Lepton-Nucleus Cross Sections in the Energy Range 150-700 MeV

Motivation
Nuclear Models
Effects of Nuclear Correlations
Benchmark Data: C(e,e')X Saclay ALS
Comparison: Theory and Experiment
Extensions to Neutrino Scattering
Summary

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Motivation

• A simple truth in neutrino physics:

\[
\text{observed event rate} = \text{(cross section)} \times \text{(flux of neutrinos)} \times \text{(detection efficiency)}
\]

Contains everything interesting: oscillation physics, exotic event rates, cross sections, etc.

The subject of this talk
MiniBooNE Neutrino Cross Section Predictions

- MiniBooNE was designed to utilize the quasi elastic neutrino-$^{12}$C charged current reaction to study neutrino oscillations.

- The L/E of the LSND result was centered in L/E around 0.5-1.4 meter/MeV.

- This corresponds to neutrino energies in the range 350 MeV to 1000 MeV at the MiniBooNE distance of 500 meters.
A “first principle” calculation is too difficult

- In systems of $A \gtrsim 10$ nucleons it is not (yet) possible to perform exact, many-body, field theory calculations. MiniBooNE uses Carbon, $A=12$… oh well…

- There are several different approaches commonly seen: shell model, TDA, RPA, shell model, etc…

- Here, I employ a simple “Fermi Gas motivated” momentum-space spectral function description, which seems to work remarkably (better than it should!). It can be corrected for nuclear effects and extended to higher energies.
LSND Excited State $\nu_\mu^{-12}\text{C Events}$

- The only “precision” measurement of quasi elastic neutrino scattering on Carbon

\[ \nu_\mu^{12}\text{C} \rightarrow \mu^{-12}N^* \]

\[ \langle \sigma \rangle_{\text{meas}} = (10.5 \pm 1.6) \times 10^{-40} \text{cm}^2 \]

This is 40-50% lower than expected!
Neutrino and Electron Scattering on $^{12}\text{C}$

Time Tested Strategy:
Use electron scattering to measure the response of the nucleus.

\[ C^* \left\{ C(15.11) \right\} \]

$^{12}\text{C}$

$^{12}\text{N}$

$\nu_e^{12}\text{C} \rightarrow e^- N$

$\nu_\mu^{12}\text{C} \rightarrow \mu^- N$

$e^{-12}\text{C} \rightarrow e^- 12\text{C}(15.11)$

$e^{-12}\text{C} \rightarrow e^- 12\text{C}^*$
Moments of Initial Nucleon Momenta

\[
f(k, q, \omega) = \frac{M_T \Omega}{(2\pi)^3} \frac{\delta(\omega + \epsilon_k + U(|k|) - \epsilon_{|k-q|} - U(|k-q|)) n_i(k)}{\epsilon_k \epsilon_{|k-q|}} m_f(1 - n_f(|k-q|))
\]

Electron scattering:

\[
\{a_1, \ldots, a_5\} = \int d^3k f(k, q, \omega) \{1, \frac{|k|^2}{M^2}, \frac{|k|^2 \cos^2 \tau}{M^2}, \frac{\epsilon_k^2}{M^2}, \frac{\epsilon_k |k| \cos^\tau}{M^2}\}
\]

Neutrino scattering:

\[
a_{1, \ldots, 7}(|q|^2, \omega) = \int d^3k f(k, q, \omega) \{1, \frac{|k|^2}{M^2}, \frac{|k|^2 \cos^2 \tau}{M^2}, \frac{\epsilon_k^2}{M^2}, \frac{\epsilon_k |k| \cos^\tau}{M^2}, \frac{|k| \cos^\tau}{M}, \frac{\epsilon_k}{M}\}
\]
Some Important Nuclear Effects

• Finite system effects
  – There are only 12 nucleons in carbon
  – Start with harmonic oscillator ground state momentum distributions

• Non-local nuclear potential
  – Also known as long range correlations
  – Due to strongly attractive scalar field (σ)
  – Implies a dispersion relation for the nucleon

• Short range correlations
  – Due to the nucleon’s hard core, i.e. repulsive vector forces (ω)
  – Causes high momentum tails to nucleon momentum distributions
Finite System with $A=12$

- Use a harmonic oscillator momentum distribution as a starting point

$$n(k) = \left(1 + \frac{3}{4} \left(\frac{k}{k_0}\right)^2\right) e^{-\left(\frac{k}{k_0}\right)^2}$$

$k_0$ controls the oscillator strength $\sim 122$ MeV/c
Long Range Correlations

Non-local nuclear potential form:
(M is the nucleon mass, k is the nucleon momentum)

\[ U(|k|) = U_0 \left(1 - a \left( \frac{k}{M} \right)^2 - b \left( \frac{k}{M} \right)^4 + \cdots \right) \approx \frac{U_0}{\left(1 + a \left( \frac{k}{M} \right)^2 + b \left( \frac{k}{M} \right)^4 \right)} \]

This fractional form has nice properties:

- approximates a scalar interaction (\(\sigma\) field)
- gives correct dispersive behavior at low k
- goes to zero at high momentum
- \(a \sim 7.9, \, b \sim 10, \, M\) is nucleon mass
Effects of Short Range Correlations

Pandharipande et. al., Phys Rev C 46, 1741 (1992), fig 4
Short Range Correlations cont.

- Dominant effect is high momentum tail in $n(k)$

$$n(k) = (1 - f_{\text{tail}}) n_{ho}(k) + f_{\text{tail}} \ n_{\text{tail}}(k)$$

$$n(k) = \left( 1 + \frac{3}{4} \left( \frac{k}{k_0} \right)^2 \right) e^{-\left( \frac{k}{k_0} \right)^2}$$

$$n_{\text{tail}}(k) = \frac{N_{\text{tail}}}{1 + \left( \frac{k}{k_{\text{tail}}} \right)^4}.$$ 

- $k_{\text{tail}} \approx 350 \text{ MeV/c}$, $f_{\text{tail}} \approx 0.30$, $N_{\text{tail}}$ normalization

Note: It is vital to include the recoil energy in the integration over $k$. 

Our Benchmark: ALS (Saclay) Data
Electron energies 150 MeV->680 MeV
Model 0 (naive)

• Simple Fermi Gas (sphere)
• No binding energy
• No nuclear potential effects
• Impulse approximation
Model 0 (naive)

\[ \frac{d\sigma}{d\Omega/d\omega} \text{ (nb/MeV/str)} \]

\[ 36^\circ \]

Theory

\[ \omega \text{ (MeV)} \]

\[ 680 \text{ MeV Experiment} \]

\[ 680 \text{ MeV Theory} \]
Model 0 (naive)

480 MeV, 60°
Model 0 (naive)

280 MeV, 145°

\[ \frac{d\sigma}{d\Omega/d\omega} \text{ (nb/MeV/str)} \]

\( \omega \) (MeV)

\( \omega \) (MeV)

Theory

280 MeV Experiment

280 MeV Theory
Model 1

- Simple Fermi Gas (sphere)
  - Include binding energy
- No nuclear potential
- Impulse approximation
Model 1 (better)

680 MeV, 36°

\[ \frac{d\sigma}{d\Omega/d\omega} \text{ (nb/MeV/str)} \]
Model 1 (better)

Theory

480 MeV, 60°
Model 1 (better)

280 MeV, 145°

\[ \frac{d\sigma}{d\Omega/d\omega} \text{ (nb/MeV/str)} \]

\( \omega \) (MeV)
Model 2

- Harmonic oscillator momentum distribution
- Include binding energy
- ‘Realistic’ nuclear potential
Model 2 (good)

\[
\frac{d\sigma}{d\Omega}/d\omega \text{ (nb/MeV/str)}
\]

\[
\omega \text{ (MeV)}
\]

680 MeV Experiment
680 MeV Theory
Model 2 (good)

\[ \frac{d\sigma}{d\Omega}/d\omega \text{ (nb/MeV/str)} \]

480 MeV, 60°
Model 2 (good)

280 MeV, 145°

Theory

280 MeV Experimental
280 MeV Theory

$d\sigma/d\Omega/d\omega$ (nb/MeV/str)

$\omega$ (MeV)
Neutrino-\(^{12}\)C Interactions

\[ B^* \rightarrow e^\nu_{e}^{12}C \]

\[ B_{gs} \rightarrow e^\nu_{e}^{12}C \]

\[ C(15.11) \rightarrow e^-^{12}C \]

\[ 13.370 \text{ MeV} \]

\[ \bar{\nu}_e^{12}C \rightarrow e^+ B \]

\[ \bar{\nu}_\mu^{12}C \rightarrow \mu^+ B \]

\[ 15.11 \text{ MeV} \]

\[ 12C \rightarrow e^-^{12}C(15.11) \]

\[ 15.11 \text{ MeV} \]

\[ e^-^{12}C \rightarrow e^-^{12}C^* \]

\[ 17.344 \text{ MeV} \]

\[ \nu_e^{12}C \rightarrow e^- N \]

\[ \nu_\mu^{12}C \rightarrow \mu^- N \]

\[ N^* \rightarrow p^{11}C \]

\[ N_{gs} \rightarrow e^\nu_e^{12}C \]
LSND Excited State $\nu_{\mu}^{-12}\text{C}$ Events

- The only “precision” measurement of quasi elastic neutrino scattering on Carbon

$$\nu_{\mu}^{12}\text{C} \rightarrow \mu^{-12}\text{N}^*$$

$$\langle \sigma \rangle_{\text{meas}} = (10.5 \pm 1.6) \times 10^{-40} \text{ cm}^2$$

This is 40-50% lower than expected!
Result of “Model 2”-like calculation for LSND inclusive

Nuclear Corrections:
- effective mass of nucleon
- binding energy change

The LSND data prefers:

\[ \frac{M}{M^*} \sim 1.4 \]
\[ E_b \sim 15.11 + 12 \text{ MeV} \approx 27.1 \text{ MeV} \]
( this corresponds to 25 MeV in electron scattering )
Conclusions
(quasi elastic neutrino scattering)

• “Model 2” seems to provide a “simple intuitive solution” to LSND’s cross section puzzle

• Simple nuclear effects are important for quasi elastic neutrino processes

• Provides a good anchor point for MiniBooNE…. but MiniBooNE requires cross sections up to ~1 GeV
Extension to Higher Energies: Model 3

- Harmonic oscillator momentum distribution
- Include binding energy
- ‘Realistic’ nuclear potential
- Add short range correlation effects (high momentum)
- Add Delta resonance
Model 3 (very good)

![Graph showing the relationship between \( \omega \) (MeV) and \( d\sigma/d\Omega/d\omega \) (nb/MeV/str). The graph includes data points and a theoretical curve. The data points are labeled as 680 MeV, 36° Data, and the theoretical curve is labeled as 680 MeV, 36° Theory. The graph is labeled with the x-axis as \( \omega \) (MeV) ranging from 0 to 500 and the y-axis as \( d\sigma/d\Omega/d\omega \) (nb/MeV/str) ranging from 0 to 30. The graph also includes a note that the 36° T is 680 MeV.]
Model 3 (very good)

\[ \frac{d\sigma}{d\Omega/d\omega} \text{ (nb/MeV/str)} \]

\[ \omega \text{ (MeV)} \]

480 MeV, 60° Data

480 MeV, 60° Theory
Model 3 (very good)

\[ \frac{d\sigma}{d\Omega d\omega} \text{ (nb/MeV/str)} \]

- 280 MeV, 145° Data
- 280 MeV, 145° Theory
TBD…

- Extend Model III to neutrinos, mostly done but needs finishing touches
- Comparison with other ee’ data, other A?
The End…
Antineutrino Running

• Good agreement with electron scattering data is not enough! Determines only the Vector part of the cross section.

• Axial part of weak current is not present in electron scattering, however the axial part can be extracted in an unambiguous way from a combination of neutrino and anti-neutrino data at MiniBooNE
Axial Part of Cross Section

\[ \left( \frac{d^2\sigma}{d\Omega_2 d\epsilon_2} \right)_{\nu/\bar{\nu}} = \frac{G_F^2 \cos^2 \theta_c k_2^2 \cos^2(\frac{\chi}{2})}{2\pi^2 M_T} \]

\[ \left\{ W_2 + \left[ 2W_1 + \frac{m_i^2}{M_T^2} W_\alpha \right] \tan^2(\frac{\chi}{2}) + \frac{(W_\beta \pm W_8) m_i^2}{M_T \epsilon_2 \cos^2(\frac{\chi}{2})} \right\}^{1/2} \]

\[ + 2 \left( \frac{W_8}{M_T} \right) \tan(\frac{\chi}{2}) \sec(\frac{\chi}{2}) \left[ q^2 \cos^2(\frac{\chi}{2}) + |q|^2 \sin^2(\frac{\chi}{2}) + m_i^2 \right]^{1/2} \]

Difference of neutrino and anti-neutrino cross section gives axial terms directly:

\[ W_8 \sim \left( a_7(|q|, \omega) + \frac{\omega}{|q|} a_6(|q|, \omega) \right) F_A(q^2) \left( F_1(q^2) + F_2(q^2) \right) \]