A description of few-particle correlations and clustering

Y. Suzuki & W. Horiuchi (Niigata)

Outline
1. Motivation
   -- Mean-field motion vs clustering
2. Explicitly correlated Gaussian (ECG) basis
3. Test of the basis
   -- Spectrum of $^4$He
4. Examples of two-nucleon correlation
   -- Momentum distribution of A=6 nuclei
5. Summary and Outlook
Alpha-clustering
- $^8\text{Be}$, $^{12}\text{C}$, $^{16}\text{C}$, $^{20}\text{Ne}$ etc.
- Tight binding of alpha particle
- Tensor force, Distortion of alpha particle
FIG. 15. Contours of constant density, plotted in cylindrical coordinates, for $^8\text{Be}(0^+)$. The left side is in the "laboratory" frame while the right side is in the intrinsic frame.

$^{16}\text{O}$ spectrum
– Coexistence of shell and cluster states

\begin{figure}
\centering
\includegraphics[width=\textwidth]{spectrum.png}
\end{figure}
Motivation

\(^{16}\text{O}\): Testing ground to study dynamics of nucleon motion from multi ph excitations (deformed) and clustering

- Still challenging despite theoretical progress
  - Green’s function Monte Carlo (A \(\sim\) 12)
  - No core shell-model
  - Coupled cluster theory
- \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) hard!
  - Measurement at energies of astrophysical interest
  - Reliable calculation with isospin mixture due to CSB forces and p-n mass difference
**Ab-Initio** Coupled-Cluster Study of $^{16}$O

M. Wloch  


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**Ab initio** No-Core Shell Model

J.P. Vary$^{1, a}$  


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**FIG. 1** (color online). The coupled-cluster energies of the ground-state (g.s.) and first-excited $3^-$ and $0^+$ states as functions of the number of oscillator shells $N$ obtained with the Idaho-A interaction.

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**Fig. 4.** Low-lying positive-parity $^{16}$O states from the CD-Bonn interaction at the $a = 2$ cluster approximation in the NCSM with $\hbar \Omega = 15$ MeV. The spectra are aligned with the experimental first-excited $0^+$ state.
FIG. 1. A comparison of experiment and the $4\hbar \omega$ $^{16}$O shell-model spectrum of $T=0$ states. The spectrum resulting from diagonalizing the same Hamiltonian in a $2\hbar \omega$ space is also shown.
Correlations in Nuclei: Seattle 26-29 Nov. 2007

Excitation of $^{12}\text{C}$

Pauli constraint

$^\text{12}\text{C}+\alpha$ model


Energy levels of $^{16}\text{O}$. (T=0)

Excitation of $^{12}\text{C}$

Pauli constraint
Can we found the $^{12}\text{C}+\alpha$ model on the dynamics of four nucleons (without assuming $\alpha$ cluster) interacting with a $^{12}\text{C}$ core?
core + few-nucleons approach

Features of the model

– Few nucleons interacting via realistic potential
– Macroscopic core-nucleon interaction
– Few nucleons receive Pauli constraint from core, otherwise move unconditionally
– Core excitation
Basis function for orbital motion

Explicitly correlated Gaussian (ECG) with angular functions specified by global vectors (GV)

- \( L, \text{parity} = (-1)^L \)

\[
\exp \left( -\frac{1}{2} \vec{x} A \vec{x} \right) \mathcal{Y}_{LM}(\vec{u}_1 \vec{x})
\]

\[
\vec{x} = \sum_{i,j=1}^{N-1} A_{ij} \vec{x}_i \cdot \vec{x}_j \quad A_{ij} \neq 0
\]

\[
\mathcal{Y}_{LM}(\vec{u}_1 \vec{x}) = |\vec{u}_1 \vec{x}|^L \mathcal{Y}_{LM}(\vec{u}_1 \vec{x})
\]

\[
\vec{u}_1 \vec{x} = \sum_{i=1}^{N-1} u_{1i} \vec{x}_i
\]

- \( L, \text{parity} = (-1)^{L+1} \)

\[
\exp \left( -\frac{1}{2} \vec{x} A \vec{x} \right) \left[ \mathcal{Y}_{L}(\vec{u}_1 \vec{x}) \mathcal{Y}_{1}(\vec{u}_2 \vec{x}) \right]_{LM}
\]

Varying \( A \) and \( u \) enables to include all correlations.

NB: \( L=0, \text{parity} = -1 \)
Unifying shell and cluster correlations

Both types of correlations can be described in a single coordinate set. Permutations of identical particles induce linear transformation of coordinates. No need of coord. trans. Only suitable choice of $A$ and $u$ needed.

\[ y = Tx \implies \tilde{y}By = \tilde{x}\tilde{T}BTx \quad \tilde{vy} = \tilde{T}vx \]

Advantage of the basis functions
Variational Solution

$$\Phi(A, u, \alpha) = A \left\{ \left[ \psi_{L}^{(\text{orbital})}(A, u) \psi_{S}^{(\text{spin})}(S_{12}, S_{123}, \ldots) \right]_{JM} \times \psi_{T_{12}, T_{123}, \ldots}^{(\text{isospin})} \right\} \alpha = (L, S, S_{12}, \ldots, T_{12}, \ldots)$$

$$\Psi_{JMT_{12}T_{123}} = \sum_{i=1}^{K} C_{i} \Phi(A_{i}, u_{i}, \alpha_{i})$$

$$H_{ij} = \langle \Phi(A_{i}, u_{i}, \alpha_{i}) | H | \Phi(A_{j}, u_{j}, \alpha_{j}) \rangle$$

$$B_{ii} = \langle \Phi(A_{i}, u_{i}, \alpha_{i}) | \Phi(A_{j}, u_{j}, \alpha_{j}) \rangle$$

$$\sum_{j=1}^{K} (H_{ij} - EB_{ij}) C_{j} = 0$$

Symmetry, Center of mass motion
Variation after projection

Correlations in Nuclei:
Seattle 26-29 Nov. 2007
$\Psi_{JM} = \sum_{LS} C_{LS}[\psi_L \psi_S]_{JM}$

L, S coupling scheme useful to know tensor correlation

Example: A=4, J=0
(L,S)=(0,0), (1,1), (2,2)
Relationship between Partial-Wave Expansion and GVR

Successive coupling: \[
[\cdots[[y_{l_1}(x_1)y_{l_2}(x_2)]_{l_{12}}y_{l_3}(x_3)]_{l_{123}}\cdots]_{LM}
\]

Small \(\ell\) values are used. Calculation of matrix elements is involved.

Global vectors: \[
y_{LM}(u_1x_1 + u_2x_2 + \cdots + u_{N-1}x_{N-1})
\]

\[
y_{LM}(ax_1 + bx_2) = \sum_{l=0}^{L} \sqrt{\frac{4\pi(2L+1)!}{(2l+1)!(2L-2l+1)!}} a^l b^{L-l} [y_l(x_1)y_{L-l}(x_2)]_{LM}
\]

Cross terms of Correlated Gaussians add additional \(\ell\) values:

\[
\exp(A_{ij}x_i \cdot x_j) \to \sum_n (x_i \cdot x_j)^n \sim \sum_{\ell=n,n-2,...} [y_{\ell}(x_i)y_{\ell}(x_j)]_{00}
\]

**G3RS vs AV8’**

$$\Psi_{JM} = \sum_{LS} C_{LS} \psi_L \psi_S J_M$$

$$E = \sum_{LS} \sum_{L'S'} C_{LS} C_{L'S'} \langle LS; JM|H|L'S'; JM \rangle$$

<table>
<thead>
<tr>
<th>$d(1^+)$</th>
<th>(0,1)</th>
<th>(2,1)</th>
<th>$P(L, S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,1)</td>
<td>4.198</td>
<td>-12.73</td>
<td>0.952</td>
</tr>
<tr>
<td>(2,1)</td>
<td>6.257</td>
<td>-12.73</td>
<td>0.048</td>
</tr>
</tbody>
</table>

Deuteron

-2.27

-2.24 MeV

<table>
<thead>
<tr>
<th>$t(\frac{1}{2}^+)$</th>
<th>(0,1/2)</th>
<th>(2,3/2)</th>
<th>(1,1/2)</th>
<th>(1,3/2)</th>
<th>$P(L, S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,1/2)</td>
<td>4.089</td>
<td>-22.50</td>
<td>-0.000</td>
<td>-0.001</td>
<td>0.930</td>
</tr>
<tr>
<td>(2,3/2)</td>
<td>10.83</td>
<td>-0.155</td>
<td>-0.070</td>
<td>0.070</td>
<td></td>
</tr>
<tr>
<td>(1,1/2)</td>
<td>0.069</td>
<td>0.012</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,3/2)</td>
<td>0.029</td>
<td>0.000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Triton

-7.70

( Arai )

-7.76

( Hiyama et al. )

AV8’ gives larger ME than G3RS for T, V_v, V_b but smaller for V_c.
The net result is similar between the two.

Tamagaki, PTP39 (1968)
Pudliner et al., PRC56(1997)
Algorithm of the SVM

Possibility of the stochastic optimization
1. increase the basis dimension one by one
2. set up an optimal basis by trial and error procedures
3. fine tune the chosen parameters until convergence

1. Generate \((A_k^1, A_k^2, \cdots, A_k^m)\) randomly
2. Get the eigenvalues \((E_k^1, E_k^2, \cdots, E_k^m)\)
3. Select \(A_k^n\) corresponding to the lowest \(E_k^n\) and Include it in a basis set
4. \(k \rightarrow k+1\)

As preparation for core+4N calculations, we focus on

1. Test of GV representation
2. Spectrum of 4N system ($^4\text{He}$)
   -- 3N+N clustering
   -- evidence for tensor correlation
3. core+2N problem ($^6\text{He}$, $^6\text{Li}$)
   -- characteristics of N-N correlation
Test of ECG: \(^4\text{He} \ 0^+\) states

(W. Coulomb)
Spectrum of $^4$He: Purely central vs realistic forces

Correlations in Nuclei:
Seattle 26-29 Nov. 2007
Realistic potential reproduces correctly splitting of levels, which are degenerate in pure central force model.
Theory predicts $0^+$ and $1^+$ states with $3N+N$ cluster structure.

$$(S, T) = (1/2, 1/2); \quad L = 0$$
Correlations in Nuclei: Seattle 26-29 Nov. 2007

Contribution of unnatural parity components

$^4\text{He}: \text{G3RS}$

**Ground state**

<table>
<thead>
<tr>
<th>$0^+_1$</th>
<th>(0,0)</th>
<th>(2,2)</th>
<th>(1,1)</th>
<th>$P(L, S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>0.919</td>
<td>-46.51</td>
<td>-0.006</td>
<td>0.885</td>
</tr>
<tr>
<td>(2,2)</td>
<td></td>
<td>21.17</td>
<td>-1.497</td>
<td>0.112</td>
</tr>
<tr>
<td>(1,1)</td>
<td></td>
<td></td>
<td>0.692</td>
<td>0.002</td>
</tr>
</tbody>
</table>

$\Delta E \sim -0.8 \text{ MeV}$

**Excited state**

<table>
<thead>
<tr>
<th>$0^+_2$</th>
<th>(0,0)</th>
<th>(2,2)</th>
<th>(1,1)</th>
<th>$P(L, S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>4.821</td>
<td>-22.54</td>
<td>-0.002</td>
<td>0.931</td>
</tr>
<tr>
<td>(2,2)</td>
<td></td>
<td>10.94</td>
<td>-0.282</td>
<td>0.068</td>
</tr>
<tr>
<td>(1,1)</td>
<td></td>
<td></td>
<td>0.139</td>
<td>0.001</td>
</tr>
</tbody>
</table>

3N+N cluster state

**Cf. triton**

<table>
<thead>
<tr>
<th>$t(\frac{1}{2}^+)$</th>
<th>(0,1/2)</th>
<th>(2,3/2)</th>
<th>(1,1/2)</th>
<th>(1,3/2)</th>
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<td></td>
<td></td>
<td>0.069</td>
<td>0.012</td>
<td>0.000</td>
</tr>
<tr>
<td>(1,3/2)</td>
<td></td>
<td></td>
<td></td>
<td>0.029</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Most distinct role of unnatural parity: $0^-$

**Experiment:**
- $E_x = -7.29$ MeV  
  (1.20 MeV above $t+p$)
- $\Gamma = 0.84$ MeV

**Calculation:**
- $-6.07$ MeV  
  (1.63 MeV above $t+p$)

---

### Total

<table>
<thead>
<tr>
<th>$0^-$</th>
<th>(1,1)</th>
<th>(2,2)</th>
<th>$P(L,S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1)</td>
<td>1.137</td>
<td>$-13.85$</td>
<td>0.954</td>
</tr>
<tr>
<td>(2,2)</td>
<td>6.644</td>
<td>0.046</td>
<td></td>
</tr>
</tbody>
</table>

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### Kinetic

<table>
<thead>
<tr>
<th>$\langle T \rangle$</th>
<th>(1,1)</th>
<th>(2,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1)</td>
<td>41.87 (43.89)</td>
<td></td>
</tr>
<tr>
<td>(2,2)</td>
<td>7.385 (160.2)</td>
<td></td>
</tr>
</tbody>
</table>

---

### Tensor

<table>
<thead>
<tr>
<th>$\langle V_t \rangle$</th>
<th>(1,1)</th>
<th>(2,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1)</td>
<td>$-13.55 (-14.21)$</td>
<td>$-13.86 (-66.09)$</td>
</tr>
<tr>
<td>(2,2)</td>
<td>0.367 (7.955)</td>
<td></td>
</tr>
</tbody>
</table>

---

### Coulomb

<table>
<thead>
<tr>
<th>$\langle V_{Coul} \rangle$</th>
<th>(1,1)</th>
<th>(2,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1)</td>
<td>0.469 (0.492)</td>
<td></td>
</tr>
<tr>
<td>(2,2)</td>
<td>0.021 (0.454)</td>
<td></td>
</tr>
</tbody>
</table>

---

### Central

<table>
<thead>
<tr>
<th>$\langle V_c \rangle$</th>
<th>(1,1)</th>
<th>(2,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1)</td>
<td>$-27.98 (-29.33)$</td>
<td></td>
</tr>
<tr>
<td>(2,2)</td>
<td>$-1.129 (-24.49)$</td>
<td></td>
</tr>
</tbody>
</table>

---

### Spin-orbit

<table>
<thead>
<tr>
<th>$\langle V_s \rangle$</th>
<th>(1,1)</th>
<th>(2,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1)</td>
<td>0.327 (0.343)</td>
<td>0.010 (0.050)</td>
</tr>
<tr>
<td>(2,2)</td>
<td>0.000 (0.006)</td>
<td></td>
</tr>
</tbody>
</table>

---

Vital to reproduce $0^-$ is coupling between natural and unnatural states, which arises from tensor force.
Momentum distribution measures correlations

• Quantity reflecting two-nucleon correlation
• Experiments on nucleon correlation
  – $^{12}$C(e,e’n$p$), $^{12}$C(e,e’pp) E. Piasetzky et al.
  • Theoretical analysis R. Schiavilla et al., PRL98 (2007).
  – Recent experiment at RIKEN T. Suda et al.

One nucleon exchange reaction

$p+^{6}$He $\rightarrow$ $^{4}$He + d + n
Correlations in Nuclei:
Seattle 26-29 Nov. 2007

Deuteron momentum distribution

D-wave fills the dip of S-wave
Magnitude of high momentum components
Effect of a short-ranged repulsion
Non-nucleonic effect in k>2.5 fm⁻¹

Correlations in Nuclei:
Seattle 26-29 Nov. 2007

Core + 2N model

$^{6}$He: $\alpha+n+n$
$^{6}$Li: $\alpha+n+p$

MN or G3RS for NN
Phenomenological pot. for $\alpha$N

W. Horiuchi, Y. S. PRC76 (2007)

### Theory

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>$^{6}$He</th>
<th>$^{6}$Li</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective</td>
<td>-0.421</td>
<td>-3.91</td>
<td>-2.20</td>
</tr>
<tr>
<td>Realistic</td>
<td>-0.460</td>
<td>-3.31</td>
<td>-2.27</td>
</tr>
<tr>
<td>Tensor (MeV)</td>
<td>-0.107</td>
<td>-12.3</td>
<td>-11.5</td>
</tr>
<tr>
<td>N-N distance (fm)</td>
<td>5.05</td>
<td>3.48</td>
<td>3.90</td>
</tr>
</tbody>
</table>

### Experiment

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>$^{6}$He</th>
<th>$^{6}$Li</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.975</td>
<td>-3.90</td>
<td>-2.22</td>
<td></td>
</tr>
<tr>
<td>5.9±1.2</td>
<td>not measured</td>
<td>3.91</td>
<td></td>
</tr>
</tbody>
</table>

Intensity interferometry experiment (Marques et al. PLB476 (2000).)
Relative momentum distribution
-- information on N-N correlation --

W. Horiuchi, Y. S. PRC 76 (2007)

Momentum distribution of $^6$He differs from that of $^6$Li. Momentum distribution of $^6$Li is similar to that of deuteron.

<table>
<thead>
<tr>
<th>$P(\ell s)$</th>
<th>$^6$He $(0^+)$</th>
<th>$^6$Li $(1^+)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(00)</td>
<td>87.5</td>
<td>0.8</td>
</tr>
<tr>
<td>(11)</td>
<td>12.5</td>
<td>3.9</td>
</tr>
<tr>
<td>(10)</td>
<td>90.3</td>
<td>5.0</td>
</tr>
</tbody>
</table>

The dip of $^6$He reflects S-wave dominance. D-wave in $^6$Li fills the dip of S-wave.
### Shrinkage due to the interaction with core

<table>
<thead>
<tr>
<th></th>
<th>(^6\text{Li} (1^+))</th>
<th>(d (1^+))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MN</td>
<td>G3RS</td>
</tr>
<tr>
<td>(E)</td>
<td>-3.91</td>
<td>-3.31</td>
</tr>
<tr>
<td>(\langle T_r \rangle)</td>
<td>17.56</td>
<td><strong>23.28</strong></td>
</tr>
<tr>
<td>(\langle v_{12}^C \rangle)</td>
<td>-13.41</td>
<td><strong>-7.71</strong></td>
</tr>
<tr>
<td>(\langle v_{12}^T \rangle)</td>
<td>—</td>
<td><strong>-12.25</strong></td>
</tr>
<tr>
<td>(\langle v_{12}^{LS} \rangle)</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>(\langle T_R \rangle)</td>
<td>13.29</td>
<td>11.49</td>
</tr>
<tr>
<td>(\langle U_1^C + U_2^C \rangle)</td>
<td>-19.00</td>
<td>-16.44</td>
</tr>
<tr>
<td>(\langle U_1^{LS} + U_2^{LS} \rangle)</td>
<td>-2.34</td>
<td>-1.69</td>
</tr>
<tr>
<td>(\sqrt{\langle r^2 \rangle})</td>
<td>3.48</td>
<td><strong>3.58</strong></td>
</tr>
</tbody>
</table>

\[ E(pn) \approx +3.3 \text{ MeV} \]
Tensor Forces and the Ground-State Structure of Nuclei

R. Schiavilla,1,2 R. B. Wiringa,3 Steven C. Pieper,3 and J. Carlson4
1Jefferson Laboratory, Newport News, Virginia 23606, USA
2Department of Physics, Old Dominion University, Norfolk, Virginia 23529, USA
3Physics Division, Argonne National Laboratory, Argonne, Illinois 61801, USA
4Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
(Received 10 November 2006; published 27 March 2007)

Correlations in Nuclei:
Seattle 26-29 Nov. 2007

Lines: n-p pair (no dip)
Dots: p-p pair

$^1_2\text{C}(e,e'np)$, $^1_2\text{C}(e,e'pp)$
Summary

• Explicitly correlated Gaussian applied to $^4\text{He}$ and $\alpha+N+N$ model for $^6\text{He}$ and $^6\text{Li}$
  -- Spectrum of $^4\text{He}$ well reproduced with realistic potential
  -- $(3N)+N$ cluster states with $0^+, 1^+$ ($T=0, 1$). Need to be examined
• Dominance of tensor correlation in $0^-$, $E_x=20.01$ MeV.
  -- Unnatural parity component described with DGV
• Relative momentum distributions in $^6\text{He}$, $^6\text{Li}$ and $d$
  -- Effect of a short-ranged repulsion at large $k$
  -- Distribution of $^6\text{He}$ differs from $^6\text{Li}$, which is similar to $d$
  -- Effect of tensor force evident at $k \sim 2$ fm$^{-1}$.

Outlook

Application to $^{12}\text{C}+(\text{few-nucleons})$ system; $^{16}\text{O}$, $^{15}\text{C}$, $^{16}\text{C}$ etc.