Time-Correlated Structure in Pulsar Timing Noise

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The Goal: To Identify Evidence of Non-rigid Body Rotation

- Timing Noise: Response of the Crust to Stochastic Torque
- All modes are excited (e.g. damped rotational modes, precession, vortex lattice modes)
- Previous studies using Fourier techniques have found no evidence of non-rigid body rotation (Boynton, Deeter, 1969; Boynton, 1981; Boynton, Deeter, et. al., 1984)
2-Component NS Model

\[ I_c \dot{\Omega}_c = N(t) - \frac{I_r}{\tau} (\Omega_c - \Omega_s) \]

\[ I_s \dot{\Omega}_s = \frac{I_r}{\tau} (\Omega_c - \Omega_s) \]

\[ |\Omega_c(\omega)|^2 = \left[ \frac{(\omega\tau)^4 + (\frac{I_s}{I})^2(\omega\tau)^2}{(\omega\tau)^4 + (\omega\tau)^2} \right] \frac{|N(\omega)|^2}{\omega^2 I_c^2} \]
Frequency Response

\[ |\Omega_c(\omega \tau)|^2 \sim (1 - \frac{I_c}{I}) \]
Observations

- We use methods in the time domain
PSR B1133+16

- Spin period: \( P = 1.19 \, s \)

- Dipole field: \( |B_d| = 5 \times 10^{12} \, G \)

- Spin-down age: \( \tau = 5 \times 10^6 \, yrs \)
The Data

Timing Residuals, PSR 1133+16
Distribution of Residuals

![Probability Distribution Function](image)
Auto-Correlation Function

• For evenly sampled data:

\[
ACF(k) = \frac{1}{(n - k) \sigma^2} \sum_{i=1}^{n-k} (y_i - \bar{y})(y_{i+k} - \bar{y})
\]

• For unevenly sampled data: we use the discrete correlation function, DCF (Edelson, Krolik, 1988)
Discrete Correlation Function

$$UDCF_{ij} = \frac{(y_i - \bar{y})(y_j - \bar{y})}{\sigma_y^2}$$

$$DCF(\tau) = \frac{1}{M} \sum_{i=1}^{M} UDCF$$

$$\tau - \frac{\Delta\tau}{2} \leq t_j - t_i \leq \tau + \frac{\Delta\tau}{2}$$

• Benefits
  - No binning required
  - Uses all data points
  - No penalty for gaps in the data
Auto-Correlation Function

\[ ACF(t) = e^{-t/\tau_d} \]
Discrete Correlation Function

PSR B1133+16


\[ S_{\text{corr}} \approx 26 \sigma \]
Significance

• From Monte Carlo simulations, the probability of the null hypothesis is less than $10^{-4}$

• Further confirmation: Lagged Dispersion Statistic
Lagged Dispersion Statistic

\[ \Delta y_{ij} = y_j - y_i \]

\[ \tau - \frac{\Delta \tau}{2} \leq t_j - t_i \leq \tau + \frac{\Delta \tau}{2} \]

\[ LD(\tau) = \frac{1}{M} \sum (\Delta y_{ij} - \bar{\Delta y})^2 \]
Lagged Dispersion

![Graph showing Lagged Dispersion](image)
Conclusions

• We find correlation timescale of 10 days in PSR B1133+16

• Interpretation: Deviations from rigid body rotation

• DCF and LD (Time Domain) are better suited for analyzing noisy data than Fourier techniques
Error Sensitivity Simulation

$\delta \phi$

$|\delta \phi(\omega)|^2$

$\omega$

(time)
Error Sensitivity Simulation

\[ \delta \phi \]

\[ |\delta \phi (\omega)|^2 \]

\[ \text{time} \]

\[ \omega \]
Error Sensitivity Simulation
Future Work

\[ I_c \dot{\Omega}_c = N(t) - \frac{I_r}{\tau} (\Omega_c - \Omega_s) \]

\[ I_s \dot{\Omega}_s = \frac{I_r}{\tau} (\Omega_c - \Omega_s) \]

- Constrain values of moment of inertia ratio using our results