Partially Screened Inner Accelerating Region of Pulsars and its Observational Consequences

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Pulsar periods are generally lengthening, with typical derivatives about

\[ \frac{dP}{dt} = \dot{P} \approx 10^{-15} \text{ s s}^{-1} \]

The energy of the radio emission of pulsars comes from the energy, which is released as the rotation of the pulsar slows down. The power, which must be released as a pulsar slows down writes:

\[ L_{\text{sd}} = -I\dot{\Omega}\ddot{\Omega} = 4\pi^2 \dot{P}P^{-3} \text{ erg s}^{-1} \]

\( I \approx 10^{45} \text{ g cm}^2 \) is the moment of inertia of the star.

\[ L_{\text{sd}} \approx 4 \times 10^{31} \frac{\dot{P}}{P^3} \quad L_{\text{sd}} \approx 10^{31} \div 10^{38} \text{ erg s}^{-1} \]
Originally, this energy loss was ascribed to the emission of large amplitude electromagnetic waves. According to the magnetic dipole radiation model the value of the surface magnetic field can be deduced as:

\[ B_0 \approx 2 \times 10^{12} \left( P \dot{P}_{-15} \right)^{0.5} \text{ G} \]

\[ B = B_0 \left( \frac{R_*}{r} \right)^3 \quad R_* = 10^6 \text{ cm} \]

On the other hand observations of the Crab Nebula show that the pulsar is the only source of energy for the Nebula activity and the neutron star should supply the nebula with the sufficiently large number of particles. So, an energy of pulsar rotation should be converted into energy of the stellar wind.
Consider a neutron star rotating about a dipole axes with angular velocity $\Omega$. The stellar matter is an excellent conductor. Therefore, the star will be polarized so as to posses an interior electric field which satisfies

$$\vec{E} + \frac{\vec{\Omega} \times \vec{r}}{c} \times \vec{B} = 0$$

The Lorentz invariant $\vec{E} \cdot \vec{B}$ vanishes in the stellar interior. In the case where the star is surrounded by a vacuum, the external value of $\vec{E} \cdot \vec{B} \neq 0$. For a typical pulsar $E \approx 10^{11}$ V/m.
The charge distribution in the magnetosphere of the rotating neutron star

\[ E_{\parallel} \approx 0 \]

\[ \vec{E} + \left( \vec{\Omega} \times \vec{r} \right) \times \vec{B} / c = 0 \]

\[ \rho_{GJ} = -\frac{\vec{\Omega} \cdot \vec{B}}{2\pi c} \]

Charge density
Radius of the light cylinder

\[ R_c = \frac{c}{\Omega} \approx 5 \cdot 10^9 P \text{ [cm]} \]

Polar cap area:

\[ A_{pc} = 6.6 \times 10^8 \times P^{-1} \text{ cm}^2 \]

\[ R_{pc} \approx 150 \text{ m} \]

Co-rotational number density:

\[ n_{GJ} = -\frac{\vec{\Omega} \cdot \vec{B}}{2\pi e c} \approx 10^{11} \text{ cm}^{-3} \]
Curvature radiation

\[ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \omega = \frac{c}{R} \gamma^3 \]

\[ \gamma \sim 10^6; \]
\[ R \sim 10^6 \text{ cm}; \]
\[ \omega \sim 3 \times 10^{22} \text{ Hz} \]
\[ \theta \sim \frac{1}{\gamma} \]
Pair Creation

In the strong magnetic field the high energy photons can be transformed into the pairs

\[ \gamma + B \rightarrow e^+ + e^- + B \]

if the energy of the photon is high enough

\[ \hbar \omega \gg 2mc^2 \sim 8 \times 10^{-7} \text{ erg} \]

\[ \omega \gg 10^{20} \text{ Hz} \]
The vacuum gap

\[ \hbar \omega = 2mc^2 \kappa / H \]

\( \kappa \) is the curvature radius of the surface magnetic field lines

\( H \) is the vacuum gap height

\[ \Delta V = 10^{13} \text{ Volt} \]

\[ \gamma_0 \sim \kappa / H \sim 10^2 \]
Positive charges then cannot be supplied at the rate that would compensate the inertial outflow through the light cylinder. As a result, a significant potential drop develops above the polar. The accelerated positrons would leave the acceleration region, while the electrons would bombard the polar cap surface, causing a thermal ejection of ions, which are otherwise more likely bound in the surface in the absence of additional heating. This thermal ejection would cause partial screening of the acceleration potential drop corresponding to a shielding factor:

\[
\Delta V = \eta \frac{2\pi}{cP} B_s H^2
\]

\[
\eta = 1 - \frac{\rho_i}{\rho_{GJ}} = 1 - \exp\left[30\left(1 - \frac{T_i}{T_s}\right)\right]
\]
The equilibrium ion charge density above the polar cap:

\[ \frac{\rho_i}{\rho_{GJ}} \approx \exp \left( 30 - \frac{\varepsilon_c}{kT_s} \right) \quad T_i = \frac{\varepsilon_c}{30k} \]
Because of the exponential sensitivity of the accelerating potential drop $\ddot{A}V$ to the surface temperature $T_s$, the actual potential drop should be thermostatically regulated. In fact, when $\ddot{A}V$ is large enough to ignite the cascading pair production, the back-flowing relativistic charges will deposit their kinetic energy in the polar cap surface and heat it at a predictable rate. This heating will induce thermionic emission from the surface, which will in turn decrease the potential drop that caused the thermionic emission in the first place. As a result of these two oppositely directed tendencies, the quasiequilibrium state should be established, in which heating due to electron bombardment is balanced by cooling due to thermal radiation. This should occur at a temperature $T_s$ slightly lower than the critical temperature above which the polar cap surface delivers thermionic flow at the corotational charge density level.

The quasi-equilibrium condition is

$$\sigma T_s^4 = \gamma m_e c^3 n, \text{ where } \gamma = \frac{e \Delta V}{m_e c^2} \text{ and } n = n_{\text{GJ}} - n_i = \eta n_{\text{GJ}}$$

$$T_s = 1.4 \times 10^6 \left(\frac{B_s}{10^{14} \text{ G}}\right)^{0.5} \left(\frac{H}{10^3 \text{ m}}\right)^{0.5} \left(\frac{\eta}{10^{-2}}\right)^{0.5} \left(\frac{P}{10^8}\right)^{-0.5} \text{ K}$$
Confidence contours (68%, 90%, and 99%), computed for BB model fits to the spectrum of PSR B1133+16

\[ P = 1.0977 \text{ s}, \quad \dot{P}_{-15} = 3.49 \]

\[ A_{pc} \approx 6 \times 10^4 \text{ m}^2 \]

Confidence contours (68%, 90%, and 99%), computed for BB model fits to the spectrum of PSR B0943+10

\[ P = 1.188 \text{ s}, \quad \dot{P}_{-15} = 3.73 \]
Cartoon of the magnetic field lines in the polar cap region. The green lines represent the pure dipole field. The red lines correspond to the last open field lines, which at high altitudes coincide with the dipole field lines. $\theta$ is the magnetic colatitude in radians.
\[ b_{bol} = \frac{A_{pc}}{A_{bol}} = \frac{B_s}{B_d} \]
Four known cases with $A_{\text{bol}} << A_{pc}$
Two known cases with $A_{\text{bol}} > A_{\text{pc}}$

For PSR J1119-6127 and 0656+14 (one of the three musketeers) the derived parameters of the radiating region are quite unusual, unlike to all other cases the surfaces of the hot-spots turned to be larger that the canonical dipolar polar cap value.
\[ \bar{L}_{cr} \equiv \frac{L_{cr}}{\gamma m_e c^2} = 5.6 \times 10^3 \frac{\gamma_6^3}{\rho_6^2} \]

\[ t = \frac{\rho}{c} \]

\[ \frac{\gamma_6^3}{\rho_6} \approx 5.4 \]
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