Schiff Moments and Nuclear Structure

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Outline

T Violation and EDMs

Existing Calculations

Reducing Uncertainty

Contribution of New Terms in Schiff Operator
Theorem (T/EDM Connection)
Nondegenerate states have static electric dipole moments iff T and P are violated.

Handwaving Proof
Lack of degeneracy implies $\langle \vec{d} \rangle \propto \langle \vec{J} \rangle$ with same proportionality constant in each $M$ substate. But $\langle \vec{J} \rangle$ and $\langle \vec{d} \rangle$ transform oppositely under time reversal of operators and state $(M \rightarrow -M)$ if T is conserved. So if T is a good symmetry, the state cannot have an EDM. If not, the state will have one.

Of course, $\mathcal{T} = \mathcal{CP}$.
EDMs Sensitive to New Physics

In standard model only one phase. Diagrams cancel to high order, e.g.:

\[ i \sin \delta_{f f'} W_{f f'} - i \sin \delta_{f f'} W_{f f'} + \ldots \]

SUSY has many phases. Low-order diagrams uncanceled, e.g.:

\[ e^{i\theta_{f f'}} \]

Thus, EDMs are insensitive to standard-model $\mathcal{CP}$ but sensitive to extra-standard-model $\mathcal{CP}$. Limits from atoms and neutrons, have already made SUSY a difficult proposition.
How Do Things Get EDMs?

Starting at most fundamental level and moving up:

- Underlying fundamental theory generates three $T$-violating $\pi NN$ vertices:

- Then neutron gets EDM from diagrams like this:
How Do Atoms Get EDMs?

- Nucleus can get one from nucleon EDM or $T$-violating $NN$ interaction:

$$W \propto \left\{ \bar{g}_0 \tau_1 \cdot \tau_2 - \frac{\bar{g}_1}{2} (\tau_1^z + \tau_2^z) + \bar{g}_2 (3\tau_1^z\tau_2^z - \tau_1 \cdot \tau_2) \right\} (\sigma_1 - \sigma_2)$$

$$- \frac{\bar{g}_1}{2} (\tau_1^z - \tau_2^z) (\sigma_1 + \sigma_2) \right\} \cdot (\nabla_1 - \nabla_2) \exp \left( \frac{-m_\pi |r_1 - r_2|}{m_\pi |r_1 - r_2|} \right)$$

- Finally, atom gets one from nucleus. Electronic shielding makes the relevant nuclear object the “Schiff moment”

$$\langle S_z \rangle \approx \langle \sum_p \left( r_p^2 z_p - \frac{5}{3Z} r_p^2 D_z + 4 \frac{\sqrt{2\pi}}{3} [r_p^2 Y_{2p} \otimes D]_0^1 \right) \rangle$$

rather than

$$\langle D_z \rangle \equiv \langle \sum_p z_p \rangle$$.

Nuclear-structure theory’s place in the chain: calculating dependence of $\langle S_z \rangle$ on the $\bar{g}$’s in heavy nuclei.
Hartree-Fock-Bogoliubov calculations with phenomenological density-dependent Skyrme interaction

\[ H_{Sk} = b_0 \left( 1 + x_0 \hat{P}_\sigma \right) \delta(r_1 - r_2) \\
+ b_1 \left( 1 + x_1 \hat{P}_\sigma \right) \left[ (\nabla_1 - \nabla_2)^2 \delta(r_1 - r_2) + \text{h.c.} \right] \\
+ b_2 \left( 1 + x_2 \hat{P}_\sigma \right) (\nabla_1 - \nabla_2) \cdot \delta(r_1 - r_2)(\nabla_1 - \nabla_2) \\
+ b_3 \left( 1 + x_3 \hat{P}_\sigma \right) \delta(r_1 - r_2) \rho^\alpha \left( \frac{r_1 + r_2}{2} \right) \\
+ ib_4 (\sigma_1 + \sigma_2) \cdot (\nabla_1 - \nabla_2) \times \delta(r_1 - r_2)(\nabla_1 - \nabla_2) \\
+ \ldots, \]

where

\[ \hat{P}_\sigma = \frac{1 + \sigma_1 \cdot \sigma_2}{2}, \]

\[ b_i, x_i, \alpha \] adjusted to fit masses, radii, etc.

Corrections to/Refinements of HFB are the frontier.
EDM Work Thus Far

Best work has been approximations (so far) to HFB with

\[ H \approx H_{Sk} + W \]

(Note: Landau-Migdal is very approximate Skyrme.)

\( T \)-violating interaction

Work to improve the calculations is in progress.
This is the nucleus with the best experimental limit.

\[ \langle S_z \rangle_{\text{Hg}} \equiv a_0 \, \bar{g} \bar{g}_0 + a_1 \, \bar{g} \bar{g}_1 + a_2 \, \bar{g} \bar{g}_2 \, (\text{e fm}^3) \]

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<td>Dmitriev, Senkov, Auerbach</td>
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Spread in Skyrme results is a crude measure of uncertainty.
Here treat $W$ as explicit perturbation:

$$\langle S_z \rangle = \sum_m \frac{\langle 0 | S_z | m \rangle \langle m | W | 0 \rangle}{E_0 - E_m} + c.c.$$  

where $|0\rangle$ is unperturbed ground state.

Ground state has nearly-degenerate partner $|\bar{0}\rangle$ with same opposite parity and same intrinsic structure, so

$$\langle S_z \rangle \rightarrow \frac{\langle 0 | S_z | \bar{0} \rangle \langle \bar{0} | W | 0 \rangle}{E_0 - E_{\bar{0}}} + c.c. \propto \frac{\langle S_z \rangle_{\text{intr.}} \langle W \rangle_{\text{intr.}}}{E_0 - E_{\bar{0}}},$$

$\langle S_z \rangle$ is large because $\langle S_z \rangle_{\text{intr.}}$ is collective and $E_0 - E_{\bar{0}}$ is small.
Spectrum of $^{225}$Ra
Ra Results

Hartree-Fock calculation (Dobaczewski et al.) with SkO′ gives

\[ \langle S_z \rangle_{Ra} = -1.5 \, \bar{g}_0 + 6.0 \, \bar{g}_1 - 4.0 \, \bar{g}_2 \text{ (e fm}^3) \]

Larger by over 100 than in $^{199}$Hg!

But “uncertanty” (i.e. variation) similar.

What can we do to reduce it?

- Improve calculations. (We’re doing this.)
- Study Skyrme functionals and related observables. (Would be nice if someone else did this.)
Currently using HFODD to

- really do fully self-consistent Skyrme-HFB in $^{199}$Hg (and $^{131}$Xe), including deformation
- add angular-momentum projection for these nuclei
- octupole-deformed light actinides
- add parity projection for light actinides
Constructing Good Skyrme Interaction

This is mission of SciDAC UNEDF collaboration. But it looks at everything and here we’re interested in very particular and sensitive nuclear properties.

What do we care about?

$W$ probes spin density. Interaction should at minimum have good spin response.

Example of study: M. Bender et al. fit some time-odd terms of SkO’ to Gamow-Teller resonance energies and strengths.
In octupole-deformed nuclei ground-state spin properties more important than response. Reflected, e.g., in spin-orbit splittings.

Recent work on Skyrme tensor and spin-orbit forces needs to be systematically pursued.
Response (in non-octupole nuclei) to Schiff operator also important. Strength distribution of isoscalar analog of Schiff operator measured in $^{208}$Pb.

Would be nice to measure response in more detail and other nuclei (e.g. $^{199}$Hg), really nice to measure single transition to parity doublet in light actinides.
New terms in Schiff Operator

Liu, Ramsey-Musolf, Haxton, Timmermans, and Dieperink [arXiv:0705.1681] write operator as

\[
S_z \approx \sum_p r_p^2 z_p - \frac{5}{3Z} \left\langle \sum_p r_p^2 \right\rangle D_z + 4 \frac{\sqrt{2\pi}}{3} \sum_p r_p^2 [Y_{2p} \otimes D]_0 + \ldots
\]

with previously accepted version (containing \( \langle \rangle \) and \( \times \)) an approximation to this.

Experimentalists: “Uh, oh, what does this mean for me?”

**Answer:** New stuff, at least up to the \( \ldots \), doesn’t change things much in heavy nuclei.
Sum Rules

Consider first term containing \( r^2 \mathcal{D} \):

\[
\langle \mathcal{O} \rangle = \sum_i \langle 0 | \sum_p r_p^2 | i \rangle \langle i | \mathcal{D}_z | 0 \rangle = \langle \mathcal{O} \rangle_0 + \langle \mathcal{O} \rangle_{\text{exc.}}.
\]

\[
\langle \mathcal{O} \rangle_0 = \langle 0 | \sum_p r_p^2 | 0 \rangle \langle 0 | \mathcal{D}_z | 0 \rangle = ZR_{\text{ch}}^2 \langle \mathcal{D}_z \rangle, \quad \langle \mathcal{O} \rangle_{\text{exc.}} = \sum_{i \neq 0} \langle 0 | \sum_p r_p^2 | i \rangle \langle i | \mathcal{D}_z | 0 \rangle.
\]

For excited states there is an energy weighted sum rule:

\[
\langle \mathcal{O} \rangle_{\text{EW}} \equiv \text{Re} \sum_i (E_i - E_0) \langle 0 | \sum_p r_p^2 | i \rangle \langle i | \mathcal{D}_z | 0 \rangle = \frac{1}{2} \langle 0 | \left[ \sum_p r_p^2, [H, \mathcal{D}_z] \right] | 0 \rangle.
\]

Evaluating in the same approximate way as usual \( E1 \) sum rule:

\[
\langle \mathcal{O} \rangle_{\text{EW}} \approx \frac{\hbar^2}{m} \langle \mathcal{D}_z \rangle.
\]

Let \( \bar{E} \equiv \langle \mathcal{O} \rangle_{\text{EW}} / \langle \mathcal{O} \rangle_{\text{exc.}} \). Then

\[
\bar{E} = 10 \text{ Mev} \quad \text{(for example)} \quad \longrightarrow \quad \langle \mathcal{O} \rangle_{\text{exc.}} \approx \frac{\langle \mathcal{O} \rangle_0}{600}.
\]
Another Sum Rule

There is also an inverse-energy-weighted sum rule:

\[ \langle O \rangle_{\text{IEW}} \equiv \text{Re} \sum_{i \neq 0} \frac{\langle 0| \sum_p r_p^2 |i\rangle \langle i| D_z |0 \rangle}{E_i - E_0} = -\frac{1}{2} \frac{d}{d\lambda} \left( \langle D_z \rangle_{H + \lambda \sum_p r_p^2} \right) \]

Assuming spherical symmetry and a Hamiltonian \( H = H_{\text{oscillator}} + \ldots \) (and varying neutron oscillator as well as proton oscillator), one gets

\[ \langle O \rangle_{\text{IEW}} \approx \frac{1}{2m\omega^2} \langle D_z \rangle, \]

where \( \omega \) is the oscillator frequency.

Again, no \( Z \) enhancement.

Assuming \( \langle O \rangle_{\text{IEW}} \approx \langle O \rangle_{\text{exc.}} / \bar{E} \), again

\[ \bar{E} = 10 \text{ Mev} \quad \longrightarrow \quad \langle O \rangle_{\text{exc.}} \approx \frac{\langle O \rangle_0}{600} \]
Quadrupole Term

- Similar considerations make excited-intermediate-state part negligible.
- Intermediate ground-state part vanishes for $J = 1/2$ nuclei.

Latter is negligible in heavy nuclei even if $J \neq 1/2$. Largest known static quadrupole moment is $\approx 700 \text{ fm}^2$. Implies that for large $Z$, ground-state part of the quadrupole term is smaller than that of the monopole term by a factor that is at least 5 (and usually much more).

Conclusion: None of the new terms included in $\vec{S}$ will be important for large $Z$.

Caution: There are other terms, most apparently suppressed by $\nu/c$, that remain unanalyzed.
THE END
Consider nondegenerate ground state $|\text{g.s.} : J, M\rangle$. Symmetry under rotations $R_y(\pi)$ for vector operator like $d_z \equiv \sum_i e_i z_i$,

$$\langle \text{g.s.} : J, M | d_z | \text{g.s.} : J, M \rangle = -\langle \text{g.s.} : J, -M | d_z | \text{g.s.} : J, -M \rangle .$$

$T$ takes $M$ to $-M$, like $R_y(\pi)$. But $\tilde{d}$ is odd under $R_y(\pi)$ and even under $T$, so for $T$ conserved

$$\langle \text{g.s.} : J, M | d_z | \text{g.s.} : J, M \rangle = +\langle \text{g.s.} : J, -M | d_z | \text{g.s.} : J, -M \rangle .$$

Together with the first equation, this implies

$$\langle d_z \rangle = 0 .$$

If $T$ is violated, argument fails because $T$ can take $|\text{g.s.} : JM\rangle$ to $|\text{ex.} : J, -M\rangle$, a state in a different multiplet.