Model Building Tools from the D4-D8 System

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Carone, JE, Tan arXiv:0704.3084
Carone, JE, Sher arXiv:0802.3702
JE, Tan arXiv:080??.????
Outline

Basic Question: What might AdS/QCD be good for besides QCD?

- Quick Review of Sakai-Sugimoto model
- The D4-D8-D8 system as an EWSB model
- Enhanced symmetries from extra dimensions
- General lessons from stringy AdS/Technicolor
Massless fluctuations of D4 branes describe non-supersymmetric SU(N) gauge theory
Adding massless chiral quarks
Sakai, Sugimoto; c.f. Kruczenski, Mateos, Myers, Winters (not chiral)

Massless fluctuations of D4 branes describe non-supersymmetric SU(N) gauge theory

Strings stretching from D4’s to D8’s are massless chiral quarks
There is a one-parameter set of D8-brane configurations that minimize the D8-brane action.
QCD bound state masses and interactions

- Probe brane limit: Assume $N_f \ll N$ (Karch, Katz)
- Determine shape of D8-brane by minimizing D8-brane action in D4-brane background
- Fluctuations of the D8-brane describe bound states of quarks; D8-brane action determines their masses, decay constants, form factors, couplings, …
This is not QCD!

KK scale \sim \text{confinement scale}: \text{can’t separate additional physics from QCD states of interest} \ (c.f. \ Polchinski-Strassler).

Quarks are massless so far \textbf{(but some debate: see Evans-Threlfall)}.

Chiral symmetry breaking scale can be separated from confining scale.

We ignore 4 dimensions on the D8-brane.

Large-N-iness reflected in spectral functions, etc.
D4 brane geometry

\[ ds^2 = \left( \frac{U}{R} \right)^{3/2} \left( \eta_{\mu\nu} dx^\mu dx^\nu - f(U) d\tau^2 \right) - \left( \frac{R}{U} \right)^{3/2} \left( \frac{dU^2}{f(U)} + U^2 d\Omega_4^2 \right) \]

\[ f(U) = 1 - \frac{U_{KK}^3}{U^3} \]

\[ R^3 = \pi g_s N_l^3_s \]

\[ \tau \text{ period: } \frac{4\pi R^{3/2}}{(3U_{KK}^{1/2})} \]
Probe brane limit \( N_f \ll N_c \)

D8-brane action

\[
S_{DBI} = -T \int d^9 x \, e^{-\phi} \sqrt{\det g_{MN}}
\]

\[
e^{\phi} = g_s \left( \frac{U}{R} \right)^{3/4}
\]

Stationary Solution:

\[
f(U) + \left( \frac{R}{U} \right)^3 \frac{U'(\tau)^2}{f(U)} = \frac{U^8 f(U)^2}{U_0^8 f(U_0)}
\]
Probe brane limit $N_f \ll N_c$

Induced metric on D8-brane:

$$ds^2 = \left(\frac{U}{R}\right)^{3/2} \eta_{\mu\nu} dx^\mu dx^\nu - \left(\frac{R}{U}\right)^{3/2} U^2 d\Omega_4^2 - \left(\frac{R}{U}\right)^{3/2} \left[ \frac{1}{f(U)} + \left(\frac{U}{R}\right)^3 \frac{f(U)}{U'(\tau)^2} \right] dU^2$$

$$f(U) = 1 - \frac{U_{KK}^3}{U^3}$$

$$R^3 = \pi g_s N l_s^3$$
Vector mesons on the D8-branes

SU($N_f$) gauge fields live on the D8-branes – identify 4D modes with vector mesons

$$ S_{D8} = -T \int_{D8 + \overline{D8}} d^4x \, dU \, d\Omega_4 \, e^{-\phi} \sqrt{\det (g_{MN} + (2\pi \alpha') F_{MN})} $$

$$ S_{D8} \approx -\frac{3}{2} \tilde{T} (2\pi \alpha')^2 R^3 U_{KK}^{-1/2} \cdot \int d^4x \, dU \, \text{Tr} \left[ \frac{1}{2} F_{\mu\nu} F^{\mu\nu} U^{-1/2} \gamma(U)^{1/2} + F_{\mu U} F^{\mu U} U^{5/2} R^{-3} \gamma(U)^{-1/2} \right] $$

$$ \gamma(U) = \frac{U^8}{U^8 f(U) - U_0^8 f(U_0)} $$
Vector mesons on the D8-branes

Solve equations of motion for modes of the vector field
Symmetric modes are identified with vector resonances
Antisymmetric modes are identified with axial vector resonances

Vector and axial vector masses alternate
What other physics might AdS/QCD describe?

Condensed matter? (Son; Hartnoll, Herzog, Horowitz)

Electroweak sector?
Holographic Technicolor
Hirn, Sanz; Piai; Agashe, Csaki, Grojean, Reece; Hong, Yee, …

AdS/QCD is like QCD.

Technicolor is like QCD.

Hence, AdS/QCD is like technicolor.
QCD and Electroweak Symmetry Breaking

Quarks are charged under electroweak symmetry, which in the quark sector is a gauged subgroup of the chiral symmetry $\times U(1)_{B-L}$.

\[ q_L q_R + q_R q_L \text{ charged under } SU(2)_W \times U(1)_Y, \]

Invariant under $U(1)_{EM}$

Even in the absence of any other source of EWSB, nonvanishing $\langle q_L q_R + q_R q_L \rangle$ breaks EW to EM, gives small mass to W, Z bosons.

But we need more EWSB -- lot’s more!
Assume a new asymptotically free gauge group factor $G_{TC}$ with $N_F$ techniquark flavors

Gauge a $SU(2) \times U(1)$ subgroup of the chiral symmetry ($\times U(1)_{\text{B-L}}$)

Identify with electroweak gauge invariance

The chiral condensate breaks the electroweak symmetry to $U(1)_{\text{EM}}$

The good: No fundamental scalars – no hierarchy problem

The bad: Estimates of precision electroweak observables disagree with experiment $\rightarrow$ walking TC may be better

The ugly: No fermion masses $\rightarrow$ Extended Technicolor
Minimal Technicolor

Gauge group: $G_{TC} \times SU(2) \times U(1)$

$SU(2)$ technifermion doublet $Q_L = (U, D)_L$

$SU(2)$ technifermion singlets $U_R, D_R$

Technifermion condensate $<U U + D D> = 4\pi f^3$

Breaks Electroweak Symmetry

(No Standard Model fermion masses yet, doesn’t satisfy electroweak constraints…)
Question: Can the D4-D8 system give us insight into how to cure the problems with technicolor models?

Answer: Yes. Well, sort of.
Gauging the EW symmetry on the D8 branes

Decompose the D8-brane gauge fields in modes

Turn on non-vanishing solutions at boundaries (zero modes)

These solutions correspond to sources for the electroweak currents

Decay constants are read off of couplings between sources and resonances

\[ \kappa = \frac{g^2 N^2}{108 \pi^3} \]
### Precision Electroweak Constraints

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
<th>Standard Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_t$ [GeV]</td>
<td>172.7 ± 2.9 ± 0.6</td>
<td>172.7 ± 2.8</td>
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<tr>
<td>$M_W$ [GeV]</td>
<td>80.450 ± 0.058</td>
<td>80.376 ± 0.017</td>
</tr>
<tr>
<td></td>
<td>80.392 ± 0.039</td>
<td></td>
</tr>
<tr>
<td>$M_Z$ [GeV]</td>
<td>91.1876 ± 0.0021</td>
<td>91.1874 ± 0.0021</td>
</tr>
<tr>
<td>$\Gamma_Z$ [GeV]</td>
<td>2.4962 ± 0.0023</td>
<td>2.4968 ± 0.0011</td>
</tr>
<tr>
<td>$\Gamma$(had) [GeV]</td>
<td>1.7444 ± 0.0020</td>
<td>1.7434 ± 0.0010</td>
</tr>
<tr>
<td>$\Gamma$(inv) [MeV]</td>
<td>499.0 ± 1.5</td>
<td>501.65 ± 0.11</td>
</tr>
<tr>
<td>$\Gamma(\ell^+\ell^-)$ [MeV]</td>
<td>83.984 ± 0.086</td>
<td>83.996 ± 0.021</td>
</tr>
<tr>
<td>$\sigma_{had}$ [nb]</td>
<td>41.541 ± 0.037</td>
<td>41.467 ± 0.009</td>
</tr>
<tr>
<td>$R_e$</td>
<td>20.804 ± 0.050</td>
<td>20.756 ± 0.011</td>
</tr>
<tr>
<td>$R_\mu$</td>
<td>20.785 ± 0.033</td>
<td>20.756 ± 0.011</td>
</tr>
<tr>
<td>$R_\tau$</td>
<td>20.764 ± 0.045</td>
<td>20.801 ± 0.011</td>
</tr>
<tr>
<td>$R_b$</td>
<td>0.21629 ± 0.00066</td>
<td>0.21578 ± 0.00010</td>
</tr>
<tr>
<td>$A_{(0,e)}$</td>
<td>0.1721 ± 0.0030</td>
<td>0.17230 ± 0.0004</td>
</tr>
<tr>
<td>$A_{FB}$</td>
<td>0.1721 ± 0.0030</td>
<td>0.17230 ± 0.0004</td>
</tr>
<tr>
<td>$A_{(0,\mu)}$</td>
<td>0.1045 ± 0.0025</td>
<td>0.01622 ± 0.00025</td>
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<tr>
<td>$A_{FB}$</td>
<td>0.1045 ± 0.0025</td>
<td>0.01622 ± 0.00025</td>
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<tr>
<td>$A_{(0,\tau)}$</td>
<td>0.0163 ± 0.0013</td>
<td>0.0163 ± 0.0013</td>
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<tr>
<td>$A_{FB}$</td>
<td>0.0163 ± 0.0013</td>
<td>0.0163 ± 0.0013</td>
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<tr>
<td>$A_{(0,b)}$</td>
<td>0.0188 ± 0.0017</td>
<td>0.0188 ± 0.0017</td>
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<tr>
<td>$A_{(0,c)}$</td>
<td>0.0992 ± 0.0016</td>
<td>0.1031 ± 0.0008</td>
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<tr>
<td>$A_{FB}$</td>
<td>0.0992 ± 0.0016</td>
<td>0.1031 ± 0.0008</td>
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<tr>
<td>$A_{(0,\ell)}$</td>
<td>0.0707 ± 0.0035</td>
<td>0.0737 ± 0.0006</td>
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<tr>
<td>$A_{FB}$</td>
<td>0.0707 ± 0.0035</td>
<td>0.0737 ± 0.0006</td>
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<tr>
<td>$A_{(0,\nu)}$</td>
<td>0.0976 ± 0.0114</td>
<td>0.1032 ± 0.0008</td>
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<tr>
<td>$A_{FB}$</td>
<td>0.0976 ± 0.0114</td>
<td>0.1032 ± 0.0008</td>
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<tr>
<td>$\chi^2_{(A_{FB})}$</td>
<td>0.2324 ± 0.0012</td>
<td>0.23152 ± 0.00014</td>
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<tr>
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<td>0.2324 ± 0.0012</td>
<td>0.23152 ± 0.00014</td>
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<tr>
<td>$A_e$</td>
<td>0.15138 ± 0.00216</td>
<td>0.1471 ± 0.0011</td>
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<tr>
<td></td>
<td>0.15138 ± 0.00216</td>
<td>0.1471 ± 0.0011</td>
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<tr>
<td>$A_\mu$</td>
<td>0.142 ± 0.015</td>
<td>0.1498 ± 0.0049</td>
</tr>
<tr>
<td>$A_\tau$</td>
<td>0.136 ± 0.015</td>
<td>0.136 ± 0.015</td>
</tr>
<tr>
<td></td>
<td>0.136 ± 0.015</td>
<td>0.136 ± 0.015</td>
</tr>
<tr>
<td>$A_b$</td>
<td>0.923 ± 0.020</td>
<td>0.9347 ± 0.0001</td>
</tr>
<tr>
<td>$A_c$</td>
<td>0.670 ± 0.027</td>
<td>0.6678 ± 0.0005</td>
</tr>
<tr>
<td>$A_s$</td>
<td>0.895 ± 0.091</td>
<td>0.8356 ± 0.0001</td>
</tr>
<tr>
<td>$g_1^2$</td>
<td>0.30005 ± 0.000137</td>
<td>0.30378 ± 0.00021</td>
</tr>
<tr>
<td>$g_2^2$</td>
<td>0.03070 ± 0.00110</td>
<td>0.03006 ± 0.00003</td>
</tr>
<tr>
<td>$g_3^2$</td>
<td>0.03070 ± 0.00110</td>
<td>0.03006 ± 0.00003</td>
</tr>
<tr>
<td>$g_4^2$</td>
<td>-0.040 ± 0.015</td>
<td>-0.0396 ± 0.0003</td>
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<tr>
<td>$g_A^e$</td>
<td>-0.507 ± 0.014</td>
<td>-0.5064 ± 0.0001</td>
</tr>
<tr>
<td>$A_{PV}$</td>
<td>-1.31 ± 0.17</td>
<td>-1.53 ± 0.02</td>
</tr>
<tr>
<td>$Q_W$(Cs)</td>
<td>-72.62 ± 0.46</td>
<td>-73.17 ± 0.03</td>
</tr>
<tr>
<td>$Q_W$(Tl)</td>
<td>-116.6 ± 3.7</td>
<td>-116.78 ± 0.05</td>
</tr>
<tr>
<td>$\Gamma(b\rightarrow\tau\nu)$</td>
<td>$3.35^{+0.50}_{-0.44} \times 10^{-3}$</td>
<td>$(3.22 \pm 0.09) \times 10^{-3}$</td>
</tr>
<tr>
<td>$\Gamma(b\rightarrow X\nu)$</td>
<td>$20.39^{+1.68}_{-0.78}$</td>
<td>$20.87 \pm 1.76$</td>
</tr>
<tr>
<td>$\Gamma(q\rightarrow q-\nu)$</td>
<td>$451.17 \pm 0.82$</td>
<td>$450.92 \pm 0.10$</td>
</tr>
<tr>
<td>$\tau_\tau [\text{fs}]$</td>
<td>$290.79 \pm 0.78$</td>
<td>$291.87 \pm 1.76$</td>
</tr>
</tbody>
</table>
Chiral Symmetry Breaking in QCD/Technicolor

\[ \bar{q}_L q_R \rightarrow \bar{q}_L U_L^\dagger U_R q_R \]

Nonvanishing \( \langle \bar{q}_L q_R \rangle \) breaks chiral symmetry to diagonal subgroup (Isospin)

\[
\begin{align*}
  i \int d^4 x \, e^{i q \cdot x} \langle J_{\mu}^a V (x) J_{\nu}^b V (0) \rangle &= \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \delta^{ab} \, \Pi_V (-q^2) \\
  i \int d^4 x \, e^{i q \cdot x} \langle J_{\mu}^a A (x) J_{\nu}^b A (0) \rangle &= \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \delta^{ab} \, \Pi_A (-q^2)
\end{align*}
\]

\[
\Pi_V (-q^2) = \sum_n \frac{g_{Vn}^2 q^2}{m_{Vn}^2 (-q^2 + m_{Vn}^2)} ,
\]

\[
\Pi_A (-q^2) = -f_\pi^2 + \sum_n \frac{g_{An}^2 q^2}{m_{An}^2 (-q^2 + m_{An}^2)}. 
\]

Goldstones
Oblique Parameters

Peskin, Takeuchi; Golden, Randall; Holdom, Terning

Oblique corrections describe influence of new physics on vacuum polarization corrections to electroweak observables

-- Parametrized by three quantities that can be calculated from matrix elements of products of electroweak currents: $S, T, U$

$S$: measures neutral gauge boson kinetic terms
$T$: measures relation between $m_W$ and $m_Z$
$U$: measures relation between $W$ and $Z$ kinetic terms

In most acceptable models, $T$ and $U$ are protected by a custodial symmetry.

Sikivie, Susskind, Voloshin, Zakharov
The trouble with technicolor

The S parameter in QCD-like technicolor theories is estimated to be too large to be consistent with precision electroweak measurements.

\[ \Pi_V(-q^2) = \sum_n \frac{g_{Vn}^2 q^2}{m_{Vn}^2 (-q^2 + m_{Vn}^2)} , \]

\[ \Pi_A(-q^2) = -f_\pi^2 + \sum_n \frac{g_{An}^2 q^2}{m_{An}^2 (-q^2 + m_{An}^2)} . \]

\[ S = -4\pi \frac{d}{dQ^2} (\Pi_V - \Pi_A) \bigg|_{Q^2=0} \]
Constraints on $S,T$

$\Gamma_Z, \sigma_{\text{had}}, R_I, R_q$

asymmetries

$M_W$

$\nu$ scattering

$Q_W$

E 158

all: $M_H = 117$ GeV

all: $M_H = 340$ GeV

all: $M_H = 1000$ GeV
The SS S Parameter – first few contributions

\[ S = 4\pi \sum_n \left( \frac{g_{Vn}^2}{m_{Vn}^4} - \frac{g_{An}^2}{m_{An}^4} \right) \]

\[ \kappa = \frac{g^2 N^2}{108\pi^3} \]
The S Parameter – sum over all modes

\[ S = -12\pi \frac{U_{KK}^{-1/2} \kappa}{R^3} \left[ U^{5/2} \gamma^{-1/2} \frac{d}{dq^2} \left( \partial_U \psi_V^0 - \partial_U \psi_A^0 \right) \right]_{U=\infty, \, q^2=0} \]

\[ g^2N=4\pi, \, N=4 \]

\[ S = 58.9 \kappa \approx 0.9 \]

Factor of 10 too big
Separating the Standard Model from New Physics

JE, Tan (to appear)

The S parameter is more precisely defined with respect to a Reference Standard Model with some Higgs mass.

\[ S = S_{TC} - S_{SM} \]
Reference Standard Model contributions to $S_{\text{SM}}$:

Define

$$R_{V,A}(s) = -12\pi \text{Im} \prod_{V,A}(s)$$

$$S = \frac{1}{3\pi} \int_{0}^{\infty} ds \frac{ds}{s} \left[ R_V(s) - R_A(s) \right]$$

$$-\frac{1}{4} \left( 1 - \left( 1 - \frac{m_H^2}{s} \right)^3 \theta(s - m_H^2) \right)$$

Reference Higgs mass

Peskin, Takeuchi

Diagram:

- $\pi^+ \pi^- \pi^+$
- $H \pi^0$
- $H, \pi^0$
1/N corrections to S-parameter

Inconsistency in AdS/QCD due to large-N with N=3:

Poles for real $q^2 \rightarrow$ implies zero-width resonances.

But, decay rates can be calculated from meson couplings, do not vanish.

Solution:

Self-consistently, we can assume location of poles predicts spectrum, but fix analytic properties of spectral functions by including hadronic loop corrections.
Include Goldstone loops, use vector meson dominance:

\[ \Pi_V = \frac{\rho}{\pi} \cdot \frac{\rho}{\pi} + \frac{\rho}{\pi} \cdot \frac{\rho}{\pi} \cdot \frac{\rho}{\pi} \cdot \rho + \ldots \]

Calculated by AdS/QCD

\[ R_V(s) \approx \frac{1}{4} \frac{m_\rho^4}{(s - m_\rho^2)^2 + m_\rho^2 \Gamma_{\rho \rightarrow \pi \pi}(s)} \]

Calculated by AdS/QCD

\[ \Gamma_{\rho \rightarrow \pi \pi}(s) = \frac{g_{\rho\pi\pi}^2}{48\pi} \frac{s}{m_\rho} \]

Result (preliminary): S parameter even larger than expected from scaled-up QCD estimates.
Other phenomenology

This model doesn’t satisfy electroweak constraints yet, but what else could be predicted in the model?

\[ f_\pi^2 = -\Pi_A(0) = (246 \text{ GeV})^2 \]

\[ m_\rho \approx 1.5 \text{ TeV} \]

\[ m_{a_1} \approx 2.2 \text{ TeV} \]

\[ M_{KK} = \frac{218}{\sqrt{\kappa}} \text{ GeV} \approx 1.8 \text{ TeV} \]

Technibaryons appear as skyrmions.

New states predicted (KK modes)
Can the model be saved?

The lightest resonances contributed negatively to $S$.

We can try to truncate the model consistently at some scale before $S$ becomes too positive.
Brane in a Box

Raise the confinement scale with respect to the chiral symmetry breaking scale:

Put the D8 branes in a box (but this isn’t string theory anymore!)

\[-1 + y_{reg} < y < 1 - y_{reg}\]

For small enough box, naively S decreases, but the electroweak sector becomes strongly coupled at the TeV scale so it is hard to calculate
Deconstruction

Deconstruct the extra dimension:

Replace gauge fields in extra dimension by a finite tower of massive resonances

Resulting theory is reminiscent of little Higgs models, analysis should be similar.
The D4-D8 system can be interpreted as a technicolor model.

The S parameter is too large, as in estimates from Real World QCD.

Still need to add Standard Model fermions, masses.

Extra states predicted (KK modes) – expect technihadron resonances + large extra dimension.
Enhanced Symmetries from Extra Dimensions

**Intriguing lesson from the D4-D8 system:**
Enhanced gauge and global symmetries are a generic feature of extra-dimensional models with multiple-UV regions.

In the D4-D8 system a single 5D SU(2) gauge field gives rise to an enhanced SU(2)×SU(2) symmetry in the low-energy 4D theory (*c.f.* Son,Stephanov).

What else might enhanced symmetries be good for?
Higgsless EWSB

Hirn, Sanz/Csaki, Grojean, Murayama, Pilo, Terning

\[
SU(2)_L \times SU(2)_R \times U(1)_{B-L}
\]

\[
U(1)_Y = T_R^3 + (B-L)/2
\]

\[
ds^2 = \frac{R_0^2}{z^2} \left( dx_\mu dx^{\mu} - dz^2 \right)
\]

Spectrum

- \( m_{KK} \sim R' \)
- \( W, Z: m^2 \sim 1/R'^2 \log(R'/\varepsilon) \)
- photon: \( m=0 \)
To reproduce Standard Model fermion couplings without extra ultralight modes, we embed the model in 6D.

SU(2) lives on a D4 brane w/ double-UV geometry

U(1)_{B-L} lives on a flat D4 brane

The two D4 branes intersect at two UV boundaries
Boundary Conditions:
Neumann + effects of localized decoupled Higgs doublet at \((z_R, 0)\)

\[
g_5 A_\mu^3(x, z = z_R) - \tilde{g}_5 B_\mu(x, \tau = 0) = 0
\]

\[
A_\mu^{1,2}(x, z = z_R) = 0
\]

\(L_\tau < R'\): Ultralight \(B_\mu\) profile approximately uniform

\[
g_5 A_\mu^3(x, z = z_R) - \tilde{g}_5 B_\mu(x, \tau = L_\tau) \approx 0
\]
WV EWSB

Ultralight spectrum:

\[ m_\gamma^2 = 0 \]

\[ m_W^2 \approx \frac{1}{R' R^2 \ln(R'/\varepsilon_L)} \]

\[ m_Z^2 \approx \frac{\ln(R'^2/\varepsilon_L \varepsilon_R)}{R'^2 \ln(R'/\varepsilon_L) \ln(R'/\varepsilon_R)} \]
Standard Model fermion couplings

Left-handed fermions localized at $z_L$
Right-handed fermions localized at $z_R$

All fermions are in SU(2) doublets
Quarks: B-L charge $1/3$
Leptons: B-L charge -1

Universal fermion couplings

Left-handed doublet
$(z_L, L_\tau)$

Right-handed fermion
$(z_R, 0)$
Standard Model fermion couplings

Massless and Ultralight modes:

\[
\bar{\Psi} \left( i \partial_{\mu} + g_5 A_\mu^a T^a + g_5 q B \right) \Psi
\]

\[
A_\mu^{1,2}(x, z) = W_\mu^{1,2}(x) \psi_W(z)
\]
\[
A_\mu^3(x, z) = A_\mu(x) \psi_0 + Z_\mu(x) \psi_Z(z)
\]
\[
B_\mu(x, \tau) \approx \frac{g_5}{g_5} A_\mu^3(x, z_R).
\]

\[
\mathcal{L} \supset \bar{\Psi} \left[ g_5 W(x) \psi_W(z_L) + g_5 A(x) \psi_0 (T^3 + q) + g_5 Z(x) (\psi_Z(z_L) T^3 + \psi_Z(z_R) q) \right] \Psi
\]
Standard Model fermion couplings

\[ S = \frac{1}{4} \int d^4 x \ F_{\mu \nu}(x)^2 \ \psi_0^2 \left( \frac{g_5}{\bar{g}_5} L_\tau + R' \log \left( \frac{R'^2}{\epsilon L \epsilon_R} \right) \right) \]

\[ - \frac{1}{4} \int d^4 x \ Z_{\mu \nu}(x)^2 \left( \frac{g_5}{\bar{g}_5} L_\tau + \int_{-R' + \epsilon_L}^{R' - \epsilon_R} dz \ w(z) \ \psi_Z(z)^2 \right) \]

\[ - \frac{1}{2} \int d^4 x \ W_{\mu \nu}^+(x) W^{- \mu \nu}(x) \int_{-R' + \epsilon_L}^{R' - \epsilon_R} dz \ w(z) \ \psi_W(z)^2 \]

\[ + \int d^4 x \ \overline{\Psi} \ (i \hat{\partial} - g_5 \mathbf{W}(x)) \ \psi_W(-R' + \epsilon_L) \]

\[ - g_5 A(x) \ \psi_0 \ (T^3 + q) \]

\[ - g_5 Z(x) \left( \psi_Z(-R' + \epsilon_L) T^3 + \psi_Z(R' - \epsilon_R) q \right) \ \Psi, \]
Standard Model fermion couplings

\[ \mathcal{L} \supset \overline{\Psi} \left[ g_5 \mathcal{W}(x) \psi_W(z_L) + g_5 A(x) \psi_0 (T^3 + q) + g_5 Z(x)(\psi_Z(z_L) T^3 + \psi_Z(z_R) q) \right] \Psi, \]

\[ g = g_5 \psi_W(z_L), \quad e = g_5 \psi_0 \]

\[ \mathcal{L}_{Z}^{SM} = \frac{g}{\cos \theta_W} \overline{\Psi} \left( T^3 - \sin^2 \theta_W (T^3 + q) \right) \Psi \]

\[ \frac{g}{\cos \theta_W} = g_5 [\psi_Z(z_L) - \psi_Z(z_R)] \quad \text{sin}^2 \theta_W = \frac{\psi_Z(z_R)}{\psi_Z(z_R) - \psi_Z(z_L)} \]

Left-handed doublet
Right-handed fermion couplings similar, also consistent with Standard Model
Hidden custodial symmetry

\[ \cos^2 \theta_W \approx \frac{\ln(R'/\epsilon_R)}{\ln(R'^2/\epsilon_L\epsilon_R)} \]

\[ m_W^2 \approx \frac{1}{R'^2 \ln(R'/\epsilon_L)} \]

\[ m_Z^2 \approx \frac{\ln(R'^2/\epsilon_L\epsilon_R)}{R'^2 \ln(R'/\epsilon_L) \ln(R'/\epsilon_R)} \]

\[ \rho = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = 1 + \mathcal{O}(1/\ln(R'/\epsilon_L, R)) \]

Both weak SU(2) gauge group and SU(2) custodial symmetry arise from a single 5D SU(2) gauge group.
**Precision Electroweak Constraints**

\[
\sin^2 \theta_W \approx \frac{\ln\left(\frac{R'}{\epsilon_L}\right)}{\ln\left(\frac{R'^2}{\epsilon_L \epsilon_R}\right)}
\]

\[
\Rightarrow \frac{\epsilon_L}{R'} \approx \left(\frac{\epsilon_R}{R'}\right) \tan^2 \theta_W
\]

For \(L_\tau \ll R'\) and \(\epsilon_R/R' = 10^{-16}\): 
\(S = 3.9, \ T = 0.04\)

Perhaps \(S\) can be reduced by adding brane kinetic terms, adding Majorana masses, or otherwise modifying the model.

Can’t simply allow fermions to propagate in the bulk because they are charged under localized gauge fields.
Standard Model Fermion Masses

\[ S_m = \int d^4x \overline{\Psi}_L(x) W(x) M \Psi_R(x) \]

\[ W(x) = P \exp \left[ i \int_{(x,z_L)}^{(x,z_R)} d\tilde{z}'^M A_M(\tilde{z}') \right] \times P \exp \left[ i \int_{(x,0)}^{(x,L_\tau)} d\tau'^N B_N(\tau') \right] \]

c.f. Aharony, Kutasov

Mass matrix M can be isospin and CP violating.
Can add Majorana masses for right-handed neutrinos.
An SU(3) GUT from multi-UV geometries?

Low-energy theory like trinification model? Glashow, Weinberg
Summary

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2. Extra states seem to be a generic feature of calculable stringy models.

3. Related models may satisfy electroweak and FCNC constraints: walking technicolor models from D-branes?
   
   • At finite temperature, the chiral and deconfining phase transitions are first order, can occur at different times – implications for electroweak baryogenesis?

   • The D4-D8 system motivates the paradigm of unification into higher-dimensional gauge theories with smaller gauge and global symmetry groups.
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