Correlations of scales in AdS/QCD

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Question - How two colorless objects correlate if they have very different sizes?

Motivation - "Experimental surprise" and "lattice puzzle"

Experiment (Belle) – resonances near the open charm threshold with unusual decay modes. The suggestion (Voloshin 2007) - these resonances are the bound states of the charmonium "inside" the excited light meson. The only decay modes of these resonances – c harmonium and light mesons
Lattice (2007, Boiko et. al.) – investigation of the internal structure of the QCD string. Technically- investigation of the correlator of two circular Wilson loops in Euclidean space $<W(r)W(R)>$ when $R$ is much larger then $r$. The unexpected dependence on $r$ is found numerically. The investigation concerns the broadening of the string shape and the correlator behaves as $r^2$. Being translated at the string width it would mean that the string becomes very thin.
We shall try to handle both problems within AdS/QCD approach.

Consider the simplest soft wall model (Karch et. al 2006) with nontrivial dilaton.

The heavy meson is considered as localized near UV boundary $z=0$ with the characteristic scale of $z \sim 1/m$. On the other hand the light meson wave function is mainly localized at the IR region and has nontrivial $z$ dependence. It can be treated in terms of modes of 5D fields.
Physics behind interaction- heavy quark contribution to the van der Waals type interaction due to the chromo-polarizability of the heavy meson

\[ H_{\text{eff}} = -\frac{1}{2} \alpha(Q\bar{Q}) E_i^a E_i^a, \]

where E - chromoelectric field. The next step- relate \( H_{\text{eff}} \) to the conformal anomaly
We replace the electric interaction with the sum of the electric and magnetic terms. This corresponds to a conservative treatment of the problem of the bound states because of the sign-definite magnetic contribution.

\[
\theta_{\mu} = \frac{\beta (g^2)}{4 g^4} (F_{\mu\nu}^a)^2 \approx -\frac{9}{32\pi^2} (F_{\mu\nu}^a)^2
\]

\[
H_W = -C \theta_{\mu} \quad C = (8 \pi^2/9) \alpha^{(Q\bar{Q})} [1 + O(g^2)]
\]
That is the interaction of the light meson $X$ with heavy meson is due to the dilaton exchange. The light meson in the model is described by the eigenvalue problem

$$
\left( -\frac{d^2}{dz^2} + z^2 + 2S - 2 + \frac{S^2 - 1/4}{z^2} \right) \psi_n(z) = m_n^2 \psi_n(z),
$$

where $S$ – meson spin, and

\[ m^2 = p^2 \]

The spectrum has Regge behavior

\[ m_n^2 = 4(n + S) \]
The heavy meson induces the potential for the light meson modes due to the dialton exchange in the bulk and the 4D eigenvalue problem for energy $w$ reads

\[
\left(-\Delta_x - \frac{\partial^2}{\partial z^2} + z^2 + 2S - 2 + \frac{S^2 - 1/4}{z^2} + V(z, \bar{x})\right) \chi_n(z, \bar{x}) = \omega_n^2 \chi_n(z, \bar{x})
\]

which involves three-dimensional Laplasian and potential is defined by the dilaton bulk-boundary propagator.
From the dilaton propagator one gets

\[ V(z, \bar{z}) = f(z) \int_0^\infty d\tau \left( \frac{1}{2\pi \tau} \right)^{3/2} \exp \left( -\frac{x^2}{2\tau} \right) \frac{e^{-\tau}}{\sinh^3 \tau} \exp \left( -\frac{z^2}{2} \frac{1}{\sinh \tau} \right) \]

The function \( f(z) \) can be found from the relation

\[ \langle \phi_n(z, x) | \theta^-_\mu | \phi_n(z, x) \rangle = 2 m_n^2 \langle \phi_n(z, x) | \phi_n(z, x) \rangle \]
which yields $f(z) = -c z^6$

And $c$ is determined by the polarizability.

The problem can be solved by variational procedure with the probe function

$$\chi_0(z, x) = z^{s+\frac{1}{2}} e^{-z^2/2} \xi(x)$$

Which obeys the equation

$$[-\Delta_x + U(x)] \xi(x) = (\omega^2 - 4S) \xi(x)$$
With the potential

\[ U(x) = -8 c(S+1)(S+2)(S+3) \int_{0}^{\infty} d\tau \left( \frac{1}{2\pi \tau} \right)^{3/2} \exp \left( -\frac{x^2}{2\tau} \right) \left( 1 - e^{-2\tau} \right)^{S+1} e^{-4\tau} \]

Assuming the scale factor \( \sigma \approx m_{\rho}^2/4 \approx 0.15 \text{ GeV}^2 \)

the bound state emerges starting from \( S=4 \) - numerical calculation.

However likely the polarizability can considerably larger decreasing the value of \( S \).
Experimentally there is no decay of Hadro-quarkonium into D-mesons. Why?

Consider the brane realization of this state.
Final state for the decay into two D mesons
This is the stringy tunneling process however there are strings in the initial state, that is *INDUCED* tunneling. Moreover it turns out to be induced tunneling near the top of the barrier.

Effective potential for the tunneling process

\[ V(r) \]
Horizontal slice of the Euclidean bounce solution corresponding to the induces decay of the hadro-quarkonium
The process can be also treated as the ionization of the heavy meson in the external field. Both approaches yield the same answer for the probability

\[ \Gamma \propto \exp \left( -2 \int |p| \, dr \right) \sim \exp \left( -A \sqrt{\frac{M_Q}{\Lambda_{QCD}}} \right) \]

Hence this process is suppressed exponentially indeed
Consider the Euclidean correlator

\[ C(R_2, R_1) = \langle W_{R_2}, W_{R_1}; h = 0 \rangle_{\text{connected}} \]

and assume very different radii. What is the connectness mechanism? Lattice tells the unexpected dependence on the small radius (2007) \((a \cdot \Lambda_{QCD})^2\).
The natural expectation was due to the general arguments concerning the scales of fields in the flux tube.

What AdS/QCD yields for this correlator at strong coupling? Naively one could expect string worldsheet with two boundaries at the Wilson loops. No such minimal surface if radii are very different (Olesen-Zarembo 2000).

The are two possibilities instead

14. Exchange by SUGRA mode-dilaton
2. ‘Complex time” solution for the string
- **Mode exchange. Consider expansion**

\[ W(R_1) = \langle W(R_1) \rangle \sum_{N} c_N(R_1) O_N, \quad c_N \propto R_1^{\Delta N} \]

and select the operator with the minimal dimension \( O = TrG^2 \) which couples to the dilaton

- **Immediately we get**

\[ \langle W(R_1)W(R_2) \rangle \propto \frac{R_1^4}{R_2^4} F(R_2) \]
That is the dilaton exchange result agrees with the common expectations. Is there any place for the quadratic dependence?

Consider the solution to the stringy equation of motion in the “complex time -z”. The solution looks as two “Euclidean regions” with real actions and the ‘Minkowskian region” between with the imaginary action. This resembles the QM situation with the resonant tunneling when penetration of the particle through TWO barriers is of order 1
The analogue in QM looks as follows
If the energy of the particle coincides with the metastable level between two barriers then the resonant tunneling happens. It corresponds to the infinite oscillations between two barriers. In the case of extended objects the resonant tunneling was applied to the production at threshold problem (Voloshin-A.G. 93). In the Minkowski region- oscillating closed string. The resonance condition can be formulated in WKB approximation
The solution for the string in AdS consists from three pieces

\[\begin{align*}
\epsilon < r < R_{\text{min}} & \sim R_1: \quad z \approx \sqrt{R_1^2 - r^2} \\
R_{\text{min}} < r < R_{\text{max}} & \sim R_2: \quad z \approx \sqrt{-2r} \\
R_{\text{max}} r < R_2 & \sim R_2: \quad z \approx \sqrt{R_2^2 - r^2}
\end{align*}\]

The total action is

\[S_{\text{tot}} = S_{\text{evkl},1}(R_2) + S_{\text{evkl},2}(R_1) + iS_{\text{mink}}(R_1, R_2)\]
The resonance condition reduces to

\[ S_{\text{mink}} = 2\pi N \propto \log \frac{R_2}{R_1} \]

And taking into account the string tension

\[ \frac{R_{IR}}{R_{UV}} = \exp\left(\frac{c}{g_{YM}}\right) \]

in the strong coupling regime
At the resonance condition the correlator of two Wilson loops becomes large and has nontrivial dependence on the small radius.

Problems
1. It is not clear if one can neglect the gravitational emission of the string during the oscillations in the Minkowski region.
2. Unusual dependence on the coupling constant. Probably the running string tension could help.
There are nontrivial correlators of the composite Wilson loops

\[ <W(D,R)W(D,r)> \]

Where each dyonic Wilson loop consists of arcs of magnetic and electric components. The correlator of such composite Wilson loops is saturated by the worldsheet of the dyonic string at any radii.
The horizontal slice of the configuration of two composite Wilson loops involving magnetic(M), electric(E) and dyonic(D) string worldsheets. In the Minkowski case it corresponds to the interaction of E and M degrees of freedom,(Minahan 98)
Conclusions

- In the Minkowski geometry correlation of “IR” and ‘UV ’s scales provides the possibility for the exotic bound state- hadro-quarkonium to exist.
- In the Euclidean geometry the exchange of SUGRA modes - no surprises.
- The possibility of the resonant tunneling in the Euclidean case - interesting nonperturbative IR/UV mixing.
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