ANISOTROPY OF FLOW AND THE ORDER OF PHASE TRANSITION IN RELATIVISTIC HEAVY ION COLLISIONS

Pasi Huovinen

University of Virginia
Lattice QCD Equation of State

$\varepsilon_c \sim 0.7 \text{ GeV/fm}^3$

$T_c = (173 \pm 15) \text{ MeV}$

Calculations with two light quarks or three light quarks

“2+1 flavor” estimate for two light and one heavier (strange) quark

Karsch & Laermann, hep-lat/0305025
Hydrodynamic description

- Equation of state explicit input

In Hydrodynamical description space-time evolution is given by

\[ \partial_{\mu}T^{\mu\nu} = 0 \quad \text{and} \quad \partial_{\mu}j^{\mu} = 0 \]

- These equations merely express local conservation of energy, momentum and baryon number

In ideal fluid approximation we assume

- local kinetic equilibrium
- local chemical equilibrium
- no dissipation

and get simple forms for energy-momentum tensor and baryon four-flow

\[ T^{\mu\nu} = (\epsilon + P)u^{\mu}u^{\nu} - g^{\mu\nu}P, \]
\[ j^{\mu} = n_Bu^{\mu} \]
Equation of State

• Conservation laws do not form a closed system of equations
• Additional equations needed:
  – Equation of State (EoS) of matter: \( P = P(T, \mu_b) \)
• Unknown \textit{microscopic dynamics} is hidden into the EoS
• How to build one?
  – Parametrized \textit{lattice} result
    * not applicable in hadronic phase below \( T_c \)
  – Ideal gas of massless partons + bag constant
  – Noninteracting gas of \textit{massive hadrons and resonances}
    * Inclusion of \textit{resonances} mimics the effect of \textit{hadron interactions} surprisingly well
  – Connect two phases using Maxwell construction \( \rightarrow \) \textit{first order phase transition}
  – Interpolate “smoothly” between them \( \rightarrow \) \textit{crossover}
Parametrized lattice EoS

- Extrapolation to finite quark masses?
- Schneider & Weise: quasiparticle model for plasma with known masses
  - Tune the model to fit lattice results with lattice masses
  - Change masses to more physical ones
Examples

- HRG+QGP: ideal gas of partons and hadron resonance gas, first order PT
- Lattice: lattice EoS + hadron resonance gas, crossover
- pure HRG: hadron resonance gas only, no PT
Beginning and end

- Hydrodynamics needs additional input to define where start and stop

- Initial state: density in the beginning
  - Choose density to reproduce final multiplicity in most central collisions
  - Shape from Glauber model calculation of number of participants and binary collisions

- At the end of evolution: convert fluid into particles
  - Freeze-out at a constant temperature surface
  - Temperature chosen to reproduce the slopes of $p_T$ distributions
Requires different freeze-out temperatures

- **HRG+QGP**: $T_{f,o} = 130$ MeV
- **Lattice**: $T_{f,o} = 141$ MeV
- **HRG**: $T_{f,o} = 135$ MeV
Non-central collisions

- Reaction zone **anisotropic** in coordinate space + rescattering

⇒ Particle distribution **anisotropic** in momentum space – even if initial particle production isotropic

- Fourier expansion of momentum distribution:

\[
\frac{dN}{dy\,p_Tdp_Td\phi} = \frac{1}{2\pi} \frac{dN}{dy\,p_Tdp_T} \left( 1 + 2v_1(y, p_T)\cos \phi + 2v_2(y, p_T)\cos 2\phi + \cdots \right)
\]

**v_1:** Directed flow:
preferred direction

**v_2:** Elliptic flow:
preferred plane
Anisotropy as function of $p_T$:

Minimum bias

- Pion $v_2(p_T)$ deviates from the data around $p_T \approx 1.5$ GeV
- Proton $v_2(p_T)$ follows the data longer, up to $p_T \approx 2.5$ GeV
- All EoSs lead to similar pion $v_2(p_T)$
- Proton $v_2(p_T)$ is sensitive to EoS:
  - Lattice EoS gives as bad fit than EoS without any phase transition!
  - An EoS with a first order phase transition is closest to the data
Two-particle correlations:

- HBT puzzle cannot be changed into a $v_2$ puzzle using these EoSs
- Lattice EoS is closest to the data, but still far away

Note that $R_{\text{out}}/R_{\text{side}}$ is largest when there is no phase transition—exactly the opposite to what was expected!
Anisotropy as function of $p_T$:

Minimum bias

- Finite size effect?
- Or this particular parametrization?
- Or assumption of chemical equilibrium?
- Or ideal fluid assumption...? (i.e. viscosity)
Lattice Eos at $T_c$
Anisotropy as function of $p_T$:

Minimum bias

- Finite size effect?
- Or this particular parametrization?
- Or assumption of chemical equilibrium?
- Or ideal fluid assumption...? (i.e. viscosity)
Chemical equilibrium

- Freeze out temperatures about $T_f \approx 110 - 140$ MeV
- According to thermal models particle ratios (approximately) correspond to a system in $T \approx 170$ MeV temperature

$\Rightarrow$ Particle ratios become wrong

- This can be cured: treat number of pions, kaons etc. as conserved quantum numbers below $T = 170$ MeV
  $\rightarrow$ several conserved currents

$\Rightarrow$ Too many (anti)protons!!!

- Use $T = 150$ MeV as chemical freeze-out temperature

$\Rightarrow$ Pion and proton yields reproduced, strange baryon yields too small.

- $P = P(\epsilon, n_b)$ changes very little
- but $T = T(\epsilon, n_b)$ changes...
and its consequences

- When particle ratios are correct nice agreement with data and calculation is lost!

- Temperature falls faster $\rightarrow$ collective anisotropy is diluted less
Viscous hydrodynamics...

...is tedious...

- Entropy no longer conserved:
  \[ \partial_{\mu} T^{\mu\nu} = 0 \]
  \[ \partial_{\mu} j^{\mu} = 0 \]
  \[ \partial_{\mu} S^{\mu} \geq 0 \]

- where
  \[ T^{\mu\nu} = (\epsilon + P + \Pi) u^{\mu} u^{\nu} - g^{\mu\nu}(P + \Pi) + q^{\mu} u^{\nu} + q^{\nu} u^{\mu} + \pi^{\mu\nu} \]
  \[ j^{\mu} = n_B u^{\mu} \]
  \[ S^{\mu} = s u^{\mu} + \frac{q^{\mu}}{T} - \frac{\alpha_0 \Pi q^{\mu}}{T} + \frac{\alpha_1 \pi^{\mu\nu} q^{\nu}}{T} \]
  \[ - \left( \beta_0 \Pi^2 - \beta_1 q^{\nu} q^{\nu} + \beta_2 \pi^{\nu\lambda} \pi_{\nu\lambda} \right) \frac{u^{\mu}}{2T} \]

- and
  \[ \Pi = \zeta \partial_{\mu} u^{\mu} \text{ is bulk pressure} \]
  \[ q^{\mu} = \kappa T \Delta^{\mu\nu} \left( \partial_{\nu} T - u_{\lambda} \partial^{\lambda} u_{\mu} u \right) \text{ is heat flow} \]
  \[ \pi^{\mu\nu} = 2 \eta \left[ \frac{1}{2} \left( \Delta^{\mu}_{\alpha} \Delta^{\nu}_{\beta} + \Delta^{\mu}_{\beta} \Delta^{\nu}_{\alpha} \right) - \frac{1}{3} \Delta^{\mu\nu} \Delta_{\alpha\beta} \right] \partial^{\alpha} u^{\beta} \]
  \[ \text{is stress tensor} \]
  \[ \zeta: \text{ bulk viscosity coefficient} \]
  \[ \eta: \text{ shear viscosity coefficient} \]
  \[ \kappa: \text{ thermal conductivity coefficient} \]
  \[ \alpha_0, \alpha_1, \beta_0, \beta_1, \beta_2 \text{ are relaxation time coefficients} \]

- These give 14 equations to solve 14 unknowns
Add some dissipation
in the form of cascade description of hadronic phase

and the problem seems to be cured.
(hydro+cascade calculation by Teaney et al. nucl-th/0110037 for √s_{NN} = 130 GeV)

• this still assumes the qgp to be ideal fluid
• but does not prove it
• and assumes first order phase transition...
Conclusions

• Transverse flow is not sensitive to the phase transition
• Anisotropic flow of protons seems to be sensitive to the order of phase transition
  – first order transition favored
• HBT puzzle not solved with these EoSs—$R_{side}$ too small.