Interplay between the cluster and shell correlations

N. Itagaki (University of Tokyo)

At INT Seattle workshop 2007
Appearance of "α-matter" in symmetric nuclear matter

"Phase" of nuclei

mean-field-like (liquid)

cluster state with geometric shape

gas like structure

--- αthreshold

Excitation energy

N. I., T. Otsuka, K. Ikeda, and S. Okabe,
PRL 04 (2004) 142501
Gas like state: Hoyle state
The second $0^+$ state of $^{12}$C is gas-like structure of 3 $\alpha$-clusters
The Hoyle state is a coherent state of many SU(3) representations

<table>
<thead>
<tr>
<th>N</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
<th>20</th>
<th>22</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{12}\text{C} (0^+_{1})$</td>
<td>54</td>
<td>30</td>
<td>11</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{12}\text{C} (0^+_{2})$</td>
<td>11</td>
<td>12</td>
<td>12</td>
<td>10</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

Probability of the harmonic oscillator wave functions for the $3\alpha$ wave function

\[
O = \frac{1}{2\pi} \int_{0}^{2\pi} d\theta \exp \left\{ i\theta \left[ \sum_{i=1}^{A} P_i \left( H_{HO}(i) - \frac{3}{2} \right) \right] - Q \right\}
\]

Alpha Cluster Condensation in $^{12}$C and $^{16}$O

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(Received 29 June 2001; published 17 October 2001)

A new $\alpha$-cluster wave function is proposed which is of the $\alpha$-particle condensate type. Applications to $^{12}$C and $^{16}$O show that states of low density close to the 3 and 4 $\alpha$-particle thresholds in both nuclei are possibly of this kind. It is conjectured that all self-conjugate $4n$ nuclei may show similar features.

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\[ \Phi = \int d\vec{R}_1 d\vec{R}_2 \cdots d\vec{R}_n \]

\[ \mathcal{A} \ G_1(\vec{R}_1) G_2(\vec{R}_2) G_3(\vec{R}_3) \cdots G_n(\vec{R}_n) \]

\[ \times \exp[-(\vec{R}_{1}^2 + \vec{R}_{2}^2 + \vec{R}_{3}^2 \cdots \vec{R}_{n}^2)/\sigma^2] \]

\[ = \mathcal{A} \prod_{i=1}^{n} \int d\vec{R}_i \ G_i(\vec{R}_i) \exp[-\vec{R}_i^2/\sigma^2], \]
FIG. 1. Contour map of the energy surface $E_{3\alpha}(R_0, b)$ for $^{12}$C. Numbers attached to the contour lines are the binding energies.
FIG. 2. Contour map of the energy surface $E_{4\alpha}(R_0, b)$ for $^{16}$O. Numbers attached to the contour lines are the binding energies.
Evidence for an alpha cluster condensed state in $^{16}$O$(\alpha, \alpha')$ at 400 MeV

T. Wakasa a E. Ihara a K. Fujita b Y. Funaki c K. Hatanaka b H. Horiuchi b M. Itoh d J. Kamiya e G. Röpke f H. Sakaguchi g N. Sakamoto b Y. Sakemi b P. Schuck h Y. Shimizu b M. Takashina i c S. Terashima e A. Tohsaki b M. Uchida i H. P. Yoshida k M. Yosoi b

Inelastic $\alpha$ scattering on $^{16}$O is studied at 400 MeV by using an ice target. Near the 4$\alpha$ breakup threshold of 14.4 MeV, a broad peak is observed at an excitation energy of 13.6±0.2 MeV with a width of 0.6±0.2 MeV. The spin-parity is estimated to be $0^+$ from the momentum-transfer dependence. The observed width is significantly larger than those of the neighboring $0^+$ states indicating a state with a well-developed $\alpha$ cluster structure. The magnitude of the cross section is sensitive to the density distribution of the constituent $\alpha$ clusters. The observed cross section is consistent with the theoretical prediction for the $\alpha$ cluster condensed state characterized by its dilute density distribution with a large root-mean-square radius of about 4.3 fm.

Fig. 3. Results of peak fitting of $^{16}$O$(\alpha, \alpha')$ at $T_\alpha = 400$ MeV and $q_{\text{cm}} = 0.2$ fm$^{-1}$ with hyper-Gaussian and Lorentzian peaks and a continuum, with the width $\Gamma$ of the new state at 13.6 MeV taken to be (a) 0.3 MeV, (b) 0.6 MeV, and (c) 0.9 MeV.
Dilute multi-$\alpha$ cluster states in nuclei

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(Received 27 October 2003; published 26 February 2004)

Gross-Pitaevskii equation for dilute $N\alpha$ nuclear systems

\[-\frac{\hbar^2}{2m}\left(1 - \frac{1}{N}\right)\nabla^2 \varphi(r) + U(r)\varphi(r) = \varepsilon \varphi(r),\]

\[U(r) = (N - 1) \int dr' |\varphi(r')|^2 v_2(r', r) + \frac{1}{2}(N - 1)(N - 2)\]

\[\times \int dr'' dr' |\varphi(r'')|^2 |\varphi(r')|^2 v_3(r'', r', r),\]
FIG. 3. Single-α-particle potentials $U_\alpha(R)$ (solid line) which are obtained by solving the Gross-Pitaevskii equation with the density-dependent potential; (a) $3\alpha$, (b) $4\alpha$, (c) $5\alpha$, (d) $8\alpha$, and (e) $10\alpha$ systems. The dashed, dot-dashed, and dotted lines demonstrate, respectively, the contribution from the two-range-Gaussian term, density-dependent term, and Coulomb potential.
How we calculate gas-like states of many alpha clusters in more microscopic way?
For the $\alpha$ clusters in the condensed state

$$W(\vec{R}_i) \propto \exp[-\vec{R}_i^2/\sigma^2].$$

Virtual Schuck wave function

N.I., M. Kimura, M. Ito, C. Kurokawa, and W. von Oertzen,
PRC 75 037303 (2007)
Hamiltonian

\[ \hat{H} = \sum_{i=1}^{\infty} \hat{t}_i - \hat{T}_{c.m.} + \sum_{i>j} \hat{v}_{ij}, \]

Two-body interaction

\[ V(r) = (W - MP^\sigma P^\tau + BP^\sigma - HP^\tau) \]
\[ (V_1 \exp(-r^2/c_1^2) + V_2 \exp(-r^2/c_2^2)), \]
r.m.s. radius of $^{12}$C

<table>
<thead>
<tr>
<th>$\sigma = 3$</th>
<th>$\sigma = 4$</th>
<th>$\sigma = 5$</th>
<th>Micro</th>
<th>Cond</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.06</td>
<td>3.60</td>
<td>4.38</td>
<td>3.47</td>
<td>3.83</td>
</tr>
</tbody>
</table>
$^{16}\text{O}$ 4$\alpha$ condensed state
$\sigma = 2, 3, 4 \text{ fm}$
$^{20}$Ne $5\alpha$ condensed state

$\sigma = 2, 3, 4$ fm
$0^+ \text{ states of } 5\alpha \text{ system}$

(a): 
$^{16}\text{O} + \alpha$

(b): 
$^{16}\text{O} + \alpha$ and virtual Schuck \[ \sigma = 2,3,4 \text{ fm} \]
TABLE I: The squared overlap between the wave function of each $0^+$ state of $^{20}$Ne (Fig. 4 (b)) and virtual Schuck wave function with $\sigma = 2, 3, 4$ fm.

<table>
<thead>
<tr>
<th>state</th>
<th>$\sigma = 2$ fm</th>
<th>$\sigma = 3$ fm</th>
<th>$\sigma = 4$ fm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+_1$</td>
<td>0.19</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>$0^+_2$</td>
<td>0.02</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>$0^+_3$</td>
<td>0.09</td>
<td>0.04</td>
<td>0.01</td>
</tr>
<tr>
<td>$0^+_4$</td>
<td>0.68</td>
<td>0.51</td>
<td>0.13</td>
</tr>
<tr>
<td>$0^+_5$</td>
<td>0.00</td>
<td>0.04</td>
<td>0.03</td>
</tr>
</tbody>
</table>
Gas-like states of alpha clusters, the next step……..

Gas state around core nucleus


Enhancement of emissions of the condensed state compared with the sequential α emissions from the compound state
Is it due to the lowering of the Coulomb barrier for the condensed state?

FIG. 1. The folded potential \((V\ \text{in the text})\) for \(^{12}\text{C}\) emission as a function of the distance \((R\ \text{in the text})\) between the \(^{12}\text{C}\) and the \(^{40}\text{Ca}\) core. The solid, dashed, and dotted lines correspond to the condensed, cluster, and ground states of \(^{12}\text{C}\), respectively.

$^{16}\text{O}+2\alpha$

$^{16}\text{O}+3\alpha$

Solid, dotted, dashed, dash-dotted $\rightarrow \sigma = 2,3,4,5$ fm
“Phase” of nuclei

- gas like structure
- cluster state with geometric shape
- mean-field-like (liquid)

N. I., T. Otsuka, K. Ikeda, and S. Okabe,
PRL 04 (2004) 142501
Cluster shell competition is important
In light neutron-rich nuclei

Strong spin-orbit interaction is needed to explain the magic numbers

We introduce a general and simple model to describe this transition
“dissolution” of the clusters

Spin-orbit interaction works to restore the J-J coupling symmetry
\[ \Phi = P^\pi P^J_{MK} \Psi, \]
\[ \Psi = A[(\psi_1 \chi_1)(\psi_2 \chi_2) \cdots], \]
\[ \psi_i = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{4}} \exp \left[ -\nu(\vec{r} - \vec{z}_i/\sqrt{\nu})^2 + \frac{\vec{z}_i^2}{2} \right], \]

4 nucleons share the same \( z \) value for each alpha cluster

The spin-orbit interaction: \((r \times p) \cdot s = (s \times r) \cdot p\)

\( r \rightarrow \) Gaussian centre parameter
\( p \rightarrow \) imaginary part of the Gaussian centre parameter

We transform the Gaussian centre parameter \( S \) as:
\[ S \rightarrow S + i \Lambda (\text{spin} \times S) \]
We give imaginary part for the Gaussian center parameters

\[
\bar{z}_i / \sqrt{v} = \bar{S}_1 + i \Lambda (\bar{e}_{\text{spin}}^i) \times \bar{S}_1,
\]

**12C case**

For the spin up n and p

\[
\bar{z} / \sqrt{v} = (\sqrt{3} R_1/2)(\bar{e}_x + i \Lambda \bar{e}_y),
\]

For the spin down n and p

\[
\bar{z} / \sqrt{v} = (\sqrt{3} R_1/2)(\bar{e}_x - i \Lambda \bar{e}_y).
\]
The Limit at $\Lambda = 1$

For the spin up $n$ and $p$

$$\frac{z}{\sqrt{v}} = (\sqrt{3}R_1/2)(\vec{e}_x + i\Lambda\vec{e}_y),$$

$$= (\sqrt{3}R_1/2) \ Y_{11}$$

$$\Psi \sim \exp[-\nu r^2] \ Y_{11}$$

For the spin down $n$ and $p$

$$\frac{z}{\sqrt{v}} = (\sqrt{3}R_1/2)(\vec{e}_x - i\Lambda\vec{e}_y),$$

$$= (\sqrt{3}R_1/2) \ Y_{1-1}$$

$$\Psi \sim \exp[-\nu r^2] \ Y_{1-1}$$
Solid and open circles, solid and open squares, solid and open triangles $R_1 = 3.0, 2.5, 2.0, 1.5, 1.0, \text{ and } 0.5 \text{ fm, respectively}$
Possible SU(3) representations for the 3α model space


<table>
<thead>
<tr>
<th>$N$</th>
<th>$(\lambda, \mu)^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>(0, 4)</td>
</tr>
<tr>
<td>9</td>
<td>(3, 3)</td>
</tr>
<tr>
<td>10</td>
<td>(6, 2), (2, 4)</td>
</tr>
<tr>
<td>11</td>
<td>(9, 1), (5, 3), (3, 4)</td>
</tr>
<tr>
<td>12</td>
<td>(12, 0), (8, 2), (6, 3), (4, 4), (0, 6)</td>
</tr>
<tr>
<td>13</td>
<td>(11, 1), (9, 2), (7, 3), (5, 4), (3, 5)</td>
</tr>
<tr>
<td>14</td>
<td>(14, 0), (12, 1), (10, 2), (8, 3), (6, 4)$^2$, (2, 6)</td>
</tr>
<tr>
<td>15</td>
<td>(15, 0), (13, 1), (11, 2), (9, 3)$^2$, (7, 4), (5, 5), (3, 6)</td>
</tr>
</tbody>
</table>

$N = 8$  

\[
\begin{array}{cccc}
S & P & P & N & N \\
P_x & P & P & N & N \\
P_z & P & P & N & N \\
\end{array}
\]

$P_z$  

\[
\begin{array}{cccc}
\square & \square & \square & \square \\
\end{array}
\]

\[
\begin{array}{cccc}
P_x & \square & \square & \square & \square \\
\end{array}
\]

$(\lambda, \mu) = (0, 4)$

Spin singlet
Group theoretical interpretation: \((^{12}\text{C}, N = 8)\)

Mixing amplitude is controlled by the parameter \(\Lambda\)
Inclusion of the spin-orbit effect → effective SU(3)

\[ H(\varepsilon) = H_0(\varepsilon) + Cl \cdot s + Dl^2 \]

\[ a_z^\dagger, \quad a_\pm^\dagger = \mp \frac{1}{\sqrt{2}} (a_x^\dagger \pm ia_y^\dagger), \]

an asymptotic single-particle Nilsson state is expressed

\[ |Nn_\Sigma\Lambda \Sigma\rangle = \frac{(a_z^\dagger)^{n_z} (a_+^\dagger)^{n_+} (a_-^\dagger)^{n_-}}{\sqrt{n_z!} \sqrt{n_+!} \sqrt{n_-!}} |0\rangle \xi_\Sigma, \]

where

\[ N = n_z + n_+ + n_-, \quad \Lambda = n_+ - n_- \]

and \( \xi_\Sigma \) is a spin wave function.

FIG. 2: Calculated energies of $^{12}\text{C}$ (upper panel), $^{14}\text{C}$ (middle panel), and $^{16}\text{C}$ (lower panel) as functions of $\Lambda$. Solid and open circles, solid and open squares, solid and open triangles correspond to $R_1 = 3.0, 2.5, 2.0, 1.5, 1.0$ and $0.5$ fm, respectively.
Importance of multi nucleon correlations

In weakly bound (low-density) systems, strongly bound subsystems become important compared with uniform distribution.

In neutron-rich nuclei, protons are deeply bound.

* Cluster distance gets shorter
* Cluster structure disappears due to the spin-orbit interaction

• Cluster correlation is important in the excited states
• Neutron-neutron correlation is important in neutron-rich nuclei
## Tensor contribution in C isotopes

<table>
<thead>
<tr>
<th>(MeV)</th>
<th>$^{11}\text{C}$</th>
<th>$^{12}\text{C}$</th>
<th>$^{13}\text{C}$</th>
<th>$^{14}\text{C}$</th>
<th>$^{16}\text{C}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>With tensor</td>
<td>-64.8</td>
<td>-89.3</td>
<td>-93.8</td>
<td>-106.2</td>
<td>-109.2</td>
</tr>
<tr>
<td>Without T</td>
<td>-65.6</td>
<td>-91.4</td>
<td>-95.5</td>
<td>-106.3</td>
<td>-111.6</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>+0.9</td>
<td>+2.1</td>
<td>+1.7</td>
<td>+0.1</td>
<td>+2.4</td>
</tr>
</tbody>
</table>

Competition between the spin-orbit and tensor contributions
Where is the cluster state?

0$^+$ states of $^{16}$C

$\Lambda = 0.8$

(Base 1) + (Base 2)

$\Lambda = 0.8$ and 0.0

(Base 1)
“Phase” of nuclei

- gas like structure
- cluster state with geometric shape
- mean-field-like (liquid)

N. I., T. Otsuka, K. Ikeda, and S. Okabe,
PRL 04 (2004) 142501
FIG. 2: The $0^+$ and $3^-$ ($K = 3$) energies of $^{12}\text{C}$ (dashed lines) and $^{14}\text{C}$ (solid lines) as a function of the $\alpha-\alpha$ distance. The equilateral triangular configuration of $3\alpha$ is assumed. The solid lines represent $^{14}\text{C}$ with the $(p_z)^2$ configuration for the excess neutrons, and the dashed lines represent $^{12}\text{C}$.
Neutron motion around 2α clusters

1 node (- parity) $\rightarrow \pi$
$K = 3/2-$ or $1/2-$

2 node (+ parity) $\rightarrow \sigma$
$K = 1/2+$

With “good” $K$ numbers
alpha + alpha + n+n model for $^{10}$Be

Triaxial components in $^{10}$Be ($\alpha + \alpha + n + n$)

If di-neutron correlation becomes important

→ Triaxial defomation
→ “K” is not good quantum number
→ inter-band transition occurs

\[
\frac{B(E2: 2^+_2 \rightarrow 2^+_1)}{B(E2: 2^+_1 \rightarrow 0^+_1)} = \frac{\frac{20}{7} \frac{\sin^2(3\gamma)}{9 - 8\sin^2(3\gamma)}}{1 + \frac{3 - 2\sin^2(3\gamma)}{\sqrt{9 - 8\sin^2(3\gamma)}}}
\]

Davydov-Filippov Model
N. Itagaki, S. Hirose, T. Otsuka, S. Okabe, and K. Ikeda

**Triaxial (SU(3))**                       **single particle-like (JJ)**

Configuration of the two valence neutrons

<table>
<thead>
<tr>
<th></th>
<th>(Px, Px)</th>
<th>(Px, Px) + i (Px,Py)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V^{LS}$</td>
<td>1000</td>
<td>1500</td>
</tr>
<tr>
<td>$M$</td>
<td>0.58</td>
<td>0.59</td>
</tr>
<tr>
<td>$B(E2, 2^+ \rightarrow 2^+)$</td>
<td>17.53</td>
<td>9.53</td>
</tr>
</tbody>
</table>

(e$^2$fm$^4$)
Conclusions: nuclei has three phases

- The study on the gas-like states can be extended beyond $^{16}$O
- Cluster-shell competition is an important feature of light nuclei
- The spin-orbit interaction is the driving force for the wandering of nuclear structure
- Tensor interaction works to reduce the spin-orbit interaction
- Cluster structure with geometric shape is stabilized by valence neutrons