Decay out of Superdeformed Bands in a Two-Level Model

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Correlations in Nuclei: From Di-Nucleons to Clusters
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Acknowledgments

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- Charles A. Stafford
- Bruce R. Barrett
Why collectivity?

“Top Down”: Collective Motion

“Bottom Up”: Microscopic Approaches
Outline

1. Superdeformed Nuclei
   - Superdeformation
   - Decay

2. Two-State Model
   - What is it?
   - Statistical Theory of $V$
   - Accuracy

3. Universality
Superdeformation

- General prediction of shell models.
- Ellipsoidal and highly deformed: \( \frac{\text{major}}{\text{minor}} \approx 2 \).
- Clear experimental signature
  - Large electric quadrupole: \( Q \approx 0.007 ZA^{2/3} \text{eb} \).
  - Little centrifugal stretching: rigid rotor spectrum.
- For very high angular momenta, SD states can be yrast.

from Wong (1998).
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Life and Death of an SD Nucleus

Typical Decay Experiment

1. Nucleus is created in a high angular momentum SD yrast state.
2. Decay via E2 transitions along SD rotational band.
3. Transition to a lower-lying ND band.
4. Decay down ND band via E1-dominated transitions.
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Modeling the Decay

Schematic Potential
- Double well.
- Function of angular momentum.

In principle, each SD state can decay to all ND states.
Interesting Questions
A shopping list

- How many states do we need to keep in the ND well?
- How important is electromagnetic broadening?
- Can we extract information about the potential barrier from a decay experiment?
- Why are the decay profiles for $A \approx 190$ so similar?

from Wilson et al. PRC, 71, 34319 (2005).
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Overview

**Basic Assumption**

Only one ND state mixes significantly with the decaying SD state.

\[ \varepsilon_N - \Gamma_N \quad \text{ND} \quad V \quad \text{SD} \quad \varepsilon_S \]


**Benefits**

- Elegant, intuitive model.
- Treats all interactions (nuclear and EM) on the same footing.
- Exactly solvable via Dyson’s Equation.
- Just four parameters: \( V, \Delta = \varepsilon_N - \varepsilon_S, \Gamma_S, \Gamma_N \).
- \( F_N \) is an experimental input.
Electromagnetic Decay Rates

- $\Gamma_S$: lifetimes, quadrupole moments.
- $\Gamma_N$:
  - Cranking model Fermi-gas level density (Åberg 1988):
    \[
    \rho(U) = \frac{\sqrt{\pi}}{48a^{1/4}} U^{-5/4} e^{2\sqrt{aU}}
    \]
  - Giant Dipole Resonance (Døssing & Vigezzi 1995):
    \[
    \Gamma_N \approx \Gamma_{E1}(U) \approx 4! \frac{4}{3\pi} \frac{e^2}{\hbar c} \frac{1}{mc^2} \frac{\Gamma_{GDR}}{E_0^4} \frac{NZ}{A} \left( \frac{U}{a} \right)^{5/2}
    \]
  - $a, E_0, \Gamma_{GDR}, U$(backshift) fit to nuclear data.
Non-Unitary Time Evolution

1. $t = 0$: The nucleus has just decayed via E2 and is localized in SD well.
2. Coherent Rabi oscillations + decoherent *virtual* interactions with EM field.
3. Nucleus escapes double-well by a *real* E1 or E2 decay.

Total wavefunction:
- $|\psi(t)\rangle = a_S(t)|S\rangle + a_N(t)|N\rangle$
- $|a_S(t)|^2 + |a_N(t)|^2 \leq 1$
- $|\psi(0)\rangle = |S\rangle$
Treat tunneling between wells as perturbation

\[ G_0^{-1} = \begin{pmatrix} E + i\frac{\Gamma_S}{2} & 0 \\ 0 & E - \Delta + i\frac{\Gamma_N}{2} \end{pmatrix} \]

\[ \hat{V} = \begin{pmatrix} 0 & V \\ V & 0 \end{pmatrix} \]

Dyson’s Equation - exact to all orders in \( \hat{V} \)

\[ G = G_0 + G_0 \hat{V} G \]

\[ G^{-1} = G_0^{-1} - \hat{V} = \begin{pmatrix} E + i\frac{\Gamma_S}{2} & -V \\ -V & E - \Delta + i\frac{\Gamma_N}{2} \end{pmatrix} \]
Complex Rabi Frequency
Stafford & Barrett *PRC* 60, 51305 (1999)

\[
P_N(t) = |G_{NS}(t)|^2 = \frac{2V^2}{|\omega|^2} e^{-(\Gamma_N+\Gamma_S)t/2} (\cosh \omega_i t - \cos \omega_r t)
\]

\[
\omega \equiv \omega_r + i\omega_i = \sqrt{4V^2 + \left[ \Delta - \frac{i}{2} (\Gamma_N - \Gamma_S) \right]^2}
\]

- \( \Gamma_N, \Gamma_S \sim 0.1\text{meV} \)
- \( V \gtrsim 1\text{eV} \)
- \( \Delta \sim D_N \equiv 1/\rho(U) \gtrsim 1\text{eV} \)

\[\Rightarrow\] The nucleus coherently oscillates \( \gtrsim 10^4 \) times before decaying!
Results

Branching ratios

\[ F_S = \frac{\Gamma_S}{\Gamma_S + \Gamma_N \Gamma_{\downarrow}/(\Gamma_N + \Gamma_{\downarrow})} = \frac{\Gamma_S}{\Gamma_S + \Gamma_{\text{out}}} \]

\[ \Gamma_{\downarrow} = \frac{2\bar{\Gamma}V^2}{\Delta^2 + \bar{\Gamma}^2}, \quad \bar{\Gamma} \equiv \frac{\Gamma_S + \Gamma_N}{2} \]

Tunneling width is a measurable quantity

\[ \Gamma_{\downarrow} = \frac{F_N \Gamma_N \Gamma_S}{\Gamma_N - F_N(\Gamma_S + \Gamma_N)} \]
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Limiting Cases of $\Gamma^\downarrow$

Return, for a moment, to the full ND spectrum. The net tunneling rate through the barrier is approximated by Fermi’s Golden Rule:

$$
\Gamma^\downarrow = 2\pi \int_{-\infty}^{\infty} V^2 \rho_S(E) \rho_N(E) dE.
$$

**Two-level limit**

$$
V \ll D_N \rightarrow \Gamma^\downarrow = \frac{2\Gamma V^2}{\Delta^2 + \Gamma^2}
$$

**Many-level limit**

$$
V \gg D_N \rightarrow \Gamma^\downarrow = 2\pi \frac{\langle V^2 \rangle}{D_N}
$$
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Gaussian Orthogonal Ensemble
A tool for calculating typical detunings

“Structureless” statistical model for ND states

- Assumes only time-reversal and rotational symmetry for nuclear Hamiltonian.
- Wigner surmise: \( P(s) = \frac{\pi}{2} s e^{-\frac{\pi}{4} s^2} \), \( s \equiv \frac{S}{D_N} \)

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\[
\mathcal{P}(\Delta_1|s) = \frac{1}{sD_N} \Theta \left( \frac{s}{2} - \frac{|\Delta_1|}{D_N} \right)
\]
\[
\mathcal{P}(\Delta_2|s) = \frac{1}{sD_N} \Theta \left( \frac{|\Delta_2|}{D_N} - \frac{s}{2} \right) \Theta \left( s - \frac{|\Delta_2|}{D_N} \right)
\]
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\[
\mathcal{P} (\Delta_{1,2}) = \int_0^\infty ds P(s) \mathcal{P}(\Delta_{1,2} \mid s)
\]

\[
\mathcal{P} (\Delta_1) = \frac{\pi}{2D_N} \text{erfc} (\sqrt{\pi} \frac{|\Delta_1|}{D_N})
\]

\[
\mathcal{P} (\Delta_2) = \frac{\pi}{2D_N} \left[ \text{erf} \left( \sqrt{\pi} \frac{|\Delta_2|}{D_N} \right) - \text{erf} \left( \frac{\sqrt{\pi}}{2} \frac{|\Delta_2|}{D_N} \right) \right]
\]
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\[
\langle |\Delta_1| \rangle = \frac{D_N}{4} \\
\langle |\Delta_2| \rangle = \frac{3D_N}{4}
\]
Statistical Theory of $V$

\[ \Gamma^\downarrow = \frac{2\Gamma V^2}{\Delta^2 + \Gamma^2} \rightarrow |\Delta| = \sqrt{\frac{2\Gamma}{\Gamma^\downarrow}} \left( V^2 - \frac{\Gamma^\downarrow \Gamma}{2} \right) \rightarrow V_{\text{min}} = \sqrt{\frac{\Gamma^\downarrow \Gamma}{2}} \]

\[ P(V) = 2P(\Delta) \left| \frac{d\Delta}{dV} \right| \]

The most one can say about $V$ with current experiments

\[ P(V \geq V_{\text{min}}) = \frac{2\pi \Gamma V}{\Gamma^\downarrow |\Delta| D_N} \text{erfc} \left( \sqrt{\pi} \frac{|\Delta|}{D_N} \right) \]

\[ \langle V \rangle = \sqrt{\frac{\Gamma^\downarrow}{2\Gamma}} \left[ \frac{D_N}{4} + O \left( \frac{\Gamma^2}{D_N} \right) \right] \]
Statistical Theory of $V$

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### Extraction of $\langle V \rangle$

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$F_N$</th>
<th>$\Gamma_S$</th>
<th>$\Gamma_N$</th>
<th>$D_N$</th>
<th>$\Gamma_{\downarrow}$</th>
<th>$\langle V \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{192}\text{Pb}(14)$</td>
<td>0.02</td>
<td>0.266</td>
<td>0.201</td>
<td>1258</td>
<td>0.0056</td>
<td>34</td>
</tr>
<tr>
<td>$^{192}\text{Pb}(12)$</td>
<td>0.34</td>
<td>0.132</td>
<td>0.200</td>
<td>1272</td>
<td>0.10</td>
<td>170</td>
</tr>
<tr>
<td>$^{192}\text{Pb}(10)$</td>
<td>0.88</td>
<td>0.048</td>
<td>0.188</td>
<td>1410</td>
<td>1.9</td>
<td>1000</td>
</tr>
<tr>
<td>$^{194}\text{Hg}(12)$</td>
<td>0.40</td>
<td>0.108</td>
<td>21</td>
<td>344</td>
<td>0.072</td>
<td>5.0</td>
</tr>
<tr>
<td>$^{194}\text{Hg}(10)$</td>
<td>0.97</td>
<td>0.046</td>
<td>20</td>
<td>493</td>
<td>1.6</td>
<td>35</td>
</tr>
</tbody>
</table>

$F_N$, $\Gamma_S$, $\Gamma_N$, and $D_N$:

- $^{194}\text{Hg}$: Lauritsen et al., *PRL* 88, 042501 (2002).

$\Gamma_{\downarrow}$ for $^{192}\text{Pb}(10)$ is the median value given $\Gamma_{\downarrow} \geq 0$ and $\sigma_{\Gamma_N} = \Gamma_N$. 
### Adding a Third Level

#### Three-level Green function

\[
G^{-1} = \begin{pmatrix}
E + i\Gamma_S/2 & -V_1 & -V_2 \\
-V_1 & E - \Delta_1 + i\Gamma_N/2 & 0 \\
-V_2 & 0 & E - \Delta_2 + i\Gamma_N/2
\end{pmatrix}
\]

#### Total ND branching ratio

\[
F_N = F_{N1} + F_{N2}
\]

- Second ND level will take some strength from each of the other levels.
- New possibility: quantum interference effects.
Three-Level Results
Levels taken at their mean detunings

\[ \frac{\Gamma_S}{\Gamma_N} = 10^{-3} \]

\[ \Gamma_S = \Gamma_N \]

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Infinite ND Band Approximation
Dzyublik & Utyuzh, PRC 68, 024311 (2003)

From Dzyublik & Utyuzh. $A \approx 190$. $\Delta_1$ is taken at its mean value in the GOE.
The Shopping List Revisited

- How many states do we need to keep in the ND well?
- How important is electromagnetic broadening?
- Can we extract information about the potential barrier from a decay experiment?
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![Graph](from Wilson et al. PRC, 71, 34319 (2005).)
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![Graph showing in-band SD intensity vs spin of initial level]

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![Graph](image)

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![Graph showing in-band SD intensity vs spin of initial level for different isotopes of Hg and Pb.](from Wilson et al. PRC, 71, 34319 (2005).)
Under what Conditions can Decay Occur?
DMC, B. R. Barrett, & C. A. Stafford, nucl-th/0702072.

Rewrite $F_S$:

$$F_S = 1 - \frac{1}{1 + \left(\frac{V_c}{V}\right)^2 + \frac{\Gamma_S}{\Gamma_N}},$$

$$V_c^2 = \left(\Delta^2 + \Gamma^2\right) \frac{\Gamma_S/\Gamma_N}{1 + \Gamma_S/\Gamma_N}.$$

- Problem separates naturally into two energy scales.
- Only when conditions are favorable in both can decay occur.
- $\Gamma_N$ is in competition with $\Gamma_S$, $V$ with $V_c$ (renormalized or effective $\Delta$).

→ Decay occurs only when $V \gtrsim V_c$ and $\Gamma_N \gtrsim \Gamma_S$. 

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Universality in the 190 Mass Region

Rapid drop in $\Gamma_S$ just before decay.

$\Gamma_S \sim T(B(E2)) \sim Q^2 I^5$

$\Gamma_N = \Gamma_S / 10$
$\Gamma_N = \Gamma_S$
$\Gamma_N = 10\Gamma_S$
$\Gamma_N = 100\Gamma_S$
$\Gamma_N = 1000\Gamma_S$
The two-level approximation yields an elegant, exactly solvable model.

The decay is governed by competition between direct decay down the SD band and a two-step series decay, through the barrier and into the ND band.

Three and infinite-level models indicate the two-level approximation is extremely accurate, especially for the $A \approx 190$ decay-out.

Making use of the GOE allows a statistical extraction of $V$.

Universality in the two-level model is a natural result of falling $\Gamma_S$. 