Neutron Star Cooling
Pairing & Magnetic Fields

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Basic Neutron Star Cooling
Troubles: Surface Effects and Pairing
Minimal Cooling
The “Magnificent Seven”: Strong Toroidal Fields?
Conclusions
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Basic Equations

Schwarzschild metric:

\[ ds^2 = -e^{2\phi} c^2 dt^2 + \frac{dr^2}{1 - 2Gm/c^2r} + r^2 d\Omega^2 \]

Proper time:

\[ d\tau = e^\phi dt \]

Proper length:

\[ dl = \frac{dr}{\sqrt{1 - 2Gm/c^2r}} \]

Energy balance:

\[ \frac{d(Le^{2\phi})}{dr} = -\frac{4\pi r^2 e^\phi}{\sqrt{1 - 2Gm/c^2r}} \left( \frac{d\epsilon}{dt} + e^\phi (q_v - q_h) \right) \quad \text{and} \quad L(r = 0) = 0 \]

\[ \frac{d\epsilon}{dt} = \frac{d\epsilon}{dT} \frac{dT}{dt} = c_v \frac{dT}{dt} \]

Energy transport:

\[ \frac{d(Te^\phi)}{dr} = -\frac{1}{\lambda} \frac{Le^\phi}{4\pi r^2 \sqrt{1 - 2Gm/c^2r}} \quad \text{and} \quad T(r = r_b) = T_b = T_b(L_b) \]

\[ L(r = r_b) = L_b = L_\gamma \quad \text{and} \quad L_\gamma = 4\pi R^2 \sigma_B T_e \]
Energy balance:

\[
\frac{dE_{th}}{dt} = C_v \frac{dT}{dt} = -L_\gamma - L_\nu + H
\]

⇒ 3 essential ingredients are needed:

- \(C_v\) = total stellar specific heat
- \(L_\gamma\) = total surface photon luminosity
- \(L_\nu\) = total stellar neutrino luminosity
Neutrino Emission

Basic mechanism: $\beta$ and inverse $\beta$ decays:

\[ n \rightarrow p + e^- + \bar{\nu}_e \quad \text{and} \quad p + e^- \rightarrow n + \nu_e \]

Energy conservation:

\[ E_{Fn} = E_{Fp} + E_{Fe} \]

Momentum conservation:

"Triangle rule": \[ p_{Fn} < p_{Fp} + p_{Fe} \]

\[ n_i = \frac{k_F^3}{3\pi^2} \Rightarrow n_n^{1/3} \leq n_p^{1/3} + n_e^{1/3} = 2n_p^{1/3} \]

\[ x_p \equiv \frac{n_p}{n_n + n_p} \geq \frac{1}{9} \approx 11\% \]

Direct URCA process in neutron stars, Lattimer, Pethick, Prakash & Haensel, 1991 PRL 66, 2701
### Fast vs Slow Neutrino Emission

<table>
<thead>
<tr>
<th>Name</th>
<th>Process</th>
<th>Emissivity (erg cm(^{-3}) s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Modified Urca cycle</strong></td>
<td>(n + n \rightarrow n + p + e^- + \bar{\nu}_e)</td>
<td>(\sim 2 \times 10^{21} R T_{9}^{8})</td>
</tr>
<tr>
<td>(neutron branch)</td>
<td>(n + p + e^- \rightarrow n + n + \nu_e)</td>
<td>Slow</td>
</tr>
<tr>
<td><strong>Modified Urca cycle</strong></td>
<td>(p + n \rightarrow p + p + e^- + \bar{\nu}_e)</td>
<td>(\sim 10^{21} R T_{9}^{8})</td>
</tr>
<tr>
<td>(proton branch)</td>
<td>(p + p + e^- \rightarrow p + n + \nu_e)</td>
<td>Slow</td>
</tr>
<tr>
<td>Bremsstrahlung</td>
<td>(n + p \rightarrow n + p + \nu + \bar{\nu})</td>
<td>(\sim 10^{19} R T_{9}^{8})</td>
</tr>
<tr>
<td>Cooper pair formations</td>
<td>(n + n \rightarrow [nn] + \nu + \bar{\nu})</td>
<td>(\sim 5 \times 10^{21} R T_{9}^{7})</td>
</tr>
<tr>
<td>Direct Urca cycle</td>
<td>(n \rightarrow p + e^- + \bar{\nu}_e)</td>
<td>(\sim 10^{27} R T_{9}^{6})</td>
</tr>
<tr>
<td>(\pi^-) condensate</td>
<td>(n + &lt; \pi^- &gt; \rightarrow n + e^- + \bar{\nu}_e)</td>
<td>(\sim 10^{26} R T_{9}^{6})</td>
</tr>
<tr>
<td>(K^-) condensate</td>
<td>(n + &lt; K^- &gt; \rightarrow n + e^- + \bar{\nu}_e)</td>
<td>(\sim 10^{25} R T_{9}^{6})</td>
</tr>
</tbody>
</table>

Direct vs Modified Urca Processes

Specific Heat

Sum over all degenerate Fermion:

$$C_v = \sum_i C_{v\ i}$$

$$C_{v\ i} = N_i(0) \frac{\pi^2}{3} k_B^2 T$$

with

$$N_i(0) = \frac{m_i^* p_{Fi}}{\pi^2 \hbar^3}$$
$T_e \approx 10^6 \left( \frac{T_{\text{int}}}{10^8 \text{K}} \right)^{1/2}$

$L_\gamma = 4\pi R^2 \sigma T_e^4$
Some Simple Analytical Solutions

\[ \frac{dE_{th}}{dt} = C_v \frac{dT}{dt} = -L_\gamma - L_\nu \]

\[ C_v = CT \quad L_\nu = NT^8 \quad L_\gamma = ST^{2+4\alpha} \]

\[ L_\gamma = 4\pi R^2 \sigma T_e^4 \text{ with } T_e \propto T^{0.5+\alpha} \]

**Neutrino Cooling Era:** \( L_\nu >> L_\gamma \)

\[ \frac{dT}{dt} = -\frac{N}{C} T^7 \implies t - t_0 = A \left[ \frac{1}{T^6} - \frac{1}{T_0^6} \right] \]

\[ T \propto t^{-1/6} \]

**Photon Cooling Era:** \( L_\gamma >> L_\nu \)

\[ \frac{dT}{dt} = -\frac{N}{S} T^{1+\alpha} \implies t - t_0 = A \left[ \frac{1}{T^\alpha} - \frac{1}{T_0^\alpha} \right] \]

\[ T \propto t^{-1/\alpha} \]
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The “$T_e - T_b$” relationship for heavy element envelopes
The “$T_e - T_b$” relationship for heavy element envelopes

... and for light element envelopes

Thermal conductivity in the liquid phase $\lambda \propto \frac{1}{Z}$

$\Delta M_{\text{Light}} \approx 10^{-7} M_{\text{Sun}}$

Troubles (1): Envelope Chemical Composition

Light elements envelope
Iron-like envelope

Log $T_e$ [K] vs. Log $T_0$ [K]

Log $L$ [erg s$^{-1}$] vs. Log $t$ [yrs]

Log $T_{e,\text{inner}}$ [K] vs. Log $t$ [yrs]

Log $T_{e,\text{outer}}$ [K] vs. Log $t$ [yrs]
Fig. 1. Energies of first excited intrinsic states in deformed nuclei, as a function of the mass number. The experimental data may be found in Nuclear Data Cards [National Research Council, Washington, D. C.] and detailed references will be contained in reference 1 above. The solid line gives the energy $\frac{5}{2}$ given by Eq. (1), and represents the average distance between intrinsic levels in the odd-$A$ nuclei (see reference 1).

The figure contains all the available data for nuclei with $150 < A < 190$ and $228 < A$. In these regions the nuclei are known to possess nonspherical equilibrium shapes, as evidenced especially by the occurrence of rotational spectra (see, e.g., reference 2). One other such region has also been identified around $A = 25$; in this latter region the available data on odd-$A$ nuclei is still represented by Eq. (1), while the intrinsic excitations in the even-even nuclei in this region do not occur below 4 Mev.

We have not included in the figure the low lying $K=0$ states found in even-even nuclei around Ra and Th. These states appear to represent a collective odd-parity oscillation.

Possible Analogy between the Excitation Spectra of Nuclei and Those of the Superconducting Metallic State, Bohr, Mottelson, Pines, 1958 PhRv 110, 936
The presence of a pairing gap in the single particle excitation spectrum results in a Boltzmann-like $\exp(-\Delta/k_B T)$ suppression of $C_v$ and $Q_\nu$:

$$C_v \rightarrow C_{v}^{\text{Paired}} = R_c C_{v}^{\text{Normal}}$$

$$Q_\nu \rightarrow Q_{\nu}^{\text{Paired}} = R_\nu Q_{\nu}^{\text{Normal}}$$

Enormous uncertainties on the actual values of $T_c$ for pairing in the core (proton $^1S_0$ and neutron $^3P_2$)
Effect of Pairing on the Cooling

Slow cooling

\[ q_\nu \sim 10^{21} T_9^8 \text{ erg cm}^{-3} \text{ s}^{-1} \]

and

Fast cooling

\[ q_\nu \sim 10^n T_9^6 \text{ erg cm}^{-3} \text{ s}^{-1} \]

controlled by pairing
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Minimal Cooling: exclude anything beyond just nucleons and leptons (i.e., no meson condensates, no hyperons, no deconfined quarks, ... no nothing) but include all uncertainties on “standard” physics.
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1. RX J0822-4247 (in SNR Puppis A)
2. 1E 1207.4-5209 (in SNR PKS 1209-52)
3. PSR 0538+2817
4. RX J0002+6246 (in SNR CTB 1)
5. PSR 1706-44
6. PSR 0833-45 (in SNR ``Vela")
7. PSR 1055-52
8. PSR 0656+14
9. PSR 0633+1748 (``Geminga")
10. RX J1856.5-3754
11. RX J0720.4--3125

A. CXO J232327.8+584842 (in SNR Cas A)
B. PSR J0205+6449 (in SNR 3C58)
C. PSR J1124--5916 (in SNR G292.0+1.8)
D. RX J0007.0+7302 (in SNR CTA 1)

a. ? (in SNR G315.4--2.3)
b. ? (in SNR G093.3+6.9)
c. ? (in SNR G084.2--0.8)
d. ? (in SNR G127.1+0.5)
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The "Magnificent Seven": Magnetic fields and surface temperature distributions,
F Haberl, 2006 astro.ph/069066

<table>
<thead>
<tr>
<th>Object</th>
<th>$dP/dt$ $10^{-13}$ ss$^{-1}$</th>
<th>$E_{\text{cyc}}$ eV</th>
<th>$B_{\text{db}}$ $10^{13}$ G</th>
<th>$B_{\text{cyc}}$ $10^{13}$ G</th>
</tr>
</thead>
<tbody>
<tr>
<td>RX J0420.0−5022</td>
<td>&lt;92</td>
<td>330</td>
<td>&lt;18</td>
<td>6.6</td>
</tr>
<tr>
<td>RX J0720.4−3125</td>
<td>0.698(2)</td>
<td>280</td>
<td>2.4</td>
<td>5.6</td>
</tr>
<tr>
<td>RX J0806.4−4123</td>
<td>&lt;18</td>
<td>430/306$^{(a)}$</td>
<td>&lt;14</td>
<td>8.6/6.1</td>
</tr>
<tr>
<td>RBS 1223</td>
<td>1.120(3)</td>
<td>300/230$^{(a)}$</td>
<td>3.4</td>
<td>6.0/4.6</td>
</tr>
<tr>
<td>RX J1605.3+3249</td>
<td>450/400$^{(b)}$</td>
<td></td>
<td></td>
<td>9/8</td>
</tr>
<tr>
<td>RX J1856.5−3754</td>
<td>&lt;60$^{(d)}$</td>
<td>750</td>
<td>&lt;24$^{(d)}$</td>
<td>15</td>
</tr>
</tbody>
</table>

$^{(a)}$ Spectral fit with single / two lines
$^{(b)}$ With single line / three lines at 400 eV, 600 eV and 800 eV
$^{(c)}$ Estimate from Hα nebula assuming that it is powered by magnetic dipole breaking (Kaplan et al. 2002c; Braje & Romani 2002; Trümper et al. 2004)
$^{(d)}$ Radio detection: Malofeev et al. (2006b)
The Composite Spectrum of RX J1856

The puzzles of RX J1856.5-3754: Neutron Star or Quark Star? Truemper, Burwitz, Haberl & Zavlin, 2004 NuPhS 132, 560

Figure 1. Blackbody fits to the optical and X-ray spectra of RX J1856.5-3754 for a two-component model (a) and a model with a continuous temperature distribution (b).
Heat Transport with Strong B

\[ \vec{F} = -\kappa \cdot \vec{\nabla}T \]

\[ \kappa = \begin{pmatrix} \kappa_{\perp} & \kappa_{\wedge} & 0 \\ -\kappa_{\wedge} & \kappa_{\perp} & 0 \\ 0 & 0 & \kappa_{\parallel} \end{pmatrix} \]

\[ \kappa_{\parallel} = \kappa_0 \]

\[ \kappa_{\perp} = \frac{\kappa_0}{1 + (\omega_B \tau)^2} \]

\[ \kappa_{\wedge} = \frac{\kappa_0 \omega_B \tau}{1 + (\omega_B \tau)^2} \]

\[ \omega_B = \frac{eB}{m_e c} = \text{electron cyclotron frequency} \]

\[ \tau = \text{electron relaxation time} \]

Temperature distribution in magnetized neutron star crusts, Geppert, Küker & Page, 2004 A&A 426, 267
Crust + Core Poloidal Field
Fig. 7. Representation of both field lines and temperature distribution in the crust whose radial scale \( (r_{p_n} \leq r \leq r_{p_b}) \) is stretched by a factor of 5, assuming \( B_0 = 3 \times 10^{12} \) G and \( T_{\text{core}} = 10^6 \) K. Left panel corresponds to a crustal field, right panel to a star-centered core field. Bars show the temperature scales in units of \( T_{\text{core}} \).
Add a Toroidal Component
Add a Toroidal Component

Temperature distribution in magnetized neutron star crusts. II. The effect of a strong toroidal component, Geppert, Küker & Page, 2006 A&A 457, 937
Composite BB Fit for RX J1856

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Fig. 10. Fit of the spectrum of RX J1856.5-3754. Dotted lines show the two blackbodies fit to the data from Trümper et al. (2004). The continuous line show our results: the star has a radius $R = 14.4$ km and $R_{\infty} = 17.06$ km for a 1.4 $M_\odot$, at a distance of 122 pcs ($N_H = 1.6 \times 10^{20}$ cm$^{-2}$ for interstellar absorption) and the observer is assumed to be aligned with the rotation axis. The magnetic field structure corresponds to model c of Figure 6 adjusted to the 14.4 km radius with $T_b = 6.8 \times 10^7$ K, resulting in $T_{\text{eff}} = 4.62 \times 10^5$ K and $T_{\text{max}} = 8.54 \times 10^5$ K.
Long Live the Magnificent Seven!
Long Live the Magnificent Seven!

The Magnificent Seven

$B_{0}^{\text{tor}} =
\begin{align*}
10^{15} \text{ G (T1)} \\
10^{15} \text{ G (T2)} \\
3 \times 10^{15} \text{ G (T1)} \\
3 \times 10^{15} \text{ G (T2)}
\end{align*}$
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Many options for fast cooling, complicated by possible pairing of nucleons (or/and hyperons, quarks).

Minimal Cooling: little evidence for fast cooling, but nevertheless we have some conspicuous cases.

Still large uncertainties on observed luminosities (and ages).

The “Magnificent Seven”: are they permeated by strong toroidal fields? Is this telling us something?
“That’s all Folks!”