Description of mixed-mode dynamics within the IVBM:

II. Orthosymplectic extension – the odd-even nuclei

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• Introduction
• Transition probabilities. Application
• The Inclusion of Spin
• Orthosymplectic extension. Algebraic structure
• Representations
• Application of the new dynamical symmetry
• Conclusions
Tensor properties

\[ Sp(12, R) \supset U(6) \supset U(3) \otimes U(2) \supset O(3) \otimes U(1) \]

\[ F^{[2]_6}_{[2]_{12}} \quad LM_{11} = \sum_{m,k} C^{LM}_{1m1k} p_m^\dagger p_k^\dagger, \]

\[ F^{[2]_6}_{[2]_{12}} \quad LM_{1-1} = \sum_{m,k} C^{LM}_{1m1k} n_m^\dagger n_k^\dagger, \]

\[ F^{[2]_6}_{[2]_{12}} \quad LM_{10} = \frac{1}{\sqrt{2}} \sum_{m,k} C^{LM}_{1m1k} (p_m^\dagger n_k^\dagger - n_m^\dagger p_k^\dagger), \]

\[ F^{[2]_6}_{[1]_{12}[0]_{02}} \quad LM_{00} = \frac{1}{\sqrt{2}} \sum_{m,k} C^{LM}_{1m1k} (p_m^\dagger n_k^\dagger + n_m^\dagger p_k^\dagger). \]

\[ A^{[1-1]_6}_{[2]_{12}[0]_{02}} \quad LM_{00} = \frac{1}{\sqrt{2}} \sum_{m,k} C^{LM}_{1m1k} (p_m^\dagger n_k^\dagger + n_m^\dagger p_k^\dagger), \]

\[ A^{[1-1]_6}_{[1]_{12}[0]_{02}} \quad LM_{00} = \frac{1}{\sqrt{2}} \sum_{m,k} C^{LM}_{1m1k} (p_m^\dagger n_k^\dagger + n_m^\dagger p_k^\dagger). \]

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Symplectic basis

\[ Sp(12, R) \supset U(6) \supset U(3) \otimes U(2) \supset O(3) \otimes (U(1) \otimes U(1)) \]

\[ | [N]_6, (\lambda, \mu); KLM; TT_0 \rangle = [F \times \ldots \times F]^{[\chi]_6}_{[\lambda]_3[2T]_2}^{LM}_{TT_0} | \Omega \rangle \]

\[ G^{[\chi]_6}_{[\lambda]_3[2T]_2}^{LM}_{TT_0} | \Omega \rangle = 0 , \quad | \Omega \rangle - \text{LWS} \]

\[ | \Omega \rangle = | 0 \rangle \quad \text{or} \quad | \Omega \rangle = u^+_k(\alpha)| 0 \rangle \]

\[ [\chi]_6 = [N]_6 \]
## Symplectic basis

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Matrix elements

Wigner–Eckart theorem

\[
\langle [N'] (\lambda', \mu'); K'L'M'; T'T_0 | T_\sigma \frac{[x]}{\sigma_3}; 2t; l =\frac{m}{l} | [N] (\lambda, \mu); KLM; TT_0 \rangle
\]

\[
= \langle [N'] (\lambda', \mu'); K'L'|T_\sigma \frac{[x]}{\sigma_3}; 2t\frac{m}{l} | [N] (\lambda, \mu); KL \rangle C_{LM}^{L'M'} C_{TT_0}^{T'T_0} C_{TT_0}^{TT_0}
\]

Reduced matrix elements

\[
\langle [N'] (\lambda', \mu'); K'L'|T_\sigma \frac{[x]}{\sigma_3}; 2t\frac{m}{l} | [N] (\lambda, \mu); KL \rangle
\]

\[
= \langle [N'] | |T_\sigma \frac{[x]}{\sigma_3}; 2t\frac{m}{l} | [N] \rangle C_{(\lambda', \mu')2T_0}^{[N']6} C_{KL}^{(\lambda, \mu)} C_{k(l)}^{(\lambda', \mu')} C_{K'L'}^{(\lambda', \mu')}
\]
Transition probabilities

Transition operator

\[ T^{E2} = e \left[ A_{[210]_3[9]_2}^{[1-1]_6} \begin{pmatrix} 0 & 20 \\ 0 & 0 \end{pmatrix} + \theta ([F \times F]_{(0,2)[0]_2}^{[4]_6} \begin{pmatrix} 0 & 20 \\ 0 & 0 \end{pmatrix} + [G \times G]_{(2,0)[0]_2}^{[-4]_6} \begin{pmatrix} 0 & 20 \\ 0 & 0 \end{pmatrix}) \right] \]

- SU(3) term
- Symplectic term

Transition rates

\[ B(E2; L_i \rightarrow L_f) = \frac{1}{2L_i + 1} \left| \langle f \| T^{E2} \| i \rangle \right|^2 \]

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Transition probabilities

Parameters:

- $e = 2.25$
- $\theta = 0.00027$

- $e = 2.1$
- $\theta = 0.00018$

Experiment vs. Theory for $^{236}U$ and $^{232}Th$.
Transition probabilities

\[ B(E2) \text{ [W.u.]} \]

\[ \theta = -0.00002 \]

\[ B(E2) \text{ [W.u.]} \]

\[ \theta = -0.00034 \]

\[ B(E2) \text{ [W.u.]} \]

\[ \theta = -0.0006 \]

Dy\(^{156}\)

\[ e = 2.52 \]

\[ e = 2.6 \]

\[ e = 2.67 \]

Experiment

Theory

Theory

Theory

IBM

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Odd mass nuclei
The Inclusion of Spin

- Consider a particle with spin $S = 1/2 \ h$

Fermion operators:

\[ \{a_i^\dagger, a_j^\dagger\} = \{a_i, a_j\} = 0, \]
\[ \{a_i, a_j^\dagger\} = \delta_{ij}. \]

Generators

\[ f_{ij} = a_i^\dagger a_j^\dagger, \]
\[ g_{ij} = a_i a_j; \quad i \neq j, \]
\[ C_{ij} = (a_i^\dagger a_j - a_j a_i^\dagger) / 2 \]

- The group of the spin - SU$^F(2)$
- Consider Core + particle picture
- Embedding $\text{SU}^F(2) \subset \text{SO}(4)$

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Orthosymplectic extension

$OSp(4/12,R) \supset SO^F(4) \otimes Sp^B(12,R)$

$\supset SU^F(2) \otimes U^B(6)$

$\supset SU^F(2) \otimes SU^B(3) \otimes U^B(2)$

$(\lambda, \mu) \quad (N, T)$

$\supset SU^F(2) \otimes SO^B(3) \otimes U^B(1)$

$L \quad T_0$

$\supset Spin^{BF}(3) \supset Spin^{BF}(2)$

$J \quad J_0$
osp(4/12,R) Lie algebra

Superoscillators:

\[ \xi_A^\dagger = \begin{pmatrix} u_{\dagger k}^j (\alpha) \\ a_j^\dagger \end{pmatrix}, \quad k = 0, \pm 1; \quad \alpha = \pm 1/2; \quad j = \pm 1/2 \]

\[ \xi_A = (\xi_A^\dagger)^\dagger \]

Generators:

\[ F_{AB} = \xi_A^\dagger \xi_B^\dagger \]
\[ G_{AB} = \xi_A \xi_B \]
\[ A_{AB} = \xi_A^\dagger \xi_B + (-1)^{\text{deg} A \cdot \text{deg} B} \xi_B \xi_A^\dagger \]

where \( \text{deg} A = 0 \) or 1 depending on whether \( A \) is a bosonic or a fermionic index

Limiting cases:

\( (A = k, \alpha) \leftrightarrow Sp(12, R), \)
\( (A = j) \leftrightarrow SO(4). \)

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Representations of osp(4/12,R)

Jordan decomposition

\[ n = n_- \oplus n_0 \oplus n_+ \]

\[ G_{AB} \quad A_{AB} \quad F_{AB} \]

\[ R = \{ | \Omega \rangle \oplus F_{AB} | \Omega \rangle \oplus F_{AB}F_{CD} | \Omega \rangle \oplus \ldots \} \]

Lowest weight state (LWS):

\[ | \Omega \rangle \]

\[ G_{AB} | \Omega \rangle = 0. \]

U(6/2) content

\[ F_{AB} \approx \begin{array}{c|c|c|c|c|c|c} \hline \hline \hline \hline \end{array} \]

\[ F_{AB}F_{CD} \approx (\begin{array}{c|c|c|c|c|c|c} \hline \hline \hline \hline \end{array})_S \]

\[ \vdots \]

\[ F_{AB} \cdots F_{CD} \approx (\begin{array}{c|c|c|c|c|c|c} \hline \hline \hline \hline \end{array})_S \]

Super Fock
Representations of \( \text{osp}(4/12, \mathbb{R}) \)

The irreducible LWS of \( \text{osp}(4/12) \):

1) \( \Omega \rangle = 0 \rangle_{SF} \)

2) \( \Omega \rangle = \xi^+_A 0 \rangle_{SF} \)

Even subalgebra: \( \text{sp}(12, \mathbb{R}) \oplus \text{so}(4) \)

Lowest weight vectors:

\[
|LWS\rangle = |\Omega_{LWS}\rangle_B \otimes |\Omega_{WLS}\rangle_F \]

\[
[N]_6 \otimes (r_1, r_2)_{GZ} \quad [N]_6 \quad (r_1, r_2)_{GZ}
\]

The IR of \( \text{osp}(4/12, \mathbb{R}) \) with the lowest weight vector \( 0 \rangle_{SF} \) has lowest weight vectors of \( \text{sp}(12, \mathbb{R}) \oplus \text{so}(4) \):

(1) \( | 0 \rangle \) \( \rightarrow \) Even-even nuclei

(2) \( u^+_k(\alpha) a^+_j 0 \rangle \)

The IR of \( \text{osp}(4/12, \mathbb{R}) \) with the lowest weight vector \( \xi^+_A 0 \rangle_{SF} \) has lowest weight vectors of \( \text{sp}(12, \mathbb{R}) \oplus \text{so}(4) \):

(3) \( u^+_k(\alpha) | 0 \rangle \approx (\square, 1) \) \( \rightarrow \) Odd–A nuclei

(4) \( a^+_j | 0 \rangle \approx (1, \square) \)

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The energy spectrum

The Hamiltonian

\[ H = aN + bN^2 + \alpha_3 T^2 + \beta_3 L^2 + a_1 T_0^2 + \gamma J^2 + \xi J_0^2 \]

The Basis

\[ | [N]_6; (N, T); KL; S; JJ_0; T_0 \rangle \]

The Energies

\[ E([N]_6; (N, T); KL; S; JJ_0; T_0 ) = \]
\[ = aN + bN^2 + \alpha_3 T(T+1) + \beta_3 L(L+1) + a_1 T_0^2 + \]
\[ + \gamma J(J+1) + \xi J_0^2 \]

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### Basis states

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<th>L</th>
<th>J</th>
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• Parity: \( \pi = (-1)^T \)
This allows us to describe both positive and negative parity states.

• Algebraic definition for yrast states:

\[
E = \min \quad \text{with respect to } N
\]

\[
N \leftrightarrow J
\]

• GSB: \( K=1/2^+ \) \( \rightarrow \) \( N = 2J - 1 \)
  \( K=3/2^- \) \( \rightarrow \) \( N = 2J + 3 \)

• Excited bands

Band head structure: \( N_{\text{ini}} \)
The energy spectrum

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GSB: $K^\pi=1/2^+$
The energy spectrum for $^{157}\text{Gd}$ shows various energy levels as indicated by the labels $3/2$, $5/2$, etc., along the y-axis. The diagram includes annotations for the Institute for Nuclear Theory, Seattle, November 2007.
Transition probabilities

Experiment vs Theory

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Conclusions

• The applicability of the model is further confirmed by the reproduction of the B(E2) behavior of the transition probabilities in the GSB for some even-even and odd-even nuclei. Analyzing the terms in the transition operator, the important role of the symplectic extension is revealed.

• The mixing of the different collective modes within the symplectic and orthosymplectic structures remains the main reason for the good reproduction of the experimental data.

• The model can be further used for the description and systematics of other collective bands.

• Critical phase/shape phenomena can be analyzed within the IVBM.

• Orthosymplectic extension of the IVBM can be used to examine the manifestation and the gross features of nuclear supersymmetry.
Thank you