Heavy Quarkonium States with the Holographic Potential

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With Hai-cang Ren, JHEP 0801:029, 2008. 0710.2639

From Strings to Things, Seattle, May 2008
OUTLINES

◆ Introduction

◆ Holographic potential model from AdS/CFT

◆ Holographic potential model with IR cut.

◆ Concluding remarks
Many interesting phenomena in QCD lie in the strongly-coupled region.

Non-perturbative methods for analysis

**Lattice:** problematic with finite chemical potential, time-dependent problems

**AdS/CFT:** Notable success in RHIC physics

Viscosity, Jet quenching, …
QCD Diagramm

- Big Bang
- Hadron Gas
- RHIC
- CERN
- Nuclear Liquid
- Supernova
- Color Superconductivity
- Neutron Stars
Heavy meson melting

- At $T < T_c$, confined, potential linearizing rises.
- At $T > T_c$, deconfined. Short range attraction range: screening length $L_{sc}$.

As $T \uparrow$, $L_{sc} \downarrow$, there exists a $T_d$: the potential no longer binding for $T > T_d$ (Melting Temperature).

Heavy quarkonium melting is an important signal of QGP.
Nonperturbative calculation of $T_d$

- Potential model
  - Lattice: F. Karsch, Brambilla...
  - AdS/CFT: Maldacena, Rey, Liu, Avramis

- Spectral function
  - Lattice: Karch
  - AdS/CFT: Hoyos, Kim et al
Holographic potential model from AdS/CFT
Extracting $V(r)$ from Wilson loop

\[ W(L_{\pm}) \equiv P e^{-i \int_0^{1/T} dt A_0(t,0,0,\pm \frac{1}{2} r)} \]
\[ e^{-\frac{1}{T} F(r,T)} = \frac{\text{tr} \ < W^\dagger(L_+)W(L_-) >}{\text{tr} < W^\dagger(L_+) > < W(L_-) >} \]

- \( F(r,t) \) is the free energy excess of a static pair of \( qq \)

The internal energy reads

\[ U(r, T) = -T^2 \frac{d}{dT} \left( \frac{F(r, T)}{T} \right) \]
• F-ansatz

\[ V(r) = F(r, T) \]

• U –ansatz

\[ V(r) = U(r, T) \]
Poential From Lattice

- Quarkonium probes non-perturbative information about medium. How?

\[ W(C) = \frac{1}{N} Tr \left[ P \exp \left( i \oint_C A \right) \right] \]

\[ \langle W(C) \rangle_T \propto \exp(-iL_{\text{time}} (E(L) - E_{\text{ren}})) \]

- Wilson loop is static time-like and in fundamental representation:  

- Wilson loop is calculable in lattice QCD

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Matsui, Satz 1986

Bielefeld Group, hep-lat/0509001
Field Theory = Gravity Theory

Gauge Theories

QCD

Holography

Quantum Gravity

String theory

the large $N$ limit

Supersymmetric Yang Mills $\downarrow \; N$ large

Gravitational theory in 10 dimensions

Calculations $\rightarrow$ Correlation functions

Quark-antiquark potential
• According to the holographic principle, the thermal average of a Wilson loop operator in 4D N=4 SUSY YM at large N_c and large 't Hooft coupling corresponds to the minimum area of the string world sheet in the 5D AdS metric with a Euclidean signature,
\[ W(C) = Pe^{-i \int_C dx^\mu A_\mu(x)} \]

\[ \text{tr} \langle W(C) \rangle = e^{-\sqrt{\lambda} S_{\text{min}}[C]} . \]
\[ ds^2 = \pi^2 T^2 y^2 (f \, dt^2 + d\vec{x}^2) + \frac{1}{y^2 f} \, dy^2 \]

*bounded by the loop C, when \( y \) goes to infinity, \( y \to 1 \) BH*

\[ f = 1 - \frac{1}{y^4}. \]
Minimizing the world sheet area (the Nambu-Goto action)

\[ S[C] = (\pi T) \int_0^{\frac{1}{T}} dt \int_0^\infty dy \sqrt{1 + \pi^4 T^4 y^4 f \left( \frac{dx_3}{dy} \right)^2} \]

\[ x_3 = \pm \pi T q \int_{y_c}^{y} \frac{dy'}{\sqrt{(y'^4 - 1)(y'^4 - y_c^4)}} \]
\[ q \quad r \quad \bar{q} \]

\[ \begin{array}{c}
\text{y} \\
- \quad + \\
\hline
I_1
\end{array} \quad \begin{array}{c}
\text{BH} \\
\hline
I_2
\end{array} \]
Free energy

\[ F(r, T) = T \min(I, 0) \]

\[ I \equiv I_1 - I_2 = \sqrt{\lambda} \left[ \int_{y_c}^{\infty} dy \left( \sqrt{\frac{y^4 - 1}{y^4 - y_c^4}} - 1 \right) + 1 - y_c \right] \]
Result of potential

\[ F(r, T) = -\frac{\alpha}{r} \phi(\rho) \theta(\rho_0 - \rho), \]

\[ \phi(\rho) = 1 - \frac{\Gamma^4}{4\pi^3} \rho + \frac{3\Gamma^8}{640\pi^6} \rho^4 + O(\rho^8). \]
Bound state & Schrödinger equation

\[-\frac{1}{2m} \nabla^2 \psi + V_{\text{eff.}}(r) \psi = -E \psi\]

\[\psi(\vec{r}) = u_l(\rho) Y_{lm}(\hat{\rho})\]

\[\frac{d^2 u_l}{d\rho^2} + \frac{2}{\rho} \frac{d u_l}{d\rho} - \left[ \frac{l(l + 1)}{\rho^2} + \mathcal{V} \right] u_l = 0\]
Numerical Results

- Value of ‘t Hooft coupling:
  \[ 5.5 < \lambda < 6\pi \]

- Upper limit: QCD value of coupling at RHIC
- Lower limit: heavy quark potential, (Gubser)
- \( M_c = 1.65\,\text{GeV}, \, M_b = 4.85\,\text{GeV}, \, T_c = 186\,\text{MeV} \)
Dissociate Temperature

<table>
<thead>
<tr>
<th>ansatz</th>
<th>$J/\psi(1S)$</th>
<th>$J/\psi(2S)$</th>
<th>$J/\psi(1P)$</th>
<th>$\Upsilon(1S)$</th>
<th>$\Upsilon(2S)$</th>
<th>$\Upsilon(1P)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$</td>
<td>143-265</td>
<td>27-50</td>
<td>31-58</td>
<td>421-780</td>
<td>80-148</td>
<td>92-171</td>
</tr>
</tbody>
</table>

$Td$ in MeV
Holographic potential model with an IR cutoff

Hard Wall : Erlich, Katz, Son, Stephanov

\[ F = -\frac{T}{16\pi G_5} \int d^4x \int_0^{z_0} dz \sqrt{g} (R - 12), \]
• In the hadronic phase
\[ ds^2 = \frac{1}{z^2} (dt^2 + d\bar{x}^2 + dz^2), \]

• In the plasma phase
\[ ds^2 = \frac{1}{z^2} (f dt^2 + d\bar{x}^2 + f^{-1} dz^2), \]
• Soft-wall model 1: Karch, Katz, Son, Stephanov

\[ F = -T \frac{1}{16\pi G_5} \int d^4x \int_{\rho_0}^{\infty} dr e^{-\frac{c}{\rho^2}} \sqrt{g}(R - 12), \]

A dilaton is introduced, Metrics are the same.

• Soft-wall model 2: Andreev, Zakharov; Kajantie

modify string frame metric by a conformal factor
\[ ds^2 = \frac{e^{b z^2}}{z^2} (f dt^2 + d\bar{x}^2 + f^{-1} dz^2) , \]

- Free energy

\[ F(r, T) = -\frac{\alpha}{r} \chi(\rho, T) \]
### Td with deformed metric

<table>
<thead>
<tr>
<th>ansatz</th>
<th>$J/\psi$</th>
<th>$\Upsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td>NA</td>
<td>235-385</td>
</tr>
<tr>
<td>$U$</td>
<td>219-322</td>
<td>459-780</td>
</tr>
</tbody>
</table>
### AdS/CFT and Lattice

$$T_d / T_c$$

<table>
<thead>
<tr>
<th>ansatz</th>
<th>$J/\psi$ (holographic)</th>
<th>$J/\psi$ (lattice)</th>
<th>$\Upsilon$ (holographic)</th>
<th>$\Upsilon$ (lattice)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
<td>NA</td>
<td>1.1</td>
<td>1.3-2.1</td>
<td>2.3</td>
</tr>
<tr>
<td>$U$</td>
<td>1.2-1.7</td>
<td>2.0</td>
<td>2.5-4.2</td>
<td>4.5</td>
</tr>
</tbody>
</table>
Summary

We calculated dissociation temperatures $T_d$ of heavy quarkonium states from holographic potential

The computed $T_d$ have remarkable features comparable with that from Lattice

$$T_d = \frac{\alpha \rho_0 M}{\pi \eta_d^2} = \frac{4\pi \rho_0}{\Gamma^4 \left(\frac{1}{4}\right) \eta_d^2} \sqrt{\lambda} M.$$  $$T_d = \frac{2.17M}{\sqrt{\lambda}}.$$

To assess the validity. At melting temperature of U satz

$$0.099 < v^2 < 0.340$$
Thanks for your attention!