Neutron Beta Decay in Effective Field Theory

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Outline

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1. Introduction

Neutron $\beta$-decay

\[ n \rightarrow p + e + \bar{\nu}_e. \]

The hadronic current

\[ J^\mu = \bar{u}_p \left[ G'_V \gamma^\mu - G'_A \gamma^\mu \gamma_5 \right] u_n, \]

and radiative corrections and weak magnetism.

“Standard calculations”:
Sirlin, Marciano, Towner, et al.’s works.
Observables of NBD

1. Decay rate (or lifetime)

\[ \Gamma = \frac{G''_V^2}{4\pi^3} (1 + 3g_A^2) \int_{m_e}^{E_{e\text{max}}} dE_e p_e E_e (E_{e\text{max}} - E_e)^2 F(Z, E_e) \left[ 1 + \frac{\alpha}{2\pi} g(E_e, E_{e\text{max}}) \right], \]

with

\[ G''_V = (G_F V_{ud})^2 (1 + \Delta V_R). \]

2. Correlation coefficients

\[ \frac{d\Gamma}{dE_e d\Omega \bar{p}_e d\Omega \bar{p}_\nu} \propto 1 + a \frac{\bar{p}_e \cdot \bar{p}_\nu}{E_e E_\nu} + \hat{n} \cdot \left( A \frac{\bar{p}_e}{E_e} + B \frac{\bar{p}_\nu}{E_\nu} + D \frac{\bar{p}_e \times \bar{p}_\nu}{E_e E_\nu} \right), \]

where (at leading order)

\[ a = \frac{1 - g_A^2}{1 + 3g_A^3}, \quad A = 2 \frac{g_A - g_A^2}{1 + 3g_A^2}, \quad B = 2 \frac{g_A + g_A^2}{1 + 3g_A^2}, \quad D = 0, \]

with \( g_A = G'_A / G''_V \).
Values of $g_A$

The neutron-spin and electron correlation coefficient $A$

$$A = \frac{2g_A(1 - g_A)}{1 + 3g_A^2}, \quad g_A = \frac{G'_A}{G'_V},$$

and a recommended value by PDG2006 (the same as PDG2004)

$$g_A = 1.2695 \pm 0.0029.$$  

This value is important for estimating the cross sections of the processes, e.g., $pp \rightarrow de^+\bar{v}$ and $\nu d \rightarrowppe(\nu d \rightarrow np\nu)$ and for test of the Goldberger-Treiman relation.
**$T$-violating coefficient $D$**

- The Standard Model, $D_{SM} < 10^{-12}$.
- Models beyond the SM (like MSSM), $D_{MSSM} \sim 10^{-7}$.
- The most recent experimental data

\[
D_{\text{exp.}} = [-2.8 \pm 6.4(\text{stat}) \pm 3.0(\text{sys})] \times 10^{-4},
\]

from Soldner et al. (2004).
- **But** from the final state interaction

\[
D_{FSI} \approx -2.3 \times 10^{-5}.
\]
CKM Unitarity

\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \Delta \]

Using \( V_{ud} \) from \( 0^+ \rightarrow 0^+ \) nuclear \( \beta \)-decay and \( V_{us} \) from PDG02, \( \Delta = 2.2\sigma \) \((0.0032 \pm 0.0014)\). \((\Delta = 0.0008 \pm 0.0011, \text{PDG06.})\)

Suggested solution:
New \( V_{us} \) values from E865, KTeV \( K_{e3}, \text{etc.} \)

From the most recent data of neutron \( \beta \)-decay \((A)\), however, \( \Delta = 2.7\sigma \) \((0.0076 \pm 0.0028)\).

Suggested solution:
New \( \tau \) value from ILL, but \( \sim 6\sigma \) discrepancy from the former exp. values.
New features in NBD

Experiment:
1) New neutron facilities under construction, e.g., at Oak Ridge and J-PARC,
2) A neutron source of “Hanaro” in Korea,
3) New experiment proposals and new detectors under investigation around the world.

Theory:
1) Model independent calculations,
2) Error estimations using EFT approach
2. **NBD in EFT**

Chiral Perturbation Theory: a low energy EFT of QCD

- SSB of chiral sym. of QCD (pions: Goldstone-bosons).
- A systematic perturbation scheme (renormalizable order by order)

\[
\mathcal{L}_{SM} \rightarrow \mathcal{L}_\chi = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \ldots
\]

\[
M \sim \sum \left( \frac{Q}{\Lambda_\chi} \right)^\nu ,
\]

where \( Q \approx m_\pi \) or \( \bar{p} \), \( \Lambda_\chi \approx 4\pi f_\pi \approx m_N \approx 1 \text{ GeV} \).

But it has a problem when a nucleon field is included.

[Gasser, Sainio, Švarc, NPB307(1988)779.]
No counting rules for loop diagrams with a relativistic nucleon.

*E.g.*, Nucleon self-energy

\[ \Sigma(p) = i \frac{3}{4} \frac{g_A^2}{f_\pi^2} \int \frac{d^4 l}{(2\pi)^4} \frac{l \cdot \gamma_5 [\gamma \cdot (l + p) + m_N] l \cdot \gamma_5}{[(l + p)^2 - m_N^2](l^2 - m_\pi^2)} \]

\[ \sim \frac{m_N^3}{(4\pi f_\pi)^2} \sim m_N \gg \frac{Q^2}{\Lambda^2} m_N. \]

**Heavy-baryon formalism**

\[ p^\mu = m_N v^\mu + k^\mu, \quad N(x) \simeq e^{im_N v \cdot x} \Psi_N(x), \ldots \]

then one has \( \gamma^\mu \to v^\mu = (1, 0), \gamma^\mu \gamma_5 \to 2S^\mu = (0, \vec{\sigma}), \) \( 1/(\gamma \cdot p - m_N) \to 1/v \cdot k, \) and

\[ \Sigma_v(k) = i \frac{3}{4} \frac{g_A^2}{f_\pi^2} \int \frac{d^4 l}{(2\pi)^4} \frac{2S \cdot l 2S \cdot l}{v \cdot (l + k)(l^2 - m_\pi^2)} \sim \frac{m_N^3}{(4\pi f_\pi)^2}. \]
Counting rules for NBD

A new scale: \( \bar{Q} \simeq m_n - m_p - m_e \ll m_\pi \)

- One-pion exchange diagram \((\bar{Q}/m_\pi)^2 \sim 10^{-5}\)
- Weak-magnetism term \(\bar{Q} \kappa_V/(2m_N) \sim 10^{-3}\)

Modified counting rules:

- Expanding parameters; \(\alpha/(2\pi), \bar{Q}/(2m_N) \sim 10^{-3}\).
- \(\tau, a, A, B\) up to NLO, \(\alpha/(2\pi), \bar{Q}/(2m_N) \sim 10^{-3}\)
- Nonzero \(D\) appears in \(\alpha \bar{Q}/(2m_N) \sim 10^{-5}\).
- Pion loops, \((m_\pi/\Lambda_\chi)^2\) corrections, in the renormalized coupling constants, \(g_A\) and \(\kappa_V\),


**Effective Lagrangian**

\[ \mathcal{L}_\beta = \mathcal{L}_{e\nu\gamma} + \mathcal{L}_{NN\gamma} + \mathcal{L}_{e\nu NN}, \]

where

\[ \mathcal{L}_{e\nu\gamma} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2\xi} (\partial \cdot A)^2 + \left(1 + \frac{\alpha}{4\pi} e_1\right) \bar{\psi}_e (i \gamma \cdot D) \psi_e - m_e \bar{\psi}_e \psi_e + \bar{\psi}_\nu (i \gamma \cdot \partial) \psi_\nu, \]

\[ \mathcal{L}_{NN\gamma} = N^\dagger \left[ 1 + \frac{\alpha}{8\pi} e_2 (1 + \tau_3) \right] i \nu \cdot DN, \]

\[ \mathcal{L}_{e\nu NN} = -\frac{G_F V_{ud}}{\sqrt{2}} \bar{\psi}_e \gamma_\mu (1 - \gamma_5) \psi_\nu \left\{ N^\dagger \tau^+ \left[ \left(1 + \frac{\alpha}{4\pi} e_V\right) \nu^\mu - 2g_A \left(1 + \frac{\alpha}{4\pi} e_A\right) S^\mu \right] N \right. \\
\left. \quad + \frac{1}{2m_N} N^\dagger \tau^+ \left[ i(\nu^\mu \nu^\nu - g^{\mu\nu}) (\vec{\partial} - \vec{\partial})_\nu - 2i \mu \nu [S^\mu, S \cdot (\vec{\partial} + \vec{\partial})] - 2ig_A \nu^\mu S \cdot (\vec{\partial} + \vec{\partial}) \right] \right\}. \]

and \( \nu = (1, 0) \) and \( 2S^\mu = (0, \vec{\sigma}) \).
Feynman diagrams for NBD

(a) v
(b) e
(c) 
(d) 
(e) 
(f) 
(g) 
(h) 
(i) 

n → p
Feynman diagrams for the $D$ coefficient

- Nonzero $D$ appears from the imaginary part of the loop diagrams in $\alpha \tilde{Q}/(2m_N)$ order ($\sim 10^{-5}$).
Results (1)

\[
\frac{d\Gamma}{dE_e d\Omega_{\bar{p}_e} d\Omega_{\bar{p}_\nu}} = \frac{(G_F V_{ud})^2}{(2\pi)^5} \frac{F(Z, E_e)|\bar{p}_e|E_\nu}{m_n[E_p + E_\nu + E_e(\bar{\beta} \cdot \hat{p}_\nu)]} |M|^2 ,
\]

where

\[
|M|^2 = m_n m_p E_e E_\nu \left( 1 + \frac{\alpha}{2\pi} e^R_V + \frac{\alpha}{2\pi} \delta^{(1)}_\alpha \right) C_0(E_e)(1 + 3\tilde{g}_A^2)
\]
\[
\times \left\{ 1 + \left( 1 + \frac{\alpha}{2\pi} \delta^{(2)}_\alpha \right) C_1(E_e)\bar{\beta} \cdot \hat{p}_\nu
\right. 
+ \left. \left( 1 + \frac{\alpha}{2\pi} \delta^{(2)}_\alpha \right) \left[ C_2(E_e) + C_3(E_e)\bar{\beta} \cdot \hat{p}_\nu \right] \hat{n} \cdot \bar{\beta}
\right. 
+ \left. \left[ C_4(E_e) + C_5(E_e)\bar{\beta} \cdot \hat{p}_\nu \right] \hat{n} \cdot \hat{p}_\nu \right\} ,
\]

and

\[
\tilde{g}_A = g_A \left[ 1 + \frac{\alpha}{4\pi} \left( e^R_A - e^R_V \right) \right],
\]
\[
e^{R}_{V,A}(\mu) = e_{V,A} - \frac{1}{2}(e_1 + e_2) + \frac{3}{2} \left[ \frac{1}{\epsilon} - \gamma_E + \ln(4\pi) + 1 \right] + 3 \ln \left( \frac{\mu}{m_N} \right),
\]

where we have employed the dimensional regularization in \( d = 4 - 2\epsilon \) for the loops.
Results (1) (Cont.)

\[
\begin{align*}
\delta_{\alpha}^{(1)} &= 3 \ln \left( \frac{m_N}{m_e} \right) + \frac{1}{2} + \frac{1 + \beta^2}{\beta} \ln \left( \frac{1 + \beta}{1 - \beta} \right) - \frac{1}{\beta} \ln^2 \left( \frac{1 + \beta}{1 - \beta} \right) + \frac{4}{\beta} L \left( \frac{2\beta}{1 + \beta} \right) \\
+ &4 \left[ \frac{1}{2\beta} \ln \left( \frac{1 + \beta}{1 - \beta} \right) - 1 \right] \left[ \ln \left( \frac{2(E_{e\text{max}} - E_e)}{m_e} \right) + \frac{1}{3} \left( \frac{E_{e\text{max}} - E_e}{E_e} \right) - \frac{3}{2} \right] \\
+ &\left( \frac{E_{e\text{max}} - E_e}{E_e} \right)^2 \frac{1}{12\beta} \ln \left( \frac{1 + \beta}{1 - \beta} \right), \\
\delta_{\alpha}^{(2)} &= \frac{1 - \beta^2}{\beta} \ln \left( \frac{1 + \beta}{1 - \beta} \right) + \left( \frac{E_{e\text{max}} - E_e}{E_e} \right) \frac{4(1 - \beta^2)}{3\beta^2} \left[ \frac{1}{2\beta} \ln \left( \frac{1 + \beta}{1 - \beta} \right) - 1 \right] \\
+ &\left( \frac{E_{e\text{max}} - E_e}{E_e} \right)^2 \frac{1}{6\beta^2} \left[ \frac{1 - \beta^2}{2\beta} \ln \left( \frac{1 + \beta}{1 - \beta} \right) - 1 \right],
\end{align*}
\]

where \( E_{e\text{max}} \) is the electron maximum energy \( E_{e\text{max}} = (m_n^2 - m_p^2 + m_e^2)/(2m_n) \) and 
\( L(x) \) is the Spence function \( L(x) = \int_0^1 \frac{dx}{x} \ln(1 - x) \).
Results (1) (Cont.)

\[ C_0(E_e) = 1 + \frac{1}{m_N(1 + 3\tilde{g}_A^2)} \left\{ (\tilde{g}_A^2 - 2\mu_V\tilde{g}_A + 1)E_{e}^{max} - \frac{m_e^2}{E_e}(1 + \tilde{g}_A^2) + 2\mu_V\tilde{g}_A(1 + \beta^2) \right\} \]

\[ C_1(E_e) = \tilde{a}\left\{ 1 + \frac{1}{m_N} \left[ \frac{(\tilde{g}_A^2 + 2\mu_V\tilde{g}_A + 1)}{1 + 3\tilde{g}_A^2} \right] \left( \frac{m_e^2}{E_e} \right) \right. \]
\[ + \frac{8\mu_V\tilde{g}_A E_{e} - 4E_{e}^{max}\tilde{g}_A(\tilde{g}_A + \mu_V))}{(\tilde{g}_A^2 - 1)(1 + 3\tilde{g}_A^2)} \right\} \right\} \]

\[ C_2(E_e) = \tilde{A}\left\{ 1 + \frac{1}{m_N} \left[ \frac{(\tilde{g}_A^2 - 1)(\tilde{g}_A + \mu_V)}{2\tilde{g}_A(1 + 3\tilde{g}_A^2)} \right](E_{e}^{max} - E_e) + \frac{E_{e}(\mu_V - 1)}{\tilde{g}_A - 1} \right. \]
\[ - \beta^2 E_e \frac{\tilde{g}_A^2 + 2\tilde{g}_A\mu_V + 1}{1 + 3\tilde{g}_A^2} \left. \right\} \right\} \]

\[ C_4(E_e) = \tilde{B}\left\{ 1 + \frac{1}{m_N} \left[ \frac{E_{e}\beta_2(\tilde{g}_A^2 - 1)(\tilde{g}_A - \mu_V)}{2\tilde{g}_A(1 + 3\tilde{g}_A^2)} \right] + \frac{(\tilde{g}_A + \mu_V)(\tilde{g}_A - 1)^2}{(\tilde{g}_A + 1)(1 + 3\tilde{g}_A^2)}(E_e - E_{e}^{max}) \right\} \]

\[ C_3(E_e) = \tilde{A}\frac{E_e(\tilde{g}_A - \mu_V)}{2m_N\tilde{g}_A}, \quad C_5(E_e) = \tilde{B}\frac{(\tilde{g}_A + \mu_V)}{2m_N\tilde{g}_A}(E_{e}^{max} - E_e), \]
\[ D_{FSI} = \frac{1}{1 + 3g_A^2} \frac{\alpha E_e}{4m_N} \frac{1}{\beta} \left\{ (1 + 3g_A^2)\left[ (\mu_V - g_A) - 3\mu_p(1 - g_A) \right] + \frac{m_e^2}{E_e^2} \left[ (3 + g_A)(\mu_V - g_A) + 3\mu_p(1 - g_A)(1 + 3g_A) \right] \right\} + \frac{1}{1 + 3g_A^2} \frac{\alpha E_\nu}{4m_N} \frac{1}{\beta} 8g_A(1 - g_A). \]

- We reproduce the terms from Callan and Treiman (1961).
- We have a new term, but it does not appear in a relativistic calculation.
3. How to fix the LECs?

How to fix the LEC $\alpha^R_V$ (usually fixed by experiment)

$$\frac{\alpha}{2\pi} e^R_V \simeq \Delta^V_R?$$

In the standard calculations (Marciano and Sirlin 86), one finds

$$\Delta^V_R = \frac{\alpha}{2\pi} \left[ -4 \ln \left( \frac{m_W}{m_Z} \right) + 3 \ln \left( \frac{m_W}{m_N} \right) + \ln \left( \frac{m_W}{m_A} \right) + A_g + 2C \right],$$

- High energy part of $W\gamma$ box diagrams
- High energy part of $Z\gamma$ box diagrams
- High energy part of axialvector current induced $W\gamma$ box diagrams where $m_A$ is a infrared cutoff
- pQCD correction $A_g$
- Low energy part of axialvector current induced diagrams $C$

→ We may compare it with a result in the EFT calculation.
Model dependent term $C$: a difference

Low-energy axialvector current induced RC $C$ in $\Delta_R^V$

- In the standard (graphical) calculations, one has
  \[
  C(Born)[\propto g_A(\mu_p + \mu_n)] = 0.881 \pm 0.030. 
  \]
- In HB$\chi$PT calculation, they are higher order terms and
  \[
  C(HB) \sim (\bar{Q}/m_N)^2 \sim 10^{-6}. 
  \]
Other contributions to $e_V^R$

- Corrections due to $\rho - \omega$ mixing and isospin breaking from light quark mass difference are negligible. [Donoghue and Wyler, PLB241(1990)243.]

- Additional corrections: the resonances, $\Delta(1232)$, $N(1440)$, etc, in the $C_{Born}$ diagrams.
What’s wrong with the new term of $D$?

HB formalism

\[
\frac{1}{i} \int \frac{d^4 l}{(2\pi)^4} \frac{l^\mu}{v \cdot (l + k)[(l - p_e)^2 - m_e^2]l^2} = v^\mu f_1 + p_e^\mu f_2 .
\]

Relativistic formalism

\[
\frac{1}{i} \int \frac{d^4 l}{(2\pi)^4} \frac{l^\mu}{[(l + p_p)^2 - m_p^2][(l - p_e)^2 - m_e^2]l^2} = p_p^\mu f'_1 + p_e^\mu f'_2 ,
\]

where \( p_p^\mu = m_p v^\mu + k \) and \( v^\mu = (1, \vec{0}) \).
4. *Discussion and conclusions*

- The subleading results of EFT reproduce well low-energy model-independent terms for $\tau, a, A, B$ in the standard calculations, and thanks to the counting rules the higher order terms will be small, $\sim 10^{-5}$.

- The high-energy and low-energy model-dependent terms in the standard calculations are replaced by the two LEC’s, $e^R_V$ and $(e^R_A - e^R_V)$. We found that the estimations of the $C$ term in the standard calculations and EFT are quite different.

- The $D$ calculation is in progress.
Appendix: A problem of HB$\chi$PT

Nucleon propagator:

\[
\frac{\gamma \cdot p + m_N}{p^2 - m_N^2} \rightarrow \frac{1}{v \cdot k} + \frac{1}{2m_N} \frac{(v \cdot k)^2 - k^2}{(v \cdot k)^2} + \cdots,
\]

where \( p^\mu = m_N v^\mu + k^\mu \).

Up to a finite order, it does not reproduce analytic structure for on-shell nucleon.

\[\rightarrow\] Manifestly Lorentz invariant baryon ChPT with infrared and on-mass shell regularization schemes.