } \omega < 0 \)

\[ \lim_{\omega \to \infty} \left( t - x \frac{\text{Re } q_n}{\omega} \right) \geq 0 \Rightarrow \frac{x}{t} < v_F \]

\[ v_F = \lim_{\omega \to \infty} \frac{\omega}{\text{Re } q_n(\omega)} \]

- Numerically \( v_F = 1 \) \( \Rightarrow \) Causality! Even for hydro modes
Remember:

There is an exact analytic result in $AdS_5$! (for $q = 0$)

[Myers, Starinets, Thomson]

$$
\rho_J(\omega) = -2 \, \text{Im} \, \tilde{G}_R(\omega, q = 0) = \frac{N_c^2 \omega^2}{16\pi} \frac{\sinh(\beta \omega/2)}{\cosh(\beta \omega/2) - \cos(\beta \omega/2)}
$$

$$
\omega_n = 2\pi T (1 \pm i)n, \ n \in \mathbb{Z}
$$

In $AdS_3$ $\omega_n = \pm q - i4\pi T(n + h)$ [Birmingham, Sachs, Solodukhin]

There are many numerical/semiclassical results

[Horowitz, Hubeny; Cardoso, Konoplya, Lemos; Cardoso, Natario, Schiappa; Nunez, Starinets; Fidkowsky, Hubeny, Kleban, Shenker;...]

$$
\omega_n \sim T(1 - i)n + \omega_0, \ n \in \mathbb{N}, \ etc
$$

- Our computation agrees with previous results
- Can we say anything from the field theory?
Dissipation in a free theory

- Massless scalar field, homogeneous source
  \[ \int d^4x j(t) \varphi^2(x) \]
- Thermal state: \( n(x) = 1/(e^x - 1) \) Bose-Einstein distribution
- 1-loop: \( \text{Im} \langle [\varphi^2, \varphi^2] \rangle \sim \) (pair creation)-(pair annihilation)

\[
\rho_\varphi(\omega, q = 0) = C \left[(1 + n(\beta \omega/2)) (1 + n(\beta \omega/2)) - n(\beta \omega/2)n(\beta \omega/2)\right]
\]

\[
\rho_\varphi(\omega, q = 0) = C \left[1 + 2n(\beta \omega/2)\right]
\]

There is also an infinite set of ’thermal resonances’: \( \omega_n = i4\pi T n, \ n \in \mathbb{Z} \)

Similar for composite operators in a gauge theory
Back to strong coupling

- Very different: elementary fields are not free at all, but...
- At $T = 0$, a non-conformal gauge theory has a simple large $N$ limit: free glueballs and mesons
- Is also the large-$N$, $T \neq 0$ theory simple?

Observation: R-current $q = 0$ spectral function at large 't Hooft coupling is similar to a free theory

$$\rho_J(\omega) = \frac{N^2 \omega^2}{16\pi} \left[ 1 + n \left( (1 - i)\beta \omega / 2 \right) + n \left( (1 + i)\beta \omega / 2 \right) \right]$$

Since width $\sim$ mass, no good quasiparticle interpretation!

$(1 \pm i)\omega \rightarrow \omega \sim$ free elementary fields

$(1 \pm i)\omega \rightarrow \pm i\omega \sim$ "glueball spectrum" ($\mathcal{CFT}$, $T \rightarrow 0$)
Non-zero momentum

In the free theory:

- Scattering with particles in the plasma produces Landau damping
- No poles, branch cuts between the points
  \[ \omega_n = \pm q + i4\pi T n, \text{ for each } n \in \mathbb{Z} \]

In AdS/CFT duals there are always poles

- Curvature corrections ('t Hooft coupling) do not modify this [Hartnoll, Kumar]
- Generic in gravity duals
- Large N effect?? [Kovtun, Yaffe]

Cartoon argument:
Singularities of thermal correlators

QNMs from AdS dual

Free theory

From Strings to Things INT Seattle, May 2008 – p. 29/30
Summary

- First hydro corrections: good for a fair interval of $\omega, q$ but need more modes close to $\omega, q \sim T$
- Diffusion modes decouple at $q \sim T$
- Sound mode becomes an ordinary mode
- Screening masses $\sim T$
- Causality recovered: $v_F = 1$

**Some possible features of $\mathcal{N} = 4$ plasma**

- Conformality: trivial scaling with $T$
- Large $\mathcal{N}$: isolated poles even at nonzero momentum (?)
- Simple description of large $\mathcal{N}$ plasma (?)