Energy Loss in Hot QCD

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Why jets and why photons?

- **What we want to study:**
  - The properties of the medium created in heavy ion collisions

- **How?**
  - See if the jets disappear.
  - See how the jets disappear.
  - See if more photons come out.
  - See what kind of photons come out.

- **What can jets and photons tell us?**
  - Medium temperature?
  - Parton momentum/energy distribution?
  - History?
Our Strategy

- Initial jet & prompt photon calculation by a proven numerical method (courtesy of P. Aurenche)
- Calculate **full leading order** in $\alpha_s$:
  - Jet energy loss rate (AMY)
  - Thermal photon production rate (AMY)
  - $\gamma$ bremsstrahlung from jets (AMY)
  - Jet-photon conversion rate
- Embed jets in a hydrodynamically evolving medium.
- Jet evolution by solving the rate equations for both hard quarks and gluons $\implies$ Jets fragment outside
Where we want to go

**Au+Au @ \( s^{1/2} = 200 \text{ GeV} \)**

Central \( \pi^0 \) (0-10%)

- PHENIX \( \pi^0 \)
- \( \alpha_s = 0.34 \)
- \( \alpha_s = 0.3 \)

\[
R_{AA} = \frac{dN_{\pi^0}}{dp_T d\gamma} = q(g) \rightarrow q + g \rightarrow \pi
\]

\( p_T \) (GeV/c)

\( dN_{\gamma}/d^2p_Tdy \) [GeV\(^{-2}\)]

**Au+Au at RHIC**

0-10% Central

\( T_i = 370 \text{ MeV} \)

**Au+Au at RHIC**

0-20% Central

\( T_i = 370 \text{ MeV} \)

S. Turbide, C. Gale, S.J. and G. Moore, PRC72:014906, 2005

Jeon  E-Loss
Where we want to go

\begin{align*}
\text{Au+Au @ } s^{1/2} = 200 \text{ GeV} \\
\text{Central } \pi^0 (0-10\%) \\
q(g) \rightarrow q + g \rightarrow \pi \\
\end{align*}

\begin{align*}
R_{AA} & \quad dN_{\gamma}/dP_T dy \quad [\text{GeV}^{-2}] \\
& \quad \text{pQCD + QGP + HG} \\
& \quad \text{pQCD+QGP (no jet-th) + HG} \\
& \quad \text{pQCD (no E-loss)} \\
\end{align*}

\begin{align*}
\text{Au+Au at RHIC} \\
0-10 \% \text{ Central} \\
T_i = 370 \text{ MeV} \\
\end{align*}

Fix $\alpha_s$ and $T$.
Achieve good description.

S. Turbide, C. Gale, S. J. and G. Moore, PRC72:014906, 2005

\begin{align*}
\text{y}_{\gamma} = 0 \\
\text{Au+Au at RHIC} \\
0-20 \% \text{ Central} \\
T_i = 370 \text{ MeV} \\
\end{align*}
Where we want to go

\[ \frac{dN}{d^2p_T dy} [GeV^{-2}] \]

\[ y_\gamma = 0 \]

Au+Au at RHIC

0-10 % Central

\[ T_i = 370 \text{ MeV} \]

With the fixed parameters

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Where we want to go

\[ \text{Au+Au @ } s^{1/2}=200 \text{ GeV} \]

Central \( \pi^0 \) (0-10\%)

\[ q(g) \rightarrow q+g \rightarrow \pi \]

\[ R_{AA} \]

\[ p_T \text{ (GeV/c)} \]

\[ dN_{\gamma}/d\gamma^2 \text{ (GeV)} \]

\[ T_i=370 \text{ MeV} \]

\[ y_{\gamma}=0 \] Au+Au at RHIC 0-10\% Central

\[ \text{Prompt + QGP + HG} \]

\[ \text{pQCD + QGP (no jet-th) + HG} \]

\[ \text{pQCD (no E-loss)} \]

\[ \text{PHENIX} \]

\[ y_{\gamma}=0 \] Au+Au at RHIC 0-20\% Central

\[ T_i=370 \text{ MeV} \]

\[ \text{Prompt + HG} \]

\[ \text{PHENIX PRELIMINARY} \]

S.Turbide, C.Gale, S.J. and G.Moore, PRC72:014906,2005

← Prediction

Jeon E-Loss
Big Picture

\[ \sum_{j} \left( f_{a/A} \otimes f_{b/A} \otimes \frac{d\sigma_{ab\rightarrow cd}}{dx} \right) \otimes (\text{E-loss module}) \otimes D_{\text{frag}} \]

- Parton-parton scattering: \( (f_{a/A} \otimes f_{b/A} \otimes \frac{d\sigma_{ab\rightarrow cd}}{dx}) \)

- \( D_{\text{frag}} \): As in vacuum but with reduced energy.

- Energy loss module – Three separate pieces
  - Gluon radiation rate: \( \frac{d\Gamma}{dt\,dk}(\epsilon, k; T) \)
  - Evolution:
    \[
    \frac{dP(\epsilon, t)}{dt} = \int dk \frac{d\Gamma}{dt\,dk} P(\epsilon + k, t) - \int dk \frac{d\Gamma}{dt\,dk} P(\epsilon, t)
    \]
  - \( T(x, t), u^\mu(x, t) \): Must be obtained independently.

- Still schematic. There are theoretical and conceptual problems to consider.
Amplitude to radiate: Need to sum over all $N$ and all $M$ and all possible radiation points. Then square it to get the radiation rate (BDMPS).
Diagrams

\[ \text{Rate} \propto \text{Im} \sum_{\text{pinching}} \mu \approx gT \]

\[ \text{: HTL resummed} \]
Why is this so hard?

**Collinear enhancement in photon & gluon radiations**


Leading order

Collinear enhancement makes these leading order as well

Need to resum all these, too (AMY)

○ : Hard Thermal Loop
Gluon ladder diagrams

Any number of gluon lines can attach like this.

Adding one more rung = $O(1)$.
Need to resum.
Equation for $F$

$$2h = i\delta E(h, p, k)F(h) + g^2 \int \frac{d^2q_\perp}{(2\pi)^2} C(q_\perp) \times$$

$$\times \left\{ (C_s - C_A/2)[F(h) - F(h-kq_\perp)] 
+ (C_A/2)[F(h) - F(h+p q_\perp)] 
+ (C_A/2)[F(h) - F(h-(p-k) q_\perp)] \right\},$$

$$\delta E(h, p, k) = \frac{h^2}{2pk(p-k)} + \frac{m_k^2}{2k} + \frac{m_{p-k}^2}{2(p-k)} - \frac{m_p^2}{2p}.$$ 

$m^2$: Medium induced thermal masses.
Gluon Radiation Rate

\[
\frac{d\Gamma_g(p, k)}{dkdt} = \frac{C_s}{16\pi p^7} \frac{1}{1 \pm e^{-k/T}} \frac{1}{1 \pm e^{-(p-k)/T}} \times \left\{ \begin{array}{ll}
\frac{1+(1-x)^2}{x^3(1-x)^2} & q \rightarrow qg \\
N_f \frac{x^2+(1-x)^2}{x^2(1-x)^2} & g \rightarrow qq \\
\frac{1+x^4+(1-x)^4}{x^3(1-x)^3} & g \rightarrow gg
\end{array} \right\} \times \int \frac{d^2h}{(2\pi)^2} 2h \cdot \text{Re} \ F(h, p, k),
\]

where \( x \equiv k/p \) is the momentum fraction in the gluon (or either quark, for the case \( g \rightarrow qq \)).

\( h \equiv p \times k \): 2-D vector. \( O(gT^2) \)
Time evolution equation

\[
\frac{dP_{q\bar{q}}(p)}{dt} = \int_k P_{q\bar{q}}(p+k) \frac{d\Gamma_{qg}^q(p+k, k)}{dkdt} - P_{q\bar{q}}(p) \frac{d\Gamma_{aq}^q(p, k)}{dkdt}
+ 2P_g(p+k) \frac{d\Gamma_{qg}^g(p+k, k)}{dkdt},
\]

\[
\frac{dP_g(p)}{dt} = \int_k P_{q\bar{q}}(p+k) \frac{d\Gamma_{qg}^q(p+k, p)}{dkdt} + P_g(p+k) \frac{d\Gamma_{gg}^g(p+k, k)}{dkdt}
- P_g(p) \left( \frac{d\Gamma_{q\bar{q}}^g(p, k)}{dkdt} + \frac{d\Gamma_{gg}^g(p, k)}{dkdt} \Theta(k - p/2) \right),
\]

- \( k \) integrals range: \((-\infty, \infty)\).
- \( k < 0 \): Absorption of thermal gluons.
- \( k > p \): Annihilation against and antiquark of energy \((k - p)\).
- \( \Theta(k - p/2) \): To prevent double counting of final states.
More on the evolution:

**Poisson ansatz** (BDMS, GLV, Wiedemann, Salgago, ...)

\[
P(p) = \int d\epsilon \ D(\epsilon, p) \ P_0(p + \epsilon)
\]

where

\[
D(\epsilon, p) = e^{- \int d\omega \ \frac{dl}{d\omega}(p,\omega)} \ \sum_{n=0}^{\infty} \ \frac{1}{n!} \ \left[ \ \prod_{i=1}^{n} \ \int d\omega_i \ \frac{dl}{d\omega_i}(p,\omega_i) \ \right] \ \delta \left( \ \epsilon - \ \sum_{i=1}^{n} \ \omega_i \ \right)
\]

Now let

\[
\frac{dl}{d\omega}(p,\omega) = \int_{t_0}^{t} dt' \ \frac{d\Gamma_{\text{Poiss.}}}{d\omega dt}(p,\omega, t')
\]
Can show that the Poisson ansatz solves (provided $\Gamma$ depends only on $\omega$):

\[
\frac{dP(p, t)}{dt} = \int d\omega \frac{d\Gamma_{\text{Poiss.}}}{d\omega dt}(\omega) P(p + \omega, t) - P(p, t) \int d\omega \frac{d\Gamma_{\text{Poiss.}}}{d\omega dt}(\omega)
\]

We solve:

\[
\frac{dP_{q\bar{q}}(p)}{dt} = \int_k P_{q\bar{q}}(p + k) \frac{d\Gamma_{qq}^g(p + k, k)}{dkdt} - P_{q\bar{q}}(p) \frac{d\Gamma_{qq}^g(p, k)}{dkdt} + 2P_g(p + k) \frac{d\Gamma_{q\bar{q}}^g(p + k, k)}{dkdt},
\]

\[
\frac{dP_g(p)}{dt} = \int_k P_{q\bar{q}}(p + k) \frac{d\Gamma_{qq}^g(p + k, p)}{dkdt} + P_g(p + k) \frac{d\Gamma_{gg}^g(p + k, k)}{dkdt} - P_g(p) \left( \frac{d\Gamma_{q\bar{q}}^g(p, k)}{dkdt} + \frac{d\Gamma_{gg}^g(p, k)}{dkdt} \Theta(k - p/2) \right)
\]
Can show that the Poisson ansatz solves (provided $\Gamma$ depends only on $\omega$):

$$\frac{dP(p, t)}{dt} = \int d\omega \frac{d\Gamma_{\text{Poiss.}}}{d\omega dt}(\omega)P(p + \omega, t) - P(p, t) \int d\omega \frac{d\Gamma_{\text{Poiss.}}}{d\omega dt}(\omega)$$

We solve:

$$\frac{dP_{q\bar{q}}(p)}{dt} = \int_k P_{q\bar{q}}(p+k)\frac{d\Gamma_{qg}^{q}(p+k, k)}{dk dt} - P_{q\bar{q}}(p)\frac{d\Gamma_{qg}^{q}(p, k)}{dk dt}$$

$$+2P_{g}(p+k)\frac{d\Gamma_{q\bar{q}}^{g}(p+k, k)}{dk dt},$$

$$\frac{dP_{g}(p)}{dt} = \int_k P_{q\bar{q}}(p+k)\frac{d\Gamma_{qg}^{q}(p+k, p)}{dk dt} + P_{g}(p+k)\frac{d\Gamma_{gg}^{g}(p+k, k)}{dk dt}$$

$$-P_{g}(p)\left(\frac{d\Gamma_{q\bar{q}}^{g}(p, k)}{dk dt} + \frac{d\Gamma_{gg}^{g}(p, k)}{dk dt} \Theta(k-p/2)\right)$$
What are we doing?

- **Full leading order** $\alpha_s$ momentum space calculation of the emission + absorption rate in fully dynamic thermal medium. Includes
  - Bremsstrahlung
  - Pair annihilation
  - Absorption from the medium
  - Thermal dispersion corrections
  - Correct and smooth transition from Bethe-Heitler to LPM

- Solve Fokker-Planck equation for the *distribution* instead of Poisson ansatz. Includes nuclear geometry and can accommodate expansion scenarios.
Caveat: Weak coupling limit. \( g \ll 1 \).

\( \tau_{coh} \ll L \)

\( \tau_{coh} \ll (d \ln T(x)/dx)^{-1} \)

One must distinguish what’s important for \( \Delta E \) and \( R_{AA} \) (BDMPS, JM).

- \( R_{AA} \) dominated by many soft emissions.
  - Fully treated in AMY.
- \( \Delta E \) dominated by rare hard emissions.
  - \( k > E_{\text{fact}} \) is not fully treated in AMY. But not important for \( R_{AA} \).

Rough estimates (Bounds for the emitted energy):

\( E_{\text{LPM}} \sim T \sim 300 \text{ MeV} \),

\( E_{\text{fact}} \approx (0.3 \text{ GeV}) \times (L/\lambda)^2 \approx 7.5 - 30 \text{ GeV} \) for \( L/\lambda = 5 - 10 \).
Baseline calculation – PP $\pi^0$

$\pi^0$, p+p at $s^{1/2} = 200$ GeV

Using P. Aurenche et al.'s program.
Results – $\delta$-function Evolution

Lesson: Use the full distribution function.

E/T at several times

Prob. distrib. $P(E)$

- $t=0$
- $t=1/T$
- $t=2/T$
- $t=3/T$
- $t=4/T$
Nuclear Modification Factor

$T_i = 370 \text{ MeV}, \frac{dN}{dy} = 1260$. 1-D Bjorken expansion.
Pb+Pb @ \( s^{1/2} = 5500 \text{ GeV} \)

Central \( \pi^0 \) (0-10%)

\( T_i = 845 \text{ MeV} \)

\( E_{i,\text{max}} = 400 \text{ GeV} \)

\( \Delta E_{\text{eff}} = \Delta E(r, \phi) \)

\[ dN/dy = 5620. \]
Understanding the ratio

Use BDMPS expression for the quenching factor for $1/p^n$ with large $n$ but with the energy range extended to $\omega < 0$:

$$R_{AA}(p) \approx \exp \left( -\left(1 - e^{-\omega n/p} \right) \int_{-\infty}^{\infty} d\omega \int_{0}^{t} dt' \Gamma(p,\omega, t) \right)$$

For $\Gamma$, use simple estimates

$$\omega \frac{dl}{d\omega dt} \approx \frac{\alpha}{\pi} \frac{N_c}{\lambda} \quad \text{for} \quad 0 < \omega < \lambda \mu^2$$

$$\omega \frac{dl}{d\omega dt} \approx \frac{\alpha}{\pi} N_c \sqrt{\frac{\mu^2}{\lambda \omega}} \quad \text{for} \quad \lambda \mu^2 < \omega < \lambda \mu^2 \left( L/\lambda \right)^2$$

$$\frac{dl}{d\omega dt} \approx \frac{\alpha}{\pi |\omega|} \frac{N_c}{\lambda} e^{-|\omega|/T} \quad \text{for} \quad \omega < 0$$
Features are roughly produced.
Flat ratio due to energy-loss and thermal absorption
BH part of the energy-loss is important.
Temperature dependence

Au+Au @ $s^{1/2} = 200$ GeV

Central $\pi^0$ (0-10%)

$\Delta E_{\text{eff}} = \Delta E(r=0)$

$R_{AA}$

$p_T$ (GeV/c)

0 2 4 6 8 10 12

0

0.2

0.4

0.6

0.8

1

PHENIX $\pi^0$

- $T_i = 1000$ MeV, $dN/dy = 1260$
- $T_i = 447$ MeV, $dN/dy = 1260$
- $T_i = 370$ MeV, $dN/dy = 1260$
- $T_i = 1000$ MeV, $dN/dy = 680$
- $T_i = 447$ MeV, $dN/dy = 680$
- $T_i = 370$ MeV, $dN/dy = 680$

$\Delta E_{\text{eff}} = \Delta E(r=0)$
What do we really learn from $R_{AA}$?

1. High $p_T$ particles are suppressed.
2. $R_{AA}$ is flat.

That’s about the end of it.

For instance, the simplest wrong explanation, $\Delta E = fE$, gives a perfectly flat $R_{AA}$ with $P(E) \propto 1/E^n$.

- Problem:
  - Too many convolutions to undo.
  - $R_{AA}(p_T)$ turned out to be featureless.
  - The magnitude largely depend on Entropy, not $T$.
  - $\hat{q} \sim T^3$. Factor of 2 change in $T \implies$ Factor of 8 change in $\hat{q}$. 

Where are we now?

- Most models have jet quenching under control in the hadronic part.
- But $R_{AA}$ too simple to be the full story.

More information? – $\gamma$ production

Need:
- Thermal photon radiation rate (AMY)
- Jet bremsstrahlung rate (AMY)
- Jet-photon conversion rate (Fries, Mueller, Srivastava)
Thermal Radiation rate:
\[
\omega \frac{dR}{d^3k} = -\frac{g^{\mu\nu}}{(2\pi)^3} \frac{1}{e^{\omega/T} - 1} \text{Im} \Pi^{R}_{\mu\nu}(k)
\]

Physical process:

Need to sum over the scattering centers and the radiation points.
Arnold, Moore and Yaffe, JHEP 0112 (2001) 009

The same formalism can be used to calculate thermal radiation \((P \sim T)\) and bremsstrahlung from jets \((P \gg T)\).
\[ \frac{dR}{dyd^2p_T} = \sum_f \left( \frac{e_f}{e} \right)^2 \frac{T^2 \alpha_s}{8\pi^2} \left[ f_q(p_\gamma) + f_{\bar{q}}(p_\gamma) \right] \left[ 2 \ln \left( \frac{4E_\gamma T}{m^2} \right) - C \right] \]

with \( C \approx 2.33 \).
Putting everything together for $\gamma$...
Using P. Aurenche et al.'s program.
Photons at RHIC

$Au + Au \ s^{1/2} = 200A \ GeV$

$T_i = 370 \ MeV$

$\gamma \ - \ Composition$

$\frac{dN}{dy \ dp_T^2} \ [GeV^2]$

$y_\gamma = 0$

- N-N
- jet-th
- jet-bremss.
- jet-fragmentation
- th-th
$\gamma$ – Our Calc vs. PHENIX Data

$y_\gamma = 0$  
Au+Au at RHIC  
0- 10 % Central

$T_i = 370$ MeV

- prompt + QGP + HG
- prompt + QGP (no jet-th) + HG
- prompt (no E-loss)
- PHENIX
$\gamma$ - Our Prediction vs. Data

$y_\gamma = 0$  
Au+Au at RHIC  
0- 20 % Central

\[
dN / d^2 p_T d y [\text{GeV}^2]\n\]

- Red: pQCD+QGP+HG
- Purple: pQCD+HG
- Blue: pQCD (no E-loss)
- Black: PHENIX PRELIMINARY

$T_i = 370 \text{ MeV}$
0-10% Central Au+Au

$s^{1/2} = 200A$ GeV

$\gamma_{\text{Total}} / \gamma_{\text{background}}$

$P_T$ (GeV/c)

Prompt+QGP, $T_i = 447$ MeV
Prompt+QGP, $T_i = 370$ MeV
Prompt $\gamma$, $T_i = 370$ MeV
$\gamma$ – Composition – LHC

Photons at LHC

$\text{Pb} + \text{Pb}, s^{1/2} = 5500A \text{ GeV}$

$T_i = 845 \text{ GeV}$

$y_\gamma = 0$

$dN_\gamma / dp_T dy [\text{GeV}^2]$

$P_T [\text{GeV}]$

- N-N
- jet-th
- jet-bremss.
- jet-fragmentation
- th-th
\( \gamma - \text{LHC prediction} \)

\[ y_\gamma = 0 \quad \text{Pb+Pb at LHC} \]

0-10 \% Central

\[ \frac{dN_\gamma}{dp_T^2 \ dy} \quad [\text{GeV}^2] \]

- Black: pQCD+QGP+HG
- Red: pQCD+QGP(no jet-th)+HG
- Green: pQCD (no E-loss)

\( T_i = 845 \text{ MeV} \)
Conclusions and Caveats

- Calculated and used the full leading order Hot QCD radiation rates for both gluons and photons.
- Important to use the full momentum distribution at any given time, not just $dE/dx$.
- Geometry and 1-D expansion included.
- Good description of existing data – pions and photons.
- For photons, jet-thermal interaction is crucial.
- LHC predictions – Should be better since pQCD should work better there.
Conclusions and Caveats

- Calculations consistent in the $g \ll 1$ limit for momenta $T < p$. Yet for quantitative calculations, we needed $\alpha_s \approx 1/3$ or $g \approx 2$! So in reality, one must sum all diagrams, not just pinching part of the ladder diagrams!
  - At this leading order, $\alpha_s$ is an overall factor. So one might hope that the structure of the solution is OK.
  - Right now, this is best we can do with perturbative calculation.
- Elastic scatterings should be incorporated.
Conclusions and Caveats

- Do better job with the medium evolution? – Need 3-D hydro code. Working on it.

- What about jet correlations? – Need to keep track of the evolution of the joint probability function of two jet energies. Much harder than single particle distributions! Will work on it.

- Most energy-loss calculations these days do get $R_{AA}$ right. Is there an experimental way to distinguish? – Photon bremsstrahlung + jet-photon conversion should be able to distinguish different scenarios. How to fish that out of all others is another matter.