Schiff Screening of Relativistic Nucleon EDMs

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Consider nondegenerate ground state \(|\text{g.s.} : J, M\rangle\). Symmetry under rotations \(R_y(\pi) \Rightarrow\) for a vector operator like \(\vec{d} \equiv \sum_i e_i \vec{r}_i\),

\[
\langle \text{g.s.} : J, M | \vec{d} | \text{g.s.} : J, M \rangle = - \langle \text{g.s.} : J, -M | \vec{d} | \text{g.s.} : J, -M \rangle .
\]

\(T\) takes \(M\) to \(-M\), like \(R_y(\pi)\). But \(\vec{d}\) is odd under \(R_y(\pi)\) and even under \(T\), so for \(T\) conserved

\[
\langle \text{g.s.} : J, M | \vec{d} | \text{g.s.} : J, M \rangle = + \langle \text{g.s.} : J, -M | \vec{d} | \text{g.s.} : J, -M \rangle .
\]

Together with the first equation, this implies

\[
\langle \vec{d} \rangle = 0 .
\]

If \(T\) is violated, argument fails because \(T\) can take \(|\text{g.s.} : JM\rangle\) to \(|\text{ex.} : J, -M\rangle\), a state in a different multiplet.
Nuclear electric dipole moment with relativistic effects in Xe and Hg atom
(Phys. Rev. C75, 035501 (2007))
Sachiko Oshima, Takehisa Fujita, and Tomoko Asaga

The atomic electric dipole moment (EDM) is evaluated by considering the relativistic effects as well as nuclear finite size effects in Xe and Hg atomic systems. ... As the results, the finite contribution to the atomic EDM comes from the first order perturbation energy of relativistic effects, and it amounts to around 0.3 and 0.4 percents of the neutron EDM $d_n$ for Xe and Hg, respectively, though the calculations are carried out with a simplified single-particle nuclear model. From this relation in Hg atomic system, we can extract the neutron EDM which is found to be just comparable with the direct neutron EDM measurement.

Relativistic corrections to dipole operator unshielded for electrons

Claim: For nucleons, the corrections suppressed by $v^2/c^2 \approx .01$
Usual screening suppression on order of $10Z^2 R_N^2 / R_A^2 \approx .001$

Atomic EDMs about 10 times more sensitive than previously thought!
Really Correct?

Relativistic dipole operator:

\[ d = \beta \sum = \beta \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix} \]

Perturbing Hamiltonian:

\[ H_1 = -\sum_{j=1}^{A} d_N^j \beta \Sigma_j \cdot E + e \sum_{i=1}^{Z} r_i \cdot E \left( -e \sum_{k=1}^{Z} R_k \cdot E \right) \]

Nucleon EDMs  Electron coords.  Proton coords.

Unperturbed Hamiltonian:

\[ H_0 = H_e^{\text{int}} + H_{\text{nuc}}^{\text{int}} + H_{e-\text{nuc}} \]

\[ H_{e-\text{nuc}} = \sum_{i=1}^{Z} \frac{Z e^2}{r_i} + \ldots \]

First order energy shift in external field:

\[ \Delta E^{(1)} = -\langle \text{g.s.} | \sum_{j=1}^{A} d_N^j \beta \Sigma_j | \text{g.s.} \rangle \cdot E_{\text{ext}} \]
But must consider atomic polarization:

\[
\Delta E^{(2)} = - \sum_n \frac{1}{E_{\text{g.s.}} - E_n} \langle \text{g.s.} | \left( \sum_{j=1}^A d_N^j \beta \Sigma_j \right) \cdot \left( \sum_{i=1}^Z \nabla_i A_0 \right) | n \rangle \\
\times \langle n | e \sum_{k=1}^Z \mathbf{r}_k \cdot \mathbf{E}_{\text{ext}} | \text{g.s.} \rangle + \text{h.c.}
\]

Electric field at nucleus from \(i^{\text{th}}\) electron

Atomic excitations

\[
A_0 = \sum_{i=1}^Z \frac{e}{r_i} = - \frac{1}{Z e} H_{e-\text{nuc}}
\]

\[
\left( \sum_{j=1}^A d_N^j \beta \Sigma_j \right) \cdot \left( \sum_{i=1}^Z \nabla_i A_0 \right) = - \frac{1}{Z e} \left[ \sum_{j=1}^A \sum_{i=1}^Z d_N^j \beta \Sigma_j \cdot \nabla_i , H_{e-\text{nuc}} \right] = - \frac{1}{Z e} \left[ \sum_{j=1}^A \sum_{i=1}^Z d_N^j \beta \Sigma_j \cdot \nabla_i , H_0 - H_{e}^{\text{int}} - H_{\text{nuc}}^{\text{int}} \right]
\]
Use:

\[
\sum_{i=1}^{Z} [d^i_N \beta \Sigma_j \cdot \nabla_i, H^\text{int}_e] = \sum_{i=1}^{Z} d^i_N \beta \Sigma_j \cdot [\nabla_i, H^\text{int}_e] = 0
\]

and

\[
\text{nuc} \langle \text{g.s.} | [d^i_N \beta \Sigma_j \cdot \nabla_i, H^\text{int}_\text{nuc}] | \text{g.s.} \rangle_{\text{nuc}} = \text{nuc} \langle \text{g.s.} | [d^i_N \beta \Sigma_j, H^\text{int}_\text{nuc}] | \text{g.s.} \rangle_{\text{nuc}} \cdot \nabla_i = 0
\]

and note that commutator with \( H_0 \) adds a factor \( E_{\text{g.s.}} - E_n \) so that you can use closure to sum over excited states. Get

\[
\Delta E^{(2)} = \frac{1}{Z} \langle \text{g.s.} | \left[ \left( \sum_{j=1}^{A} \sum_{i=1}^{Z} d^i_N \beta \Sigma_j \cdot \nabla_i \right), \sum_{k=1}^{Z} r_k \cdot E_{\text{ext}} \right] | \text{g.s.} \rangle = \langle \text{g.s.} | \sum_{j=1}^{A} d^j_N \beta \Sigma_j | \text{g.s.} \rangle \cdot E_{\text{ext}} = -\Delta E^{(1)}
\]
Why Aren’t Electron EDMs Shielded?

\[ \text{nuc} \langle \text{g.s.} | \left[ d^i_N \beta \Sigma_j \cdot \nabla_i , H^\text{int}_{\text{nuc}} \right] | \text{g.s.} \rangle_{\text{nuc}} = 0 \]

but if \( j \) labels electrons instead of nucleons

\[ \langle n | \left[ d^i_e \beta \Sigma_j \cdot \nabla_i , H^\text{int}_e \right] | \text{g.s.} \rangle \neq 0 . \]

Why?

Because \( \beta \Sigma \) does not commute with \( \alpha \cdot p \), which is in free Dirac Hamiltonian. Doesn’t matter for nucleons because expectation value is all that occurs, and it is killed by commutator with Hamiltonian.

Reason only expectation value occurs for nucleons: Gradient acts on electrons, leaving positive-parity nuclear operator \( \beta \Sigma \), which cannot excite states of opposite parity. But those are needed because deexcitation is by negative-parity operator \( \sum_k R_k \cdot E \).

Differences are due to monopole approximation for \( H_{e-nuc} \), i.e. point-like approximation for nucleus.
THE END