Neutrino-nucleus $\sigma$: measurements vs. calculations

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Outline:  

a) review of known weak processes at low and intermediate energies  
b) successes and difficulties  
c) relevant methods of description as the neutrino energy increases
Why we would like to know weak rates on complex nuclei at low or intermediate energies? \((E_\nu \sim 0 – 150 \text{ MeV})\)

The list is long …..

a) Nucleosynthesis (r-process in particular)
b) Description of the development prior to the core collapse (electron captures)
c) SN explosion (shock reheating by neutrinos)
d) Neutrino nucleosynthesis
e) Detection of SN neutrinos on earth
f) ……. etc., etc.
The elementary process, \( \nu \) scattering (both charged and neutral currents) on a nucleon is simple since only (or almost only) the elastic channel is open, the energy is not enough for pion production.

Thus the difficulties are associated only with the many-body aspects of the process. There are (semi)trivial aspects of it, existence of thresholds, Fermi motion, Pauli blocking and final state interactions (FSI) of the struck nucleons and outgoing charged leptons. Some of them, but not all, are analogous to issues involved in the electron scattering.

The aspects that are quite different from the well developed area of electron scattering include e.g. the presence of axial currents, parity nonconservation, emphasis on total cross section, the fact that we typically deal with two different nuclei (initial and final) etc.
• The simplest system – a single nucleon

The neutron decay and the antineutrino capture on a proton are governed by the same hadronic matrix element:

\[ n \rightarrow p + e^- + \bar{\nu}_e \quad \text{(neutron decay)} \]
\[ \bar{\nu}_e + p \rightarrow n + e^+ \quad \text{(inverse neutron decay)} \]

Knowing the neutron lifetime, \( \tau = 885.7(0.8) \), fixes the cross section for the relevant energies. The (relatively) small corrections of order \( E_{\nu}/M_p \) and \( \alpha/\pi \) can be accurately evaluated (see Vogel & Beacom, Phys. Rev. D60,053003 (1999) and Kurylov, Ramsey-Musolf & Vogel, Phys.Rev.C67,035502(2003))

In this way the cross section of the inverse neutron decay can be evaluated with the accuracy of \( \sim 0.2\% \) at these energies. (At higher energies the uncertainties in the nucleon form factors must be included.)
What about $\nu$ interaction with complex nuclei?

At lowest energies we must consider **exclusive** scattering to specific bound (or resonance) nuclear states. At somewhat higher energies we are typically interested in the **inclusive** scattering, summing over all possible nuclear final states.

The initial state is usually the nuclear ground state. However, in various astrophysics applications the temperature might be high enough that excited states are populated as well.

Essentially absent is the truly elastic (NC) scattering, never observed as yet. Note that at low energies ($E_\nu < 50$ MeV) such scattering is coherent with the maximum nuclear recoil energy $\sigma_{\text{tot}} \sim G_F^2 E_\nu^2 N^2/4\pi$, $T_{\text{max}} = 2 E_\nu^2/(M_N + 2 E_\nu)$, thus very difficult to observe.
Neutrino interaction with deuterons at low energy (SNO):

There are no bound states, the only open channel is the deuteron disintegration. Consider the CC scattering
\[ \nu_e + d \rightarrow p + p + e^- \]

The tree level cross section at low energies is
\[ \frac{d\sigma}{dE}_{\text{tree}} = \frac{2G_F^2}{\pi} V_{ud}^2 g_A^2 M_p p E_p |I(p^2)|^2, \]
where \( p \) is the relative momentum of protons
\[ p^2 = M_p(E_\nu + \Delta - E_e), \]
since \( E_\nu + M_d = E_e + 2M_p + p^2/M_p, \)
\[ \Delta = M_d - 2M_p, \] and
\[ I(p^2) = \int u_{\text{cont}}^*(pr) u_d(r) \, dr, \]
This integral depends on the pp scattering length, effective radius, and on the deuteron binding energy.
It is peaked at low values of \( p^2/M_p \) and is about 1 MeV wide in that variable.
With deuterons there are many possible reactions now:
\[ \nu_e + d \rightarrow p + p + e^- \ (CC) \]
\[ \nu + d \rightarrow n + p + n \ (NC) \]
the corresponding reactions with antineutrinos as well as
\[ p + p \rightarrow d + \nu_e + e^+ \ (pp \ in \ the \ Sun) \]
\[ p + p + e^- \rightarrow d + \nu_e \ (pep \ in \ the \ Sun) \]

For all these reactions we should also consider the two-body currents (pion exchange currents in the traditional language). In the effective field theory all corresponding unknown effects can be lumped together in one unknown parameter \( L_{1A} \) (isovector two-body axial current) that must be fixed experimentally.

The cross section is of the form
\[ \sigma(E) = a(E) + b(E)L_{1A}, \] where the functions \( a(E), b(E) \) are known, and \( b(E)L_{1A} \) contributes ~`a few’ %. 
Fixing the parameter $L_{1A}$

There are several ways to do this. One can use reactor data ($\bar{\nu}_e$CC and NC), solar luminosity + helioseismology, SNO data, and tritium beta decay. Here is what you get:

- Reactors: $3.6(5.5) \text{ fm}^3$ (Butler, Chen, Vogel)
- Helioseismology: $4.8(6.7) \text{ fm}^3$ (Brown, Butler, Guenther)
- SNO: $4.0(6.3) \text{ fm}^3$ (Chen, Heeger, Robertson)
- Tritium $\beta$ decay: $6.5(2.4) \text{ fm}^3$ (Schiavilla et al.)

All these values are consistent, but have rather large uncertainties. To reduce them substantially, one would have to measure one of these cross sections to $\sim 1\%$. This is very difficult. The two-body currents are not considered further, for heavier nuclei.
Exclusive reaction $\nu_e + ^{12}\text{C} \rightarrow ^{12}\text{N}_{\text{g.s.}} + \text{e}^-$

This is an example of a process where the cross section can be evaluated with little uncertainty. We can use the known $^{12}\text{N}$ and $^{12}\text{B}$ $\beta$ decay rate, as well as the exclusive $\mu$ capture on $^{12}\text{B}$ and the $M1$ form factor for the excitation of the analog $1^+, T=1$ state at 15.11 MeV in $^{12}\text{C}$. This fixes the cross section value for (almost) all energies, for both $\nu_e$ and $\nu_\mu$. 
\[ A=12 \text{ triad} \]

\[ \beta^+ \]

\[ Q_{\beta^+} = 16.32 \text{ MeV} \]

\[ Q_{\beta^-} = 13.37 \text{ MeV} \]
Experiment and theory agree very well, provided adjustments are made to describe the $\beta$ decay, M1 strength, $\mu$ capture etc.

Exp. results (in $10^{-42}$ cm$^2$):
9.4 $\pm 0.4 \pm 0.8$ (KARMEN $\nu_e$, 98, DAR)
8.9 $\pm 0.3 \pm 0.9$ (LSND $\nu_e$, 01, DAR)
56 $\pm 8 \pm 10$ (LSND $\nu_\mu$, 02, DIF)
10.8 $\pm 0.9 \pm 0.8$ (KARMEN, NC, DAR)

Calculations: 9.3 , 63, 10.5 (CRPA 96)
8.8 , 60.4, 9.8 (shell model, 78)
9.2 , 62.9, 9.9 (EPT , 88)

(DAR means `decay at rest' $E < 52$ MeV,
DIF means `decay in flight', $E \sim 180$ MeV )
Cross section for $^{12}$C($\nu_{\mu},\mu$)$^{12}$N$_{gs}$ in $10^{-42}$cm$^2$, see Engel et al, Phys. Rev. C54, 2740 (1996)
$\nu_e + ^{12}\text{C} \rightarrow ^{12}\text{N}^* + e^-$ (inclusive reaction)

Here the final state is not fixed and not known, one cannot use (at least not simply as before) the known weak processes to fix the parameters of the nuclear models.

The measurement is also more difficult since the experimental signature is less specific (units as before $10^{-42}\text{cm}^2$) (DAR spectrum):

- $4.3 \pm 0.4 \pm 0.6$ (LSND, 01)
- $5.7 \pm 0.6 \pm 0.6$ (LSND, 97)
- $5.1 \pm 0.6 \pm 0.5$ (KARMEN, 98)
- $3.6 \pm 2.0$ ($E225$, 92)
Evaluation of the $\sigma$ for inclusive $^{12}\text{C}(\nu_e,e^-)N^*$ with DAR spectrum

This is dominated by negative parity multipoles, calculation becomes more difficult. Here is what various people get (in $10^{-42}\text{cm}^2$):

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kolbe 95</td>
<td>5.9-6.3</td>
<td>CRPA</td>
</tr>
<tr>
<td>Singh 98</td>
<td>6.5</td>
<td>local density app.</td>
</tr>
<tr>
<td>Kolbe 99</td>
<td>5.4-5.6</td>
<td>CRPA, frac. filling</td>
</tr>
<tr>
<td>Hayes 00</td>
<td>3.8-4.1</td>
<td>SM, 3hw, extrapolated</td>
</tr>
<tr>
<td>Volpe 00</td>
<td>8.3</td>
<td>SM, 3hw</td>
</tr>
<tr>
<td>Volpe 00</td>
<td>9.1</td>
<td>QRPA</td>
</tr>
</tbody>
</table>

The agreement between different calculations, and with the experiment, is less than perfect.
More challenge, inclusive $^{12}\text{C}(\nu_\mu,\mu^-)\text{N}^*$ with DIF

Exp: LSND 02, (10.6± 0.3 ± 1.8)x10^{-40}cm^2  
Calc: 17.5 – 17.8 (Kolbe, CRPA, 99)  
16.6 ±1.4 (Singh, loc.den.app.,98)  
15.2  (Volpe, SM, 00)  
20.3  (Volpe, QRPA, 00)  
13.8  (Hayes, SM, 00)

Thus most calculations overestimate the cross section, with SM results noticeably smaller than CRPA or QRPA. The reason for that remains a mystery, at least to me.

Note: More recent calculations, e.g. Meucci et al, 2004 give 11.15 in agreement with exp. using Green’s function approach, while Maieron et al. 2003 obtain 15-20 using various approximations and FSI. Also, Nieves et al. 2004 obtain 11.9
CRPA also describes quite well electron scattering with similar momentum transfer

\[ ^{12}\text{C}(e,e') \text{ cross section} \]

\[ \text{momentum transfer [MeV/c]} \]

\[ \text{d}^2\sigma/\text{d}q\text{d}\omega [\text{n barn/} 1\text{sr MeV}] \]

\[ \omega [\text{MeV}] \]

\[ E_e = 148.5 \text{ MeV} \]

Kolbe et al., 96
Another example, reaction $^{127}\text{I}(\nu_e,e^-)^{127}\text{Xe}_{\text{bound states}}$

This was proposed by Haxton,88 as a radiochemical solar $\nu$ detection reaction, similar to $^{37}\text{Cl}$. The cross section, unlike $^{37}\text{Cl}$, was based only on difficult calculations. To calibrate it, an experiment with DAR spectrum was performed at LAMPF, giving

$= (2.75 \pm 0.84(\text{stat}) \pm 0.24(\text{syst})) \times 10^{-40} cm^2 \ (\text{Distel 02})$

Calculations, performed earlier (Engel 94) gave

$= 2.1 \ (\text{for } g_A=1.0) \text{ or } 3.1 \ (\text{for } g_A=1.26) \ .$

The agreement is good, but the experimental uncertainty is large, and the issue of $g_A$ renormalization remains unresolved.
The issue of GT quenching in the shell model (F. Nowacki, 2004, private comm.)

### β decay systematics

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>(^{128}\text{Sn})</th>
<th>(^{130}\text{Sn})</th>
<th>(^{132}\text{Sb})</th>
<th>(^{132}\text{Te})</th>
<th>(^{133}\text{Te})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transition</td>
<td>(0^+ \rightarrow 1^+)</td>
<td>(0^+ \rightarrow 1^+)</td>
<td>(4^+ \rightarrow 3, 4, 5^+)</td>
<td>(0^+ \rightarrow 1^+)</td>
<td>(\frac{3}{2}^+ \rightarrow \frac{1}{2}, \frac{3}{2}, \frac{5}{2}^+)</td>
</tr>
<tr>
<td>(T_{1/2}) exp.</td>
<td>59.07m</td>
<td>3.72m</td>
<td>2.79m</td>
<td>3.2d</td>
<td>12.5m</td>
</tr>
<tr>
<td>(T_{1/2}) calc. (0.74)</td>
<td>32.21m</td>
<td>2.47m</td>
<td>1.56m</td>
<td>1.73d</td>
<td>6.42m</td>
</tr>
<tr>
<td>Renorm.</td>
<td>0.54</td>
<td>0.6</td>
<td>0.55</td>
<td>0.54</td>
<td>0.53</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(^{134}\text{Te})</th>
<th>(^{135}\text{Xe})</th>
<th>(^{136}\text{Cs})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0^+ \rightarrow 1^+)</td>
<td>(\frac{3}{2}^+ \rightarrow \frac{1}{2}, \frac{3}{2}, \frac{5}{2}^+)</td>
<td>(5^+ \rightarrow 4, 5, 6^+)</td>
</tr>
<tr>
<td>41.8m</td>
<td>9.14h</td>
<td>13.16d</td>
</tr>
<tr>
<td>29.19m</td>
<td>7.07h</td>
<td>8.1d</td>
</tr>
<tr>
<td>0.62</td>
<td>0.63</td>
<td>0.57</td>
</tr>
</tbody>
</table>
Related process: $\mu$ capture

- Capture of the muon from an orbit $\mu^- + ZA \rightarrow \nu_\mu + (Z-1)A^*$

  In this process the momentum transfer is $|q| \sim m_\mu \sim 100 \text{ MeV}$.

  Total capture rate, which was determined for almost all stable nuclei, is thus analogous to the charged current neutrino scattering with a similar $q$.

- The average nuclear excitation energy is 10-20 MeV, and the nuclear response is dominated by the giant dipole resonance, not by the GT transitions (at least for heavier nuclei).

- RPA calculations give good agreement with the measured rate if no correction for quenching is applied to other than pure GT transitions.
FIG. 1: (color online) Comparison between the measured total muon capture rates [4] denoted by squares □, the calculated rates with all corrections (empty circles ○), and the calculated rates without the BCS and relativistic corrections (diamonds ◊). The insert, in larger scale, shows the same results for light nuclei. When the measurements are for the natural abundance of a given element, the calculation represent the corresponding combination of the individual isotopes.
FIG. 2: (color online) Ratios of the calculated and measured total muon capture rates vs. the atomic number.
At low $E_v$ (say 10-20 MeV above threshold) the shell model is an obvious method of choice.

The procedure involves several basic steps:

- Define a valence space
- Derive an effective interaction

\[ \mathcal{H}\psi = E\psi \rightarrow \mathcal{H}_{\text{eff}}\psi_{\text{eff}} = E\psi_{\text{eff}} \]

- Build and diagonalize the Hamiltonian matrix.

In principle, all the spectroscopic properties are described.
When using effective hamiltonian we should be using also effective operators. This is almost never done, at best effective charges are used.

Hilbert space
- Hamiltonian: $\mathcal{H}\Psi = E\Psi$
- Transition operator: $\langle \Psi | O | \Psi \rangle$

Valence space
- $\mathcal{H}_{\text{eff}} \Psi_{\text{eff}} = E \Psi_{\text{eff}}$
- $\langle \Psi_{\text{eff}} | O_{\text{eff}} | \Psi_{\text{eff}} \rangle$

Theory exists but it does not work well

Need some phenomenology from experimental data:
- Energies of states of (semi) magic nuclei
- Systematics of $\text{B(E2)}$ transitions
- $\text{GT} \text{ transitions}$
FIG. 1: Comparison of experimental $M1$ strength distribution $\langle B(M1) = \langle f\|O(M1)|i\rangle^2/(2J_i + 1) \rangle$ in $^{52}$Cr (bottom) with the shell-model result (top). The inset shows the decomposition into spin (bottom) and orbital (top) parts. Note the different scales of the ordinate for the spin and orbital pieces, respectively.

FIG. 2: Neutrino-nucleus cross sections, calculated from the M1 data (solid lines) and the shell-model $GT_0$ distributions (dotted) for $^{50}$Ti (multiplied by 0.1), $^{52}$Cr, and $^{54}$Fe (times 10). The long-dashed lines show the cross sections from the M1 data, corrected for possible strength outside the experimental energy window.
$^{48}\text{Ca}(p,n)^{48}\text{Sc}$ Strength Function

- **SM calc.**
- **data**

![Graph showing $B(\text{GT})$ vs. Energy (MeV) with data and SM calculation plotted.](image)
SM from Haxton 87, CRPA from Kolbe et al. 94, note that $E_\nu \sim 3T$

**FIG. 1.** Comparison of the CRPA (full lines) and shell model (dashed lines) cross sections. The upper panel is for the $\bar{\nu}_e$ induced reaction and the lower one is for the reaction induced by $\nu_e$. 
By combining CRPA which describes the initial excitation with the statistical model that describes the decay of the final continuum, one can obtain a prediction for various branching ratios:

<table>
<thead>
<tr>
<th>neutrino reaction</th>
<th>$\sigma, T = 5 \text{ MeV}$</th>
<th>$\sigma, T = 8 \text{ MeV}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>total</td>
<td>1.05 (00)</td>
<td>9.63 (00)</td>
</tr>
<tr>
<td>$^{16}\text{O}(\bar{\nu}, e^+)^{16}\text{N}(\text{gs})$</td>
<td>3.47 (-1)</td>
<td>2.15 (00)</td>
</tr>
<tr>
<td>$^{16}\text{O}(\bar{\nu}, n)^{15}\text{N}(\text{gs})$</td>
<td>5.24 (-1)</td>
<td>4.81 (00)</td>
</tr>
<tr>
<td>$^{16}\text{O}(\bar{\nu}, e^+ n \gamma)^{15}\text{N}^*$</td>
<td>1.47 (-1)</td>
<td>1.90 (00)</td>
</tr>
<tr>
<td>$^{16}\text{O}(\bar{\nu}, e^+ np)^{14}\text{C}^*$</td>
<td>4.56 (-3)</td>
<td>1.38 (-1)</td>
</tr>
<tr>
<td>$^{16}\text{O}(\bar{\nu}, e^+ nn)^{14}\text{N}^*$</td>
<td>5.50 (-3)</td>
<td>1.81 (-1)</td>
</tr>
<tr>
<td>$^{16}\text{O}(\bar{\nu}, e^+ \alpha)^{12}\text{B}^*$</td>
<td>1.07 (-2)</td>
<td>1.91 (-1)</td>
</tr>
<tr>
<td>$^{16}\text{O}(\bar{\nu}, e^+ n\alpha)^{11}\text{B}^*$</td>
<td>6.20 (-3)</td>
<td>2.16 (-1)</td>
</tr>
</tbody>
</table>
As the energy increases, the relative role of the GT $1^+$ multipole that is well described by the SM diminishes. Higher multipoles are then described by RPA.

FIG. 9. Total $(\nu_e,e^-)$ (solid) and $(\bar{\nu}_e,e^+)$ (dashed) cross sections on $^{40}$Ar, calculated within the RPA approach. The short-dashed line shows the forbidden contributions to the $(\nu_e,e^-)$ cross sections.
From general considerations one can identify three different energy ranges with different demands on the details with which the nuclear structure should be treated:

i) For relatively low neutrino energies, comparable with the nuclear excitation energy, the model of choice is the nuclear shell model. The shell model calculations are indeed able to reproduce the allowed (Fermi, Gamow-Teller) response.

ii) The Random Phase Approximation (RPA) has been developed to describe the collective excitation of a nucleus. The RPA is the methods of choice at intermediate energies where the reaction rate is sensitive dominantly to the total strength and the energy of the giant resonances.

iii) At yet higher incoming energies neutrinos scatter `quasi-freely' on individual nucleons. The most popular way of including Fermi motion, Pauli blocking and approximately binding effects is the Relativistic Fermi Gas model. However, more sophisticated approaches that include residual interaction also exist.
Comparison of Fermi gas model (full lines) and the CRPA (dashed lines)
Brief comment on angular distribution:

As neutrino energy increases, the typical CC quasi-elastic angular distribution turns more and more towards forward angles, as expected.
Conclusions on comparison of cross sections:

1) Comparison between calculated and measured cross sections on complex nuclei is sporadic due to lack of data.

2) Different calculations agree on a ~30% level for the energies relevant for Supernovae.

3) However, if some (even small) number of parameters could be adjusted (as for $^{12}\text{C} \rightarrow ^{12}\text{N}_{\text{gs.}}$) a much better description will likely result.

4) We need more and accurate data (perhaps coming from SNS soon), and more theoretical activity.