Lessons Learned from Studies of CP Violation in the B-Meson System

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What is the mechanism of CP violation in Nature? A status report.

- CP Violation in the SM
  - There is one CP-violating parameter in the CKM matrix.
- Why do we think there could be CP violation beyond the SM?

The Case of the Missing Anti-Matter

- How can we test the CKM mechanism of CP violation?
  - Enter “the” Unitarity Triangle.
- How do we study CP violation in the B system?
- What do we now know about the mechanism of CP violation?
- How well can we test the CKM mechanism of CP violation?
The Cabibbo-Kobayashi-Maskawa (CKM) Matrix

The decay $K^{-} \to \mu^{-}\bar{\nu}_{\mu}$ occurs: the quark mass eigenstates mix under the weak interactions. By convention

$$
\begin{pmatrix}
    d' \\
    s' \\
    b'
\end{pmatrix}_{\text{weak}} = V_{\text{CKM}}
\begin{pmatrix}
    d \\
    s \\
    b
\end{pmatrix}_{\text{mass}};
V_{\text{CKM}} =
\begin{pmatrix}
    V_{ud} & V_{us} & V_{ub} \\
    V_{cd} & V_{cs} & V_{cb} \\
    V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
$$

In the Wolfenstein parametrization (1983)

$$
V_{\text{CKM}} =
\begin{pmatrix}
    1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\
    -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
    A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + O(\lambda^4)
$$

where $\lambda \equiv |V_{us}| \approx 0.22$ and is thus “small”. $A$, $\rho$, $\eta$ are real.

All CP-violating phenomena are encoded in $\eta$.

To test the SM picture of CP violation we must test the relationships it entails.
Studies of $b$-quark decay allows us to probe

$$V^*_{ub} V_{ud} + V^*_{cb} V_{cd} + V^*_{tb} V_{td} = 0.$$  
—a relationship predicated by the unitarity of the CKM matrix. 

All terms are $O(\lambda^3)$. Enter “the” unitarity triangle...
Testing the Standard Model of CP Violation

Different CP-violating phenomena exist (or are believed to exist) in the B meson system:

- CP violation in $B - \bar{B}$ mixing

$$
\begin{array}{ccc}
\bar{B}^0 & W & B^0 \\
\bar{d} & & \bar{b}
\end{array}
\begin{array}{ccc}
b & t & d \\
\end{array}
$$

- CP violation in the interference of $B - \bar{B}$ mixing and direct decay

- CP violation in direct decay

Note $|B^0(\tau)\rangle$... a state which is “tagged” as a $B^0$ meson at proper time $\tau = 0$ has a finite probability of being $\bar{B}$ at proper time $\tau$. 
Enter the asymmetric B-factory, to facilitate the study of the time-dependence of CP violation. There are asymmetric B-factories at SLAC and KEK.
Strong Interaction Obfuscation

Want to learn about underlying CKM parameters, but strong-interaction dynamics can confound this goal. Consider

\[ A_{\text{direct}} = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} \]

A : \( M \rightarrow h_1 h_2 \) and \( \bar{A} : \bar{M} \rightarrow \bar{h}_1 \bar{h}_2 \) An interference effect...

\[
A = A_1 + A_2 \equiv A_1 [1 + re^{i\delta} e^{i\phi}]
\]

\[
\bar{A} = \bar{A}_1 + \bar{A}_2 \equiv \bar{A}_1 [1 + re^{i\delta} e^{-i\phi}]
\]

so that \( A_{\text{direct}} = \frac{-2r \sin \delta \sin \phi}{1 + 2r \cos \delta \cos \phi + r^2} \)

\( A_{\text{direct}} \) determines a combination of \( r, \delta, \phi \). Note \( \delta \) strong phase, \( \phi \) weak phase.

Flavor symmetries (SU(2), SU(3)) can be used to relate \( r \) and \( \delta \) of various decays in an approximate way. Precision studies ultimately demand better?
Studying direct CP violation in the B-meson system

Direct CP violation can be studied in a variety of ways:

- **Partial rate asymmetry:**
  \[ |A(B \to h_1 h_2)|^2 - |A(\bar{B} \to \bar{h}_1 \bar{h}_2)|^2 \neq 0 \]

- **“ε’ in the B system”:** cf. \( B(t) \to \psi K_S \) and \( B(t) \to \pi^+ \pi^- \) [Wolfenstein, 1984]
  \[
  \Gamma(B^0(t) \to f_{CP}) \propto e^{-\Gamma t} \left[ \frac{1+|\lambda_{f_{CP}}|^2}{2} + \frac{1-|\lambda_{f_{CP}}|^2}{2} \cos(\Delta m t) - \text{Im}\lambda_{f_{CP}} \sin(\Delta m t) \right]
  \]
  where \( \lambda_{f_{CP}} \equiv \eta_{f_{CP}}(q/p)(A(\bar{B} \to f_{CP})/A(B \to f_{CP})) \).
  Note \(-\lambda_{\psi K_S} \neq \lambda_{\pi^+ \pi^-}\) implicitly signals direct CP violation.

- **Angular distribution in** \( B \to V_1 V_2 \) [Sinha & Sinha, 1998]

- **Population asymmetry in**
  \[ |A(B \to f_{CP})|^2 + |A(\bar{B} \to f_{CP})|^2 \] [SG, 2003; SG & Tandean, 2004]

Direct CP violation in the B-meson system established through the partial-rate asymmetry in the “self-tagged” modes \( B(\bar{B}) \to K^\pm \pi^\mp \).
The failure of mirror symmetry in the Dalitz plot of the untagged decay rate signals the presence of direct CP violation.
Time-Dependent Studies to CP-Eigenstates

\[ \Gamma(B^0(t) \to f_{CP}) \propto e^{-\Gamma t} \left[ \frac{1 + |\lambda_{f_{CP}}|^2}{2} \cos(\Delta m t) - \text{Im}\lambda_{f_{CP}} \sin(\Delta m t) \right] \]

where \( \lambda_{f_{CP}} \equiv \eta_{f_{CP}} (q/p)(A(\bar{B} \to \bar{f}_{CP})/A(B \to f_{CP})) \).

If the decay amplitude can be characterized by an unique weak phase, the strong dynamics cancels entirely!

Enter the “golden” mode \( B \to \psi K_S \ldots \text{Im}\lambda_{\psi K_S} \) measures \( \sin(2\beta) \).

Note \( \sin(2\beta) = 0.675 \pm 0.026 \) (WA) from \( \psi K_S \) and related modes.
**sin(2β) from Penguin Modes**

The penguin modes $B \rightarrow \phi K_S$, $B \rightarrow \eta' K_S$, $B \rightarrow f_0 K_S$, etc. also measure $\sin(2\beta)$ in the SM. [Grossman, Worah (1996)]

Many possible modes exist.

N.B. the SM corrections to the $\sin(2\beta)$ measurement are not uniformly small.

However, $S(\phi K_S) - S(\psi K_S) = 0.02 \pm 0.01$. 

![Diagram](image-url)
Naive average yields $\sin(2\beta) = 0.52 \pm 0.05$ (HFAG)– a deviation of 2.6$\sigma$! (cf. M. Neubert, Moriond-EW Mar 07, 0.50 $\pm$ 0.06)

Discounting differences as statistical fluctuations yields a “global” value of $\sin(2\beta) = 0.647 \pm 0.024$
The possibility of non-SM CP violation is gradually being relegated to a smaller and smaller role. Nevertheless, intriguing discrepancies remain, of which $\sin(2\beta)$ from tree and penguin modes is one.
Testing the CKM paradigm, 2006

Can also compare combined average $\sin(2\beta) = 0.647 \pm 0.024$ with the value deduced from $|V_{ub}|$ and $|V_{td}|$ alone, to yield $\sin(2\beta) = 0.794 \pm 0.045$, for a deviation of $2.9\sigma$. [M. Neubert, Moriond EW Mar 07]

Can be used to set limits on new physics in $B_d - \bar{B}_d$ mixing.
New Physics in $B_d - \bar{B}_d$ Mixing

$$\Delta m_d = \Delta m_d^{\text{SM}} r_d^2 e^{i2\theta_d}$$
Can CP-Violating Observables in the B-Meson System Yield CKM parameters at the $O(1\%)$ Level?

This is less daunting than it may seem.

Consider, e.g., $A_{CP}(t)$ in $B(\bar{B}) \rightarrow J/\psi K_S$. The $b \rightarrow su\bar{u}$ “pollution” is suppressed by $O(\lambda^2)$ and by loop effects.

This yields $\sin(2\beta)$ up to an (estimated) correction of $-(2 \pm 2) \cdot 10^{-4}$, [Boos, Mannel, Reuter, 2004] which can be tested with $B_s \rightarrow J/\psi K_S$ data. [Fleischer, 1999]

Flavor symmetries can also be used to probe CKM angles.

Here we consider the use of isospin symmetry to determine $\alpha$ from $B \rightarrow \pi\pi \,(n\pi)$ decay. Our goal is to assess all isospin-breaking effects.
Trees and Penguins in $b \rightarrow dq\bar{q}$ ($B \rightarrow \pi\pi$, etc.)

- $\Delta l = 3/2$

- $\Delta l = 1/2$

- $\Delta l = 1/2, 3/2$

An assumption of isospin symmetry can separate tree and strong penguin contributions in $B \rightarrow \pi\pi, \rho\pi, \rho\rho$. 
The CKM angle $\alpha$ (or $\gamma$) can be determined under an assumption of isospin symmetry from the analysis of $B \rightarrow \pi\pi$ [Gronau, London, 1990], $B \rightarrow \rho\pi$ [Snyder, Quinn, 1993], and $B \rightarrow \rho\rho$ modes.

Here we focus on $B \rightarrow \pi\pi$.

$A_{\text{CP}}(t)$ in $B(\bar{B}) \rightarrow \pi^-\pi^+$ decay yields $\sin(2\alpha_{\text{eff}})$.

In the Standard Model $\alpha = \pi - \beta - \gamma$; $\gamma \leftrightarrow$ tree-level decay. Penguins make $\Delta \alpha = \alpha_{\text{eff}} - \alpha \neq 0$.

Under isospin, two pions have $I = 0, 2$ only; $B \rightarrow \pi\pi$ amplitude $A_I$.

\[
A_{B^0 \rightarrow \pi^+\pi^-} \equiv A_0 + \frac{1}{\sqrt{2}} A_2 , \quad A_{B^0 \rightarrow \pi^0\pi^0} \equiv A_0 - \sqrt{2} A_2 , \quad A_{B^+ \rightarrow \pi^+\pi^0} \equiv \frac{3}{2} A_2 ,
\]

QCD penguins yield $I = 0$ only $\implies$ must separate out $A_0/A_2$ ($\bar{A}_0/\bar{A}_2$).

Can do so with $B(B(\bar{B}) \rightarrow \pi^i\pi^j)$ data.
$\pi^0 - \eta, \eta'$ Mixing: Summary

[SG, 2005]

$$\Delta \alpha = \frac{1}{2} (\bar{\phi}' - \phi') + \frac{1}{2} (\bar{\zeta} - \zeta) + \frac{1}{2} \left[ (\bar{\phi} - \phi) - (\bar{\phi}' - \phi') \right],$$

Last term vanishes if $\xi$ and $\bar{\xi}$ are real.

Up to $\mathcal{O}(\Lambda_{QCD}/m_b)$:

$$\delta(\Delta \alpha) = 1.2^\circ [\xi] + 1.5^\circ [P_{ew}] + 1.1^\circ [P_{\pi^0 - \eta, \eta'}] + \cdots \approx 4^\circ,$$

$$\sigma_{\alpha}^{IB} = 0.4^\circ [\xi] + 0.3^\circ [P_{ew}] + 0.2^\circ [P_{\pi^0 - \eta, \eta'}] + 1.1^\circ [\text{bound}] + \cdots \approx 2^\circ,$$

There is no central limit theorem for theoretical error.

What has been omitted?!

$\implies$ Long-distance electromagnetic effects.
\[ \Delta \alpha = \frac{1}{2} (\phi' - \phi') + \frac{1}{2} (\zeta - \zeta) + \frac{1}{2} [(\phi - \phi) - (\phi' - \phi')] , \]

Last term vanishes if \( \xi \) and \( \bar{\xi} \) are real.

Up to \( \mathcal{O}(\Lambda_{QCD}/m_b) \):

\[ \delta(\Delta \alpha) = 1.2^\circ [\xi] + 1.5^\circ [P_{ew}] + 1.1^\circ [P_{\pi^0-\eta,\eta}'] + \cdots \approx 4^\circ , \]

\[ \sigma_{\Delta \alpha}^{IB} = 0.4^\circ [\xi] + 0.3^\circ [P_{ew}] + 0.2^\circ [P_{\pi^0-\eta,\eta}'] + 1.1^\circ [\text{bound}] + \cdots \approx 2^\circ , \]

There is no central limit theorem for theoretical error.

What has been omitted?!

\[ \xrightarrow{\cdots} \text{Long-distance electromagnetic effects.} \]
$\pi^0 - \eta, \eta'$ Mixing: Summary

\[ \Delta \alpha = \frac{1}{2} (\bar{\phi}' - \phi') + \frac{1}{2} (\bar{\zeta} - \zeta) + \frac{1}{2} [(\bar{\phi} - \phi) - (\bar{\phi}' - \phi')] , \]

Last term vanishes if $\xi$ and $\bar{\xi}$ are real.

Up to $O(\Lambda_{QCD} / m_b)$:

\[ \delta(\Delta \alpha) = 1.2^\circ [\xi] + 1.5^\circ [P_{\text{ew}}] + 1.1^\circ [P_{\pi^0-\eta,\eta'}] + \cdots \approx 4^\circ , \]

\[ \sigma_{\alpha}^{\text{IB}} = 0.4^\circ [\xi] + 0.3^\circ [P_{\text{ew}}] + 0.2^\circ [P_{\pi^0-\eta,\eta'}] + 1.1^\circ [\text{bound}] + \cdots \approx 2^\circ , \]

There is no central limit theorem for theoretical error.

What has been omitted?!

$\implies$ Long-distance electromagnetic effects.
\[ \pi^0 - \eta, \eta' \text{ Mixing: Summary} \]

\[ \Delta \alpha = \frac{1}{2} (\phi' - \phi) + \frac{1}{2} (\zeta - \zeta) + \frac{1}{2} \left[ (\bar{\phi} - \phi) - (\bar{\phi}' - \phi') \right] , \]

Last term vanishes if \( \xi \) and \( \bar{\xi} \) are real.

Up to \( O(\Lambda_{\text{QCD}}/m_b) \):

\[ \delta(\Delta \alpha) = 1.2^\circ [\xi] + 1.5^\circ [P_{\text{ew}}] + 1.1^\circ [P_{\pi^0 - \eta, \eta'}] + \cdots \approx 4^\circ , \]

\[ \sigma^{\text{IB}}_\alpha = 0.4^\circ [\xi] + 0.3^\circ [P_{\text{ew}}] + 0.2^\circ [P_{\pi^0 - \eta, \eta'}] + 1.1^\circ [\text{bound}] + \cdots \approx 2^\circ , \]

There is no central limit theorem for theoretical error.

What has been omitted?!

\[ \Rightarrow \text{Long-distance electromagnetic effects.} \]
Detailed numbers can change as data is updated. Have used CP-averaged branching ratios throughout.

- How does the analysis rely on QCD factorization framework?
  Relies on factorization formula; also follows from SCET in leading power. Scalar penguins do not appear.
  \( \mathcal{O}(\Lambda_{QCD}/m_b) \) effects likely incur 10-20% corrections.
  Important to the extent that \( X_{\eta^{(r)}} \), \( \overline{X}_{\eta^{(r)}} \) are not real.
  \( \implies \) Signalled if \( X_{\eta^{(r)}} \neq \overline{X}_{\eta^{(r)}} \).

- Does the use of the Feldmann-Kroll-Stech framework for \( \eta, \eta' \) matter?
  Have also employed two-angle formalism of Leutwyler. No difference incurred at current empirical precision. [Frère, Escribano, 2005]

- How can electromagnetic corrections be included?
  Recall \( K \to \pi \pi \). Treat in simultaneous chiral and electromagnetic expansion.
Isospin breaking effects will differ for different $n\pi$ modes. Here other corrections can also appear; can mimic the violation of isospin.

- $B \rightarrow \rho\pi$
  
- Other resonances can populate $B \rightarrow 3\pi$ Dalitz plot. Non-$\rho$ states yield small impact in neutral B modes.
  
- Inclusion of $\rho', \rho''$? Analyzed assuming fixed $P/T$ ratio.

- Failure of 2-body unitarity in corners of Dalitz plot?
  
- Could impact empirical determination of strong phases.

- $\rho^0 - \omega$ mixing
  
- $\rho^0 - \omega$ mixing can be removed via cuts in $M_{\pi\pi}$.

- $\pi^0 - \eta, \eta'$ mixing
  
- $\alpha$ can be fixed through $B^0, \bar{B}^0$ data only. $\delta A_{5/2,2}$ always appears with $A_{3/2,2}$; no error accrues if $\delta A_{5/2,2}$ spawned from $\Delta I = 3/2$ operators in isospin-perfect limit. [Gardner, Meißner, 2002]
**Isospin Breaking in** \( B \to \rho\pi, B \to \rho\rho \)

**B \to \rho\rho** decays analyzed in the manner of \( B \to \pi\pi \).

- \( B \to \rho\rho \)
  - Other resonances can populate \( B \to 4\pi \) Dalitz plot.
  - \( \rho^0 - \omega \) mixing
    - \( \rho^0 - \omega \) mixing can be removed via cuts in \( M_{\pi\pi} \).
  - \( I = 1 \) amplitude
    - Emerges even if isospin is unbroken. Follows from finite \( \rho \) width.

[Falk, Ligeti, Nir, Quinn, 2003]

Current empirical assays assume it negligible.
Decays are analyzed under an assumption of isospin symmetry to determine $\alpha$. 
Production Asymmetry in \( e^+ e^- \rightarrow B^+ B^-, B^0 \bar{B}^0 \)

Isospin Symmetry: \( e^+ e^- \rightarrow \Upsilon(4S) \) yields \( B^+ B^- \) and \( B^0 \bar{B}^0 \) pairs equally.

This can be tested. [Babar, hep-ex/0107025]

\[
\frac{\mathcal{B}(B^+ \rightarrow (c\bar{c})K)}{\mathcal{B}(B^0 \rightarrow (c\bar{c})K)} = 1.17 \pm 0.07 \pm 0.04
\]

Assuming isospin and using \( \tau_{B^+}/\tau_{B^0} = 1.062 \pm 0.029 \) yields

\[
R^{+/0} \equiv \frac{\mathcal{B}(\Upsilon(4S) \rightarrow B^+ B^-)}{\mathcal{B}(\Upsilon(4S) \rightarrow B^0 \bar{B}^0)} = 1.10 \pm 0.06 \pm 0.05
\]

“compatible with unity at two standard deviations”

Theory yields \( R^{+/0} - 1 \gtrsim 0.1 \). [Kaiser, Manohar, Mehen, 2002]

Yield of \( B^+ B^- / B^0 \bar{B}^0 \) should also vary across \( \Upsilon(4S) \). [Voloshin, 2003]

Production asymmetry is unlikely to be unity.
The CKM mechanism of CP violation drives the pattern of results found in the B-meson system. Non-SM sources of CP violation are not excluded, but merely relegated to a more minor role. Precision studies may yet reveal new physics!
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