The transition temperature in QCD

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for the RBC–Bielefeld Collaboration

--- results from QCDOC ---

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On the machines

US/RBRC QCDOC
20,000,000,000,000 ops/sec

BI – apeNEXT
5,000,000,000,000 ops/sec

critical temperature
equation of state
hadron properites in matter

http://quark.phy.bnl.gov/~hotqcd
Outline

- **The Lattice Setup**
  - The p4-action

- **T>0 Simulations**
  - $\beta_c$ determination and ambiguities
  - Volume dependence

- **T=0 Simulations**
  - Scale setting

- **The transition temperature**
  - Mass and continuum extrapolation
  - Comparison with other groups

Publications

- **Phys.Rev.D74:054507,2006**
  - [hep-lat/0608013]

- **Quark Matter 2006**
  - F. Karsch

- **Lattice 2006**
  - T. Umeda, [hep-lat/0610019]
  - M. Cheng, [hep-lat/0610001]

- **SEWM 2006**
  - T. Umeda and C. Schmidt, [hep-lat/0609032]

- **Panic 2005**
  - F. Karsch, [hep-lat/0601013]
  - C. Schmidt, [hep-lat/0601032]

- **Lattice 2005**
The Lattice Setup

**Goal:** QCD Thermodynamics with realistic quark masses and controlled extrapolation to the continuum limit

- Remove $O(a^2)$ cutoff errors and improve rotation symmetry by adding irrelevant operators to the standard staggered action

- Improve flavor symmetry inherent to the staggered formulation by smearing the one link term
  
  **RBC-Bielefeld:** standard + p4-term + 3 link smearing $(p4fat3)$
  
  **MILC:** standard + Naik-term + 3,5,7 link smearing $(asqtad)$
  
  **Wuppertal:** standard + exp. 3 link smearing $(stout)$

- Use newly developed RHMC algorithm which has not step-size errors

- Perform simulations with (3-4) different values of light quark masses corresponding to $150 \text{ MeV} < m < 500 \text{ MeV}$ and 2 different lattice spacings $(N_T=4,6)$ to perform chiral and continuum extrapolations

Previous results with p4-action:
2-flavor QCD, $N_T = 4$, $m_\pi = 770$ MeV
The Lattice Setup

The p4-Action (fermionic part): an improved staggered fermion Action

- Remove cutoff-effects and improve rotation symmetry by adding irrelevant operators
- Improve flavor symmetry by smearing the one link term

\[
S_F(N_\tau, N_\sigma) = \sum_{n,\tilde{n}} \sum_\mu \eta(n_\mu) \chi_n \left( \frac{3}{8} \frac{1}{1 + 6\omega} \right) + \omega \sum_{\nu \neq \mu} \left[ \begin{array}{c}
\downarrow \\
\uparrow
\end{array} \right] + \frac{1}{48} \sum_{\nu \neq \mu} \left[ \begin{array}{c}
\downarrow \\
\uparrow
\end{array} \right] \chi_{n'} + m_q \sum_n \chi_n \chi_n
\]

[Karsch, Heller, Sturm (1999)]
The Lattice Setup

The p4-Action (gluonic part): Symanzik improvement scheme

- Remove cut-off effects of order $O(a^2)$
  (tree-level improvement $O(g^0)$)

\[
S_G(N_\tau, N_\sigma) = \sum_n \sum_{\mu, \nu > \mu} \left( \frac{5}{3} \left[ 1 - \frac{1}{3} \Re Tr \left[ \begin{array}{c}
\end{array} \right] \right] - \frac{1}{6} \left[ 1 - \frac{1}{6} \Re Tr \left[ \begin{array}{c}
\end{array} \right] \right] \right)
\]

[Weisz, Wohlert (1984)]
Properties of the p4-Action: the rotational symmetry

- The free quark propagator is rotational invariant up to order $O(p^4)$
- Rotational symmetry of the heavy quark potential improved

Dispersions relation:

[Karsch, Heller, Sturm (1999)]
Properties of the p4-Action: the rotational symmetry

- The free quark propagator is rotational invariant up to order $O(p^4)$
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Dispersions relation:

[Image of a graph showing dispersions relations for different groups: RBC-Bielefeld, MILC, Wuppertal. The graph includes various lines and labels for different cases such as $p_y = p_z = 0$ and $p_x = p_y = p_z = 0$. The reference at the bottom indicates Karsch, Heller, Sturm (1999).]
Properties of the p4-Action: the cut-off effects

- Bulk thermodynamic quantities (pressure, energy density, ...) show drastically reduced cut-off effects

Continuum limit of the pressure for the free lattice gas:
**T>0 Simulations**

**Strategy:**

- Simulations at many temperatures ($\beta$-values) in a narrow temperature range around $T_c$

- Apply the Ferrenberg-Swendsen re-weighting to combine information from several independent runs

- Determine the location of the transition from several different susceptibilities: disconnected part of light and strange quark chiral susceptibility, Polyakov-loop and quark number susceptibility, ...

- Runs on different volumes to confirm small finite size effects and little influence on the location of the transition
T>0 Simulations

- Weak volume dependence
- All susceptibilities show consistent peak position:
  - chiral sus.,
  - Polyakov-loop sus.,
  - quartic quark number sus.

High statistics: about 40,000 trajectories for each point, about 200,000 trajectories enter the re-weighting

\[ N_T = 4 \]

2.5% error \( \Leftrightarrow 5 \text{ MeV} \)
T>0 Simulations

\[ N_T = 6 \]

- Weak volume dependence
- All susceptibilities show consistent peak position:
  - chiral sus.,
  - Polyakov-loop sus.,
  - quartic quark number sus.
- High statistics: up to 60,000 trajectories for each point, about 260,000 trajectories enter the re-weighting

4.0% error \(\Leftrightarrow 8\) MeV
Ambiguities in locating the crossover point

Differences in locating peaks in light ($\beta_l$), strange ($\beta_s$) and Polyakov loop ($\beta_L$) susceptibilities.

Differences in the location of pseudo-critical couplings are taken into account as systematic error.

2.5% ($N_\tau = 4$) or 4% ($N_\tau = 6$) error band ⇔ 5 or 8 MeV.
Strangeness fluctuations

- Strangeness fluctuations provide another observable to locate the crossover point
- Quartic strangeness fluctuations are strongly peaked in the transition region

\[d_4^S = \frac{1}{VT^3} \frac{\partial^4 \ln Z}{\partial (\mu_s/T)^4}\]

\(T_c\) determined from peak in chiral susceptibility

differences of pseudo-critical couplings deduced from peak in strangeness and Polyakov loop fluctuations

\[\beta_c(\chi_L) - \beta_c(\chi_s) = 0.001(2)\]

similar for light quark number fluctuations:

\[\beta_c(\chi_L) - \beta_c(\chi_{u,d}) = 0.005(3)\]
T=0 Simulations

Scale setting from the static quark potential

use $r_0$ or string tension to set the scale for $T_c = 1/N_\tau a(\beta_c)$

$$V(r) = -\frac{\alpha}{r} + \sigma r, \quad r^2 \frac{dV(r)}{dr} \bigg|_{r=r_0} = 1.65$$

no significant cut-off dependence
when cut-off varies by a factor 4

i.e. from the transition region
on $N_\tau = 4$ lattices to that
on $N_\tau = 16$ lattices !!

we use $r_0 = 0.469(7)$ fm determined from quarkonium spectroscopy

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i.e. from the transition region on $N_\tau = 4$ lattices to that on $N_\tau = 16$ lattices !!

- $r_0 \sqrt{\sigma}$ fluctuates about 3% in this interval
- no hint for large cut-off dependence

we use $r_0 = 0.469(7) \text{ fm}$ determined from quarkonium spectroscopy

The transition temperature

Combined chiral and continuum extrapolation

$$(r_0 T_c)_{N_f} = (r_0 T_c)_{\text{cont.}} + b (m_{PS} r_0)^d + c/N_f^2$$

$(d=1.08 (O(4), 2\text{nd ord.}), d=2 (1\text{st ord.}))$

\[ T_c r_0 \]

,$\eta=2+1$,

\[ m_{ps} r_0 \]

\[ \eta_f=4 \text{ (squares, triangles)} \quad 6 \text{ (circles)} \]

\[ T_c/\sqrt{\sigma} \]

,$\eta=2+1$,

\[ m_{ps} r_0 \]

\[ \eta_f=4 \text{ (squares, triangles)} \quad 6 \text{ (circles)} \]

$\Rightarrow \quad r_0 T_c = 0.456(7)^{+3}_{-1}$, $T_c/\sqrt{\sigma} = 0.408(7)^{+3}_{-1}$ at phys. point

$\Rightarrow \quad T_c = 192(7)(4) \text{ MeV}$

(1st error: stat. error on $\beta_c$ and $r_0$; 2nd error: $N_f^{-2}$ extrapolation)
The transition temperature

Preliminary results on the energy density

The Tc determination is consistent with the crossover region of the energy density

Note: T-scale is independent from Tc determination

The band marks $T=192 \pm 11$ MeV

RBC-Bielefeld preliminary and MILC (C. Bernard et al., hep-lat/0610017)
The transition temperature

Comparison with other groups

- RBC-Bielefeld (p4fat3) vs. MILC (asqtad) and Wuppertal (stout)

  - asqtad results agree with p4fat3 results within statistical errors

  - stout results for Nt=4 and Nt=6 are about 15% lower,
    Tc from Nt=8,10 covers 151-176 MeV

MILC data for $T_c r_1$ rescaled with $r_0 / r_1 = 1.4795$
The transition temperature

Comparison with other groups

- RBC-Bielefeld (p4fat3) vs. Wuppertal (stout)

- stout results for Nt=4,6 are about 15% lower, but show similar cut-off dependence

- stout results from different observables for Nt=8,10 are no longer consistent with each other

The transition temperature

Comparison with other groups

- RBC-Bielefeld (p4fat3) vs. Wuppertal (stout)

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\[ \chi_m = m_{u,d}^2 \frac{\partial^2}{\partial m_{u,d}^2} (f(T) - f(0)) \]

Peak position of the dimensionless quantity \( \chi_m / T^4 \) was used

*Is the peak position affected by the renormalization procedure?*

overall scale set with \( r_0 = 0.469 \text{ fm} \)
The transition temperature

Comparison with other groups

- RBC-Bielefeld (p4fat3) vs. Wuppertal (stout)

When plotting

\[ \chi_{\psi \psi} r^2_0 = r^2_0 \frac{\partial^2}{\partial m^2_{u,d}} (f(T) - f(0)) \]

Tc from different observables seem to agree for the stout data*

*we thank S. Katz for providing us with the Wuppertal data
Conclusions

- We studied the thermodynamics of QCD with realistic quark masses and performed a continuum extrapolation based on 2 lattice spacings (Nt=4,6)

- Using an improved staggered action we find a consistent crossover temperature from several observables

- At the physical point of 2+1 flavor QCD we find a crossover temperature of

  \[ T_c = 192 (7) (4) \text{ MeV} \]

- An analysis of the energy density shows a rapid crossover at this temperature

- This calculation is consistent with an analysis based on the asqtad action performed by MILC (some details still have to be resolved)

- Results from the Wuppertal group yield a critical temperature which is at least 15% lower.

- A calculation with Nt=8 is important to confirm our continuum extrapolation