Shear Viscosity in an Anisotropically Expanding QGP

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- QGP Properties: key findings
- The sQGP: pro’s and con’s
- Anomalous Viscosity

work in collaboration with M. Asakawa, B. Mueller & C. Nonaka
- C. Nonaka & S.A. Bass: nucl-th/0607018
Lattice Gauge Theory predictions: QCD bulk properties
LGT predicts a phase-transition to a state of deconfined nearly massless quarks and gluons: QGP
- range for $T_c$: 170-190 MeV
- at finite baryon-density, there exists a tri-critical point and a mixed phase
different DOF introduce different correlations between conserved charges:

\[ C_{BS} = -3 \frac{\langle BS \rangle - \langle B \rangle \langle S \rangle}{\langle S^2 \rangle - \langle S \rangle^2} = -3 \frac{\langle BS \rangle}{\langle S^2 \rangle} \]

QP-QGP: \( C_{BS} = 1 \) vs. BS-QGP: \( C_{BS} \approx 0.61 \)

\[ T > T_C: C_{BS} = 1 \]

- mesonic as well as baryonic bound-states (involving quarks) above \( T_C \) ruled out via \( \chi_{BS} \) and \( \mu_B \)-derivatives thereof

- Lattice confirms quasi-quark nature of flavor carriers in the deconfined phase

\[ \chi_{us} = -\frac{1}{V} \frac{\partial^2 F}{\partial \mu_u \partial \mu_s} \]
\[ \chi_{ss} = -\frac{1}{V} \frac{\partial^2 F}{\partial \mu_s^2} \]
\[ C_{BS} = 1 + \frac{\chi_{ds} + \chi_{us}}{\chi_{ss}} \]

Gavai & Gupta, PRD 73, 14006, 2006
Relativistic Heavy-Ion Collisions: probing the QGP at RHIC
**Jet-Quenching: Basic Idea**

What is a jet?

- fragmentation of hard scattered partons into collimated “jets” of hadrons

- p+p reactions provide a calibrated probe, well described by pQCD

What happens if partons traverse a high energy density colored medium?

- transport coefficient $q$ is sensitive to density of (colored) charges

\[
\hat{q} = \rho \int q^2 dq^2 \frac{d\sigma}{dq^2} \equiv \rho \sigma \left\langle k_T^2 \right\rangle = \frac{\mu^2}{\lambda_f}
\]
• suppression can be experimentally quantified in terms of $R_{AA}$ ratio:

$$R_{AA} \equiv \frac{d^2 N^{AA}}{dy dp_T} \cdot \left\langle N^{AA}_{\text{coll}} \right\rangle \bigg/ \frac{d^2 N^{pp}}{dy dp_T} \cdot \left\langle N^{AA}_{\text{coll}} \right\rangle$$

RHIC data shows values for q-hat far larger than expected even for a QGP!

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Relativistic Fluid Dynamics & Parton Recombination
Elliptic Flow at RHIC

Au+Au; √s_{NN} = 200 GeV; Mid-rapidity

- Hydro model
  - π
  - K
  - p
  - Λ

- STAR
  - h^+ + h^-
  - K_S^0
  - Λ + Λ

- PHENIX
  - π^+ + π^-
  - K^- + K^-
  - p + p

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Viscosity of an expanding QGP #9
Recombination: Parton Number Scaling of $v_2$

- In leading order of $v_2$, recombination predicts:

$$v_2^M(p_t) \approx 2v_2^p \left( \frac{p_t}{2} \right)$$

$$v_2^B(p_t) \approx 3v_2^p \left( \frac{p_t}{3} \right)$$

![Diagram](image)

- Smoking gun for hadron formation from deconfined quarks
- Measurement of partonic $v_2$
Hydro evolution from QGP to hadrons

- Full 3-d Hydrodynamics
- QGP evolution
- Hadronization
- Cooper-Frye formula
- UrQMD
- hadronic rescattering
- t fm/c
- C. Nonaka & S.A. Bass: nucl-th/0607018
- see also Hirano and Nara

Equation: \( \eta = 0 \)

- Good agreement with data for \( \eta_{\text{QGP}} = 0 \)

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Current Picture of QGP Structure

LGT & Parton Recombination:
• at \( T_C \), QGP degrees of freedom carry the quantum numbers of quarks and recombine to form hadrons

Jet-Qenching & Elliptic Flow:
• QGP produced at RHIC has very large opacity
• behaves like a nearly ideal fluid (vanishing viscosity)

Conclusion: the strongly interacting QGP (sQGP)
• combine QP nature & large opacity: very high gluon densities
• ideal fluid: vanishing mean free path of quarks and gluons
  (i.e. very large parton cross sections)
The sQGP Dilemma

- Microscopic transport theory shows that assuming quasi-particle q & g degrees of freedom would require unphysically large parton cross sections to match elliptic flow data.

- Even for $\lambda \approx 0.1 \text{ fm}$ (close to uncertainty bound) dissipative effects are large.

- Gluon densities needed for jet-quenching calculations may be too large compared to measured entropy.

- Does a small viscosity have to imply that matter is strongly interacting?

- Paradigm shift needed: consider effects of (turbulent) color fields.
(Anomalous) Viscosity
What is viscosity?

- shear and bulk viscosity are defined as coefficients in the expansion of the stress tensor in gradients of the velocity field:

\[ T_{i k} = \varepsilon u_i u_k + P(\delta_{i k} + u_i u_k) - \eta \left( \nabla_i u_k + \nabla_k u_i - \frac{2}{3} \delta_{i k} \nabla \cdot u \right) + \varsigma \delta_{i k} \nabla \cdot u \]

- microscopically, \( \eta \) is given by the rate of momentum transport:

\[ \eta \approx \frac{1}{3} n \bar{p} \lambda_f = \frac{\bar{p}}{3\sigma_{tr}} \]

- the unitary limit on cross sections suggests that \( \eta \) has a lower bound:

\[ \sigma_{tr} \leq \frac{4\pi}{\bar{p}^2} \quad \Rightarrow \quad \eta \geq \frac{\bar{p}^3}{12\pi} \]
Anomalous Viscosity:

- Any contribution to the shear viscosity not explicitly resulting from momentum transport via a transport cross section.

- **Plasma physics:**
  - A.V. = Large viscosity induced in nearly collisionless plasmas by long-range fields generated by plasma instabilities.

- **Astrophysics - dynamics of accretion disks:**
  - A.V. = Large viscosity induced in weakly magnetized, ionized stellar accretion disks by orbital instabilities.

- **Biophysics:**
  - A.V. = The viscous behavior of non-homogeneous fluids or suspensions, e.g., blood, in which the apparent viscosity increases as flow or shear rate decreases toward zero.

*(From: http://www.biology-online.org/dictionary)*
Can the QGP viscosity be anomalous?

- can the extreme *opaqueness* of the QGP be explained without invoking super-strong coupling?
- *Expanding* plasmas (such as the QGP at RHIC) *always* have anisotropic momentum distributions
  - properties of *anisotropic* plasmas:
    - plasma *turbulence*: random, nonthermal excitation of coherent field modes with power spectrum similar to the vorticity spectrum in a turbulent fluid \([P(k) \sim 1/k^2]\).
    - plasma turbulence arises naturally in plasmas with an *anisotropic* momentum distribution (Weibel-type instabilities).
  - soft color fields generate *anomalous transport coefficients*, which may give the medium the character of a nearly *perfect fluid* even at moderately weak coupling.

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Turbulent Color Fields

- Weibel Instabilities
- calculated in HTL formalism

see for example:
Mrowczynski, PLB 314, 118 (1993)
Mrowczynski, PLB 393, 26 (1997)
Anomalous Viscosity in QCD
Vlasov-Boltzmann Eqn for Color Charges

\[ \nu^\mu \frac{\partial}{\partial x^\mu} f(\mathbf{r}, \mathbf{p}, t) + g F^a \cdot \nabla_p f^a (\mathbf{r}, \mathbf{p}, t) + C[f] = 0 \]

with parton distribution functions:

\[ f(\mathbf{r}, \mathbf{p}, t) = \int dQ \tilde{f}(\mathbf{r}, \mathbf{p}, Q, t) \]

\[ f^a (\mathbf{r}, \mathbf{p}, t) = \int dQ Q^a \tilde{f}(\mathbf{r}, \mathbf{p}, Q, t) \]

- solution for Vlasov term is given by:

\[ f^a (\mathbf{p}, x) = -ig \frac{C_2}{N_c^2 - 1} \int \frac{d^4 k}{(2\pi)^4} \int d^4 x' U_{ab} (x, x') \frac{e^{ik \cdot (x-x')}}{v \cdot k + i\varepsilon} F^b (x') \cdot \nabla_p f (\mathbf{p}) \]

- resulting Vlasov force term:

\[ g F^a (x) \cdot \nabla_p f^a (\mathbf{p}, x) = -ig \frac{C_2}{N_c^2 - 1} F^a (x) \cdot \nabla_p \int \frac{d^4 k}{(2\pi)^4} \int d^4 x' U_{ab} (x, x') \frac{e^{ik \cdot (x-x')}}{v \cdot k + i\varepsilon} F^b (x') \cdot \nabla_p f (\mathbf{p}) \]
Turbulent Color Fields

- Effect on color charges can be described via ensemble–average (w/ \( \bar{f} \equiv \langle f \rangle \)):
  \[
  \left\langle F_i^a (x) U_{ab} (x, x') F_j^b (x') f (p) \right\rangle = \left\langle F_i^a (x) U_{ab} (x, x') F_j^b (x') \right\rangle \bar{f} (p)
  \]
  
- Correlation function of fields depend only on |x-x'| and fall off rapidly:
  \[
  \left\langle \mathcal{E}_i^a (x) U_{ab} (x, x') \mathcal{E}_j^b (x') \right\rangle = \left\langle \mathcal{E}_i^a \mathcal{E}_j^a \right\rangle \Phi^{(el)}_r (|t-t'|) \tilde{\Phi}^{(el)}_\sigma (|x-x'|)
  \]
  \[
  \left\langle \mathcal{B}_i^a (x) U_{ab} (x, x') \mathcal{B}_j^b (x') \right\rangle = \left\langle \mathcal{B}_i^a \mathcal{B}_j^a \right\rangle \Phi^{(mag)}_r (|t-t'|) \tilde{\Phi}^{(mag)}_\sigma (|x-x'|)
  \]
  \[
  \left\langle \mathcal{E}_i^a (x) U_{ab} (x, x') \mathcal{B}_j^b (x') \right\rangle = 0 \quad \text{(electric and magnetic fields are uncorrelated)}
  \]

- With Gaussian correlators \( \Phi^{el} \) and \( \Phi^{mag} \)

- Solution for Vlasov force term:
  \[
  \left\langle g F^a (x) \cdot \nabla_p f^a \right\rangle = -\frac{g^2 C_2}{N_c - 1} \left[ \tau^{el}_m \left\langle \mathcal{E}_i^a \mathcal{E}_j^a \right\rangle \frac{\partial^2}{\partial p_i \partial p_j} + \tau^{mag}_m \left\langle \mathcal{B}_i^a \mathcal{B}_j^a \right\rangle (v \times \nabla_p)_i (v \times \nabla_p)_j \right] \bar{f} (p)
  \]
  \[
  \equiv -\nabla_p \cdot D(p) \nabla_p \bar{f} (p)
  \]

- With the memory time: \( \tau^{el/mag}_m = \frac{1}{2} \int_{-\infty}^{\infty} dt' \Phi^{(el/mag)}_r (|t-t'|) \tilde{\Phi}^{(el/mag)}_\sigma (|v(t-t')|) \)
Shear Viscosity from Linear Response

ensemble-average: diffusive Vlasov-Boltzmann (Fokker-Planck) Equation

\[
\nu^\mu \frac{\partial}{\partial x^\mu} \overline{f}(\mathbf{r}, \mathbf{p}, t) - \nabla_p \cdot D \cdot \nabla_p \overline{f}(\mathbf{r}, \mathbf{p}, t) + \left\langle C[f] \right\rangle = 0
\]

Chapman-Enskog formalism: small perturbation of thermal equilibrium \( f_0 \):

\[
f(\mathbf{p}, \mathbf{r}) = f_0(\mathbf{p}) + \delta f(\mathbf{p}, \mathbf{r}) = f_0(\mathbf{p}) \left[ 1 + f_1(\mathbf{p}, \mathbf{r}) (1 \pm f_0(\mathbf{p})) \right]
\]

with local perturbation:

\[
f_1(\mathbf{p}, \mathbf{r}) = -\frac{\Delta(\mathbf{p})}{E_p T^2} p_i p_j (\nabla u)_{ij}
\]

- \( \Delta(\mathbf{p}) \) measures magnitude of deviation from equilibrium

- extract viscosity via comparison of E-p tensor: \( T_{ik} = \int \frac{d^3 p}{(2\pi)^3 E_p} p_i p_k f(\mathbf{p}, \mathbf{r}) \)

  with macroscopic definition of the viscous stress:

  \[
  T_{ik} = T_{ik}^{(0)} + \delta T_{ik} = P \delta_{ik} + \varepsilon u_i u_k - 2\eta (\nabla u)_{ik} - \zeta \delta_{ik} \nabla \cdot \mathbf{u}
  \]

  with: pressure \( P \), energy density \( \varepsilon \), shear viscosity \( \eta \) and bulk visc. \( \zeta \)

- comparison yields for shear viscosity:

\[
\eta = -\frac{1}{15 T} \int \frac{d^3 p}{(2\pi)^3 E_p^2} \frac{p^4}{E_p} \Delta(\mathbf{p}) \frac{\partial f_0}{\partial E_p}
\]
Diffusive Vlasov-Boltzmann Eqn: solution

\[ \nu^\mu \frac{\partial}{\partial x^\mu} \tilde{f}(\mathbf{r}, \mathbf{p}, t) - \nabla_\mathbf{p} \cdot D \cdot \nabla_\mathbf{p} \tilde{f}(\mathbf{r}, \mathbf{p}, t) + \langle C[f] \rangle = 0 \]

**drift term:**

\[ \nu^\mu \frac{\partial}{\partial x^\mu} f_0(\mathbf{p}) = f_0(1 \pm f_0) \left[ \frac{p_i p_j}{E_p T} \left( \nabla u \right)_{ij} - \frac{1}{3} m_D \tau_{m}^{el} \frac{\langle \mathcal{E}^2 \rangle E_p}{T^2} \left( \frac{\partial \mathcal{E}}{\partial T} \right) \right] \]

(...): effective color conductivity

**force term:**

\[ -\nabla_\mathbf{p} \cdot D(\mathbf{p}) \nabla_\mathbf{p} \tilde{f}(\mathbf{p}) = - \frac{g^2 C_2}{N_c^2 - 1} \left[ \tau_{m}^{el} \langle \mathcal{E}_i^{a} \mathcal{E}_j^{a} \rangle \frac{\partial^2}{\partial p_i \partial p_j} + \tau_{m}^{mag} \langle B_i^{a} B_j^{a} \rangle \left( \mathbf{v} \times \nabla_\mathbf{p} \right)_i \left( \mathbf{v} \times \nabla_\mathbf{p} \right)_j \right] \tilde{f}(\mathbf{p}) \]

**collision term:**

- vanishes in equilibrium, 1st contribution linear in \( f_1 \): \( \langle C[f] \rangle = \langle I[f_1] \rangle = I[\tilde{f}_1] \)

\[ I[f_1] = \int \frac{d^3 k}{(2\pi)^3} d\sigma_{12} v_{rel} f_0(\mathbf{p}) f_0(\mathbf{k}) \left[ f_1(\mathbf{p}) + f_1(\mathbf{k}) - f_1(\mathbf{p'}) - f_1(\mathbf{k'}) \right] \]

and differential cross section \( d\sigma \) (leading log: use ME’s w/ highest IR div.)

[see Arnold, Moore & Yaffe: JHEP 0011,1 (2000)]
Example: transverse color magnetic fields

additional approximation: \( \langle B_i^a B_j^a \rangle = \frac{1}{2} \left( \delta_{ij} - \delta_{iz} \delta_{jz} \right) \langle B^2 \rangle; \quad \langle E_i^a E_j^a \rangle \approx 0 \)

- **Drift term:**
  \[ \nu^\mu \frac{\partial}{\partial x^\mu} f_0(p) = f_0(1 \pm f_0) \left[ \frac{p_i p_j}{E_p T} (\nabla u)_{ij} \right] \]

- **Force term:**
  \[ -\nabla_p \cdot D(p) \nabla_p \tilde{f}(p) = -\frac{3 C_2 \bar{\Delta} g^2 \langle B^2 \rangle \tau_{m}^{\text{mag}}}{(N_c^2 - 1) E_p T^2} f_0(1 \pm f_0) p_i p_j (\nabla u)_{ij} \]

Ignoring the collision term, the contribution of the Vlasov term yields:

- **momentum anisotropy:**
  \[ \bar{\Delta}(p) = \frac{(N_c^2 - 1) E_p T^2}{3 C_2 g^2 \langle B^2 \rangle \tau_{m}^{\text{mag}}} \]

- **anomalous viscosity:**
  \[ \eta_{A}^{(\text{gluon})} = \frac{16 \zeta(6) (N_c - 1)^2}{\pi N_c} \frac{T^6}{g^2 \langle B^2 \rangle \tau_{m}^{\text{mag}}} \quad \eta_{A}^{(\text{quark})} = \frac{62 \zeta(6) N_c^2 N_f}{\pi^2} \frac{T^6}{g^2 \langle B^2 \rangle \tau_{m}^{\text{mag}}} \]
solution of the full Vlasov-Boltzmann eqn. coincides w/ minimum of a quadratic functional $W[f_1]$ and can be obtained via a variational principle:

$$W[f_1] = \int \frac{d^3 p}{(2\pi)^3} f_1(p) \left[ \nu^\mu \frac{\partial f_0(p)}{\partial x^\mu} + \frac{1}{2} \left( -\nabla \cdot D \cdot \nabla f + \delta f(p) + I[f_1] \right) \right] = \text{min}$$

- minimum defines optimal solution for $f_1$ or $\Delta(p)$ in transport eqn
- simple ansatz: $\bar{\Delta}(p) = A|p|/T, \quad A = (A_q, A_g)$
- minimization results in set of linear eqn. for $A_q$ & $A_g$: $(a_A + a_C) A = r$

with:

$$r = \frac{32 \zeta(5)}{3 \pi^2} \left( \frac{N_c^2 - 1}{15} \right) \left( \frac{N_c N_f}{8} \right)$$

$$a_A = \frac{32 \zeta(4)}{5 \pi^2} \frac{g^2 \langle B^2 \rangle \tau_{magg}}{T^3} \begin{pmatrix} N_c & 0 \\ 0 & \frac{7}{8} N_f \end{pmatrix}$$

$$a_C = \frac{\pi \left( N_c^2 - 1 \right)}{45} g^4 \ln g^{-1} \left[ \frac{2}{9} \left( 2N_c + N_f \right) \begin{pmatrix} N_c & 0 \\ 0 & \frac{7}{8} N_f \end{pmatrix} + \frac{\pi^2 N_f \left( N_c^2 - 1 \right)}{128 N_c} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \right]$$

[in absence of turbulent fields, this reduces to result of Arnold, Moore & Yaffe: JHEP 0011,1 (2000)]

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Viscosity of an expanding QGP #25
Shear Viscosity: full expression

- use result from linear response, insert ansatz for $\Delta(p)$ and perform momentum integration:

$$\eta = \frac{24 \zeta(5) T^3}{3 \pi^2} \left( (N_c - 1) A_g + \frac{15}{8} N_c N_f A_q \right) = \frac{3}{4} r \cdot A$$

- result of variational calculation then yields: $\eta = \frac{3}{4} r \cdot (a_A + a_C)^{-1} \cdot r$

$\triangleright$ full expression of shear viscosity can be rewritten as:

$$\eta^{-1} = \eta_A^{-1} + \eta_C^{-1}$$

(additivity of relaxation rates)

with $\eta_A = \frac{3}{4} r \cdot a_A^{-1} \cdot r$ and $\eta_C = \frac{3}{4} r \cdot a_C^{-1} \cdot r$
Anomalous Viscosity Estimate


- perturbation of equilib. distribution: \( f_1(p) = -\frac{\xi}{2E_pT} \left( p_z^2 - \frac{p_x^2}{3} \right) \)

- connection to momentum anisotropy \( \Delta \): \( \xi = 2\Delta \frac{\nabla u}{T} \)

- comparison with linear response yields: \( \xi = \frac{15\eta|\nabla u|}{2T_{00}} = 10\frac{\eta|\nabla u|}{sT} \)

- case of transverse B-fields, make ansatz: \( g^2 \langle B^2 \rangle = b_0 g^4 T^4 \xi^n \)

- memory time determined by coherence length of fields: \( \tau_m = \frac{d_0}{gT\sqrt{\xi}} \)

\( \geq \) self-consistent anomalous shear viscosity:

\[
\eta_A = \overline{c}_0 \left( \frac{T}{g^2|\nabla u|} \right)^{3/5} \quad < \quad \eta_C \approx \frac{5}{g^4 \ln g} \quad \text{collisional viscosity (HTL weak-coupling limit)}
\]
Collisional vs. Anomalous Viscosity

<table>
<thead>
<tr>
<th>viscosity:</th>
<th>$\eta \sim \eta_A$</th>
<th>?</th>
<th>$\eta \sim \eta_C$</th>
<th>$\eta \sim \eta_{HG}$</th>
</tr>
</thead>
</table>

- relaxation rates are additive
- sumrule for viscosities:
  \[
  \frac{1}{\eta} = \frac{1}{\eta_A} + \frac{1}{\eta_C}
  \]

- smaller viscosity dominates in system with 2 viscosities!

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Viscosity of an expanding QGP #28
Summary

• Heavy-Ion collisions at RHIC have produced a state of matter which behaves similar to an ideal fluid

• a small viscosity does not necessarily imply strongly interacting matter!

  ➢ (turbulent) color fields induce an anomalous viscosity, which keeps the total shear-viscosity small during the QGP evolution

  ➢ perfect liquidity in the weak coupling limit

• note that the early Universe with its much slower isotropic expansion most likely did not possess an anomalous viscosity

  ➢ QGP @ RHIC ≠ QGP @ Big Bang!
The End