Origin of hadron mass and phases of QCD at finite density

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Contents of the Talk

- The folklore phase diagram of QCD
- Chiral symmetry and origin of hadron mass
- Quarkyonic matter at finite chemical potential and not high $T$
- Generalized NJL ('t Hooft) Model
- Inclusion of a finite chemical potential
- Chiral symmetry restoration
- Meson spectra
- Gap in quarkyonic matter
- Possible phase diagram
A source: NJL type models. Not confining! Why it was considered to be reasonable? Because there was a "self-evident" opinion that a hadron mass arises from spontaneous breaking of chiral symmetry in vacuum.

(i) BAG model; (ii) Casher; Banks & Casher argument
If quark is confined, then chiral symmetry must be spontaneously broken.

At $T = \mu = 0$ consistent with the ’t Hooft anomaly matching conditions.
But does it imply that the WHOLE mass originates from SBCS?
Evidence for chiral symmetry breaking in the nucleon

- No approximately degenerate chiral partner
- Large axial charge, $G_A = 1.26$
- Large pion-nucleon coupling constant, $g \sim 13$

Implications:

- Origin of the nucleon mass is spontaneous breaking of chiral symmetry in the vacuum:
  $$<\bar{q}q> \rightarrow M_N$$
- Nonlinear realization of chiral symmetry in the nucleon

The axial rotation transforms nucleon into itself plus pion
Low and high lying baryon spectra.

Low-lying spectrum: spontaneous breaking of chiral symmetry dominates physics.

High-lying spectrum: parity doubling indicates onset of a new physical regime - EFFECTIVE chiral symmetry restoration.

Mass of excited baryons comes mostly from the chirally symmetric dynamics and baryons decouple from the quark condensate of the vacuum.

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The high-lying mesons are from LEAR. They must be confirmed. Still missing states must be found.

Large symmetry: $N = n + J$ plus chiral symmetry.
An alternative: $N = n + L$ without chiral symmetry is not consistent with the Lorentz symmetry and unitarity.
Chiral parity doublet

A free $I = 1/2$ chiral doublet $B$ in the $(0, 1/2) + (1/2, 0)$ representation:

$$B = \begin{pmatrix} B_+ \\ B_- \end{pmatrix}.$$  \hspace{1cm} (1)

The axial rotation mixes the positive and negative parity components:

$$B \rightarrow \exp \left( i \frac{\theta^a A^a}{2} \sigma_1 \right) B.$$  \hspace{1cm} (2)

A chiral-invariant Lagrangian

$$\mathcal{L}_0 = i \bar{B} \gamma^\mu \partial_\mu B - m_0 \bar{B} B$$  \hspace{1cm} (3)

- A nonzero chiral-invariant (!) mass $m_0$.
- $g^A_+ = g^A_- = 0$, while the off-diagonal axial charge, $|g^A_+| = |g^A_-| = 1$.
- Pion decouples: $G_{\pi B_+ B_-} = 0$.

B. W. Lee, 1972: "We dismiss this model as physically uninteresting"
Two possibilities to satisfy chiral symmetry:

1. **Standard scenario (ground states of baryons):** (i) fermions are massless in the Wigner-Weyl mode; (ii) no independent chiral partners; (iii) fermion mass is generated in the Nambu-Goldstone mode only due to spontaneous breaking of chiral symmetry in the vacuum (via the coupling of the fermion with the chiral condensate); (iv) $G^A \sim 1$.

2. **Chiral restoration scenario (excited states):** (i) parity-doubled fermions are massive already in the Wigner-Weyl mode, this mass is manifestly chiral-invariant; (ii) the role of the chiral condensate in the vacuum in the Nambu-Goldstone mode is to lift a degeneracy of the opposite-parity fermions; (iii) effective chiral restoration means that these opposite-parity fermions almost entirely decouple from the chiral condensate and most of their mass is chiral-invariant; (iv) $G^A \sim 0$. 

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\[ B_\pm \rightarrow N\pi \text{ decays} \]

If a state is a member of an approximate chiral multiplet, then its decay into \( N\pi \) must be suppressed, \( (f_{BN\pi}/f_{NN\pi})^2 \ll 1 \). If, on the contrary, this excited nucleon has no chiral partner and hence its mass is due to chiral symmetry breaking in the vacuum, then it should strongly decay into \( N\pi \), \( (f_{BN\pi}/f_{NN\pi})^2 \sim 1 \).

<table>
<thead>
<tr>
<th>Spin</th>
<th>Chiral multiplet</th>
<th>Representation</th>
<th>( (f_{B_+N\pi}/f_{NN\pi})^2 )</th>
<th>( (f_{B_-N\pi}/f_{NN\pi})^2 )</th>
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</thead>
<tbody>
<tr>
<td>1/2</td>
<td>( N_+ (1440) - N_- (1535) )</td>
<td>( (1/2, 0) \oplus (0, 1/2) )</td>
<td>0.15</td>
<td>0.026</td>
</tr>
<tr>
<td>1/2</td>
<td>( N_+ (1710) - N_- (1650) )</td>
<td>( (1/2, 0) \oplus (0, 1/2) )</td>
<td>0.0030</td>
<td>0.026</td>
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<tr>
<td>3/2</td>
<td>( N_+ (1720) - N_- (1700) )</td>
<td>( (1/2, 0) \oplus (0, 1/2) )</td>
<td>0.023</td>
<td>0.13</td>
</tr>
<tr>
<td>5/2</td>
<td>( N_+ (1680) - N_- (1675) )</td>
<td>( (1/2, 0) \oplus (0, 1/2) )</td>
<td>0.18</td>
<td>0.012</td>
</tr>
<tr>
<td>7/2</td>
<td>( N_+ (?) - N_- (2190) )</td>
<td>?</td>
<td>?</td>
<td>0.00053</td>
</tr>
<tr>
<td>9/2</td>
<td>( N_+ (2220) - N_- (2250) )</td>
<td>?</td>
<td>0.0000022</td>
<td>0.0000020</td>
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<tr>
<td>11/2</td>
<td>( N_+ (?) - N_- (2600) )</td>
<td>?</td>
<td>?</td>
<td>0.000000064</td>
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<tr>
<td>3/2</td>
<td>( N_- (1520) )</td>
<td>no chiral partner</td>
<td>2.5</td>
<td></td>
</tr>
</tbody>
</table>

A 100% correlation of decays with the parity doublet patterns!
Takahashi-Kunihiro lattice results for $N(1535)$

Unquenched QCD with two dynamical flavors: arXiv:0801.4702
Low and high lying hadron spectra.

Mass generation mechanisms in the low- and high-lying hadrons are essentially DIFFERENT

Chiral symmetry breaking in the vacuum is almost irrelevant to dynamics in excited hadrons. Mass has mostly a manifestly chirally symmetric origin.

In contrast to: Gell-Mann - Levy sigma model, Nambu - Jona-Lasinio mechanism and many QCD-like microscopical models.
The McLerran - Pisarski proposal for a not high T (2007)

Standard picture for a finite chemical potential above the critical one:

In the large $N_c$ world at large chemical potential a pressure $\sim N_c$. For a pure hadronic gas it must be $\sim 1$. For a deconfining quark-gluon matter it must be $\sim N_c^2$. Then the deconfining quark-gluon matter should not exist. Instead - QUARKYONIC phase with confined hadrons on top of the quark Fermi sea.

Above the chiral restoration point: confined but chirally symmetric hadron ?!? It is in conflict with all previous models and intuition.

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Generalized NJL ('t Hooft) model.

In 1+1 't Hooft model the only interaction is the Coulomb (linear) potential.


Chiral symmetry breaking is via the Schwinger-Dyson (gap) equation. Infrared regularization is required.

\[ S = S_0 + S_0 \Sigma S \]
Generalized NJL (’t Hooft) model

The gap equation:

\[ i\Sigma(p^\dagger) = \hbar \int \frac{d^4k}{(2\pi)^4} V_{CONF}(p^\dagger - k) \gamma_0 \frac{1}{S_0^{-1}(k_0, k) - \Sigma(k) \gamma_0}. \]

The self-energy consists of the scalar (chiral symmetry breaking) \( M(p^\dagger) \sim A_p \) part and vector \( B_p \) (chiral symmetric) parts: \( \Sigma(p^\dagger) = A_p + (\gamma_5 \hat{p})(B_p - p). \)

They come entirely from quantum fluctuations - loops!

Then the dispersive law is: \( E_p = A_p \sin \phi_p + B_p \cos \phi_p; \quad \tan \phi_p = \frac{A_p}{B_p} \)

\begin{align*}
\phi_p & \quad 0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \quad 1.2 \quad 1.4 \quad 1.6 \\
p & \quad 0 \quad 1 \quad 2 \quad 3 \\
M(p) & \quad 0.0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad 0.6 \quad 0.7 \quad 0.8 \\
p & \quad 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2
\end{align*}

chiral angle dynamical mass

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Inclusion of a finite chemical potential

We have to remove from the gap equation all occupied levels below $P_f$ - Pauli blocking.

PROBE QUARK

$E < 0$

$E > 0$

$p$

$p_f = 0$

$p_f = 0.02$

$p_f = 0.04$

$p_f = 0.06$

$p_f = 0.08$

$p_f = 0.1$

chiral angle dynamical mass

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Above the critical Fermi momentum, $P_f > P_f^{cr}$, there is no nontrivial solution of the gap equation. Chiral symmetry gets restored:

$$\varphi_p = 0; \quad M(p) = 0; \quad <\bar{q}q> = 0$$
Chiral symmetry restoration

\[ \varphi_p = 0 \rightarrow M(p) = 0; \quad < \bar{q}q > = 0 \]

Then in the self-energy operator,

\[ \Sigma(p^\uparrow) = A_p + (\gamma^\uparrow p)[B_p - p], \]

\[ A_p = 0; \quad B_p \rightarrow \text{infrared divergent} \]

Quarks are still confined, because a single-quark Dirac operator is still infrared-divergent.

A single quark is removed from the spectrum at any chemical potential.

There are no single quark excitations of the quarkyonic matter.
Meson spectra

Given a quark Green function from a gap equation, solve a Bethe-Salpeter equation at finite chemical potential.

Below the chiral restoration point we should expect 4 Goldstone bosons and chiral symmetry breaking in hadron spectra.

Above the chiral symmetry restoration point - multiplets of $SU(2)_L \times SU(2)_R$.

\[ J = 0 \]

\[
\begin{align*}
(1/2, 1/2)_a &: 1, 0^{-+} \leftrightarrow 0, 0^{++} \\
(1/2, 1/2)_b &: 1, 0^{++} \leftrightarrow 0, 0^{-+},
\end{align*}
\]

<table>
<thead>
<tr>
<th>Even $J &gt; 0$</th>
<th>Odd $J &gt; 0$</th>
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</thead>
<tbody>
<tr>
<td>$(0, 0)$ : $0, J^{-+} \leftrightarrow 0, J^{++}$</td>
<td>$(0, 0)$ : $0, J^{++} \leftrightarrow 0, J^{-+}$</td>
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<tr>
<td>$(1/2, 1/2)_a$ : $1, J^{++} \leftrightarrow 0, J^{++}$</td>
<td>$(1/2, 1/2)_a$ : $1, J^{+-} \leftrightarrow 0, J^{--}$</td>
</tr>
<tr>
<td>$(1/2, 1/2)_b$ : $1, J^{++} \leftrightarrow 0, J^{-+}$</td>
<td>$(1/2, 1/2)_b$ : $1, J^{--} \leftrightarrow 0, J^{+-}$</td>
</tr>
<tr>
<td>$(0, 1) \oplus (1, 0)$ : $1, J^{++} \leftrightarrow 1, J^{--}$</td>
<td>$(0, 1) \oplus (1, 0)$ : $1, J^{--} \leftrightarrow 1, J^{++}$</td>
</tr>
</tbody>
</table>
Spectra below the chiral restoration point

\[
J = 0
\]

\[
\begin{array}{c|c|c|c}
\hline
& (1/2, 1/2)_a & (1/2, 1/2)_b \\
\hline
10 & 10 & 10 & 10 \\
9 & 9 & 9 & 9 \\
8 & 8 & 8 & 8 \\
7 & 7 & 7 & 7 \\
6 & 6 & 6 & 6 \\
5 & 5 & 5 & 5 \\
4 & 4 & 4 & 4 \\
3 & 3 & 3 & 3 \\
2 & 2 & 2 & 2 \\
1 & 1 & 1 & 1 \\
\hline
\end{array}
\]

\[
1, 0^{--} \quad 0, 0^{++} \quad 0, 0^{--} \quad 1, 0^{++}
\]

\[
J = 2
\]

\[
\begin{array}{c|c|c|c}
\hline
& (0, 0) & (1/2, 1/2)_a & (1/2, 1/2)_b & (0, 1) \oplus (1, 0) \\
\hline
10 & 10 & 10 & 10 & 10 \\
9 & 9 & 9 & 9 & 9 \\
8 & 8 & 8 & 8 & 8 \\
7 & 7 & 7 & 7 & 7 \\
6 & 6 & 6 & 6 & 6 \\
5 & 5 & 5 & 5 & 5 \\
4 & 4 & 4 & 4 & 4 \\
3 & 3 & 3 & 3 & 3 \\
2 & 2 & 2 & 2 & 2 \\
1 & 1 & 1 & 1 & 1 \\
\hline
\end{array}
\]

\[
0, 2^{--} \quad 0, 2^{++} \quad 1, 2^{++} \quad 0, 2^{++} \quad 1, 2^{++}
\]
Spectra above the chiral restoration point

### $J = 0$

<table>
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<tr>
<th></th>
<th>$(\frac{1}{2}, \frac{1}{2})_a$</th>
<th>$(\frac{1}{2}, \frac{1}{2})_b$</th>
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<td>0, 0$^++$</td>
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### $J = 2$

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<th>$(0, 0)$</th>
<th>$(\frac{1}{2}, \frac{1}{2})_a$</th>
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A quark - quark-hole excitation of the quark Fermi sea is impossible (a single quark Dirac operator is always infrared-divergent). Hadronic excitation - possible.

It is valid both below and above chiral restoration point.
In the chiral symmetry broken phase there always exists a gapless excitation of the quark Fermi sea: one can excite an arbitrary amount of the Goldstone bosons.

Above the chiral restoration point the system is still confined. No Goldstone excitations, no single quark excitations. THEN THERE IS A GAP.

red - pion mass; green - sigma mass
There is neither a Cooper pairing nor a condensation of a Fermi system into a quasibosonic system, because it requires an attractive pairing interaction between fermions with NEGATIVE POTENTIAL ENERGY.

A confining interaction with POSITIVE POTENTIAL ENERGY cannot destabilize a Fermi surface. Below the Fermi surface there are NO confined hadrons.

A mechanism of a gap formation is based on simultaneous properties of confinement and manifest chiral symmetry.

The gap increases with the chemical potential and can be arbitrary large.

There cannot be dissipation of energy.
Possible phase diagram

- QUARK – GLUON MATTER
- QUARK FERMI SYSTEM WITH LARGE GAP
- HADRONIC
  - CONFINED
  - BROKEN CHIRAL SYMMETRY
- QUARKYONIC
  - CONFINED
  - RESTORED CH. SYMMETRY

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Self-energy integrals; Infrared divergences

\[ S = S_0 + \Sigma_{S_0} + \Sigma_{S_0} \Sigma_{S_0} + \ldots = S_0 + \Sigma_{S_0} S \]

\[ \Sigma = \Sigma_{S_0} + \ldots = S \]

\[ \Sigma(p) = A_p + (\gamma \hat{p})[B_p - p], \]

\[ A_p = \frac{1}{2} \int_{p_f}^{\infty} \frac{d^3 k}{(2\pi)^3} V(p - \hat{k}) \sin \varphi_k, \]

\[ B_p = p + \frac{1}{2} \int_{p_f}^{\infty} \frac{d^3 k}{(2\pi)^3} (\hat{p} \cdot \hat{k}) V(p - \hat{k}) \cos \varphi_k. \]

\[ A_p \cos \varphi_p - B_p \sin \varphi_p = 0. \]