An introduction to AdS/CFT

D. T. Son (INT)

3 aspects of this talk :

- Thermodynamics of strongly coupled plasma
  ($\mathcal{N} = 4$ supersymmetric Yang-Mills theory)

- Viscosity at strong coupling

- Attempt on building hadron models

Not in this talk :

high-energy scattering
jet quenching
} today afternoon

. . . . . .

Some literature:

Horowitz and Polchinski "Gauge/gravity duality" gr-qc/0602037

general, philosophical, basic ideas

Klebanov's TASI lectures "Introduction to AdS/CFT correspondence" hep-th/0009139

genesis of the idea, computation of entropy correlators

Gorsky "Gauge theories as string theory: the first results" hep-th/0602184

Wilson loop, viscosity

but mostly on anomalous dimensions

AdS space: 2D illustration

Sphere in projective coordinates

$$ds^2 = \frac{dx^2 + dy^2}{(1 + x^2 + y^2)^2}$$

Euclidean AdS_2 $\quad ds^2 = \frac{dx^2 + dy^2}{(1 - x^2 - y^2)^2}$ $\quad$ constant negative curvature

$$z + i\tau = \frac{1 - x - iy}{1 + x + iy} \Rightarrow ds^2 = \frac{dz^2 + d\tau^2}{z^2}$$

In more than 2 dimensions, Minkowski signature

$$ds^2 = \frac{R^2}{z^2}\left(-dt^2 + d\vec{x}^{\,2} + dz^2\right)$$

↑

"warp factor"

Original AdS/CFT correspondence:

between $\mathcal{N}=4$ SYM

Maldacena
Gubser, Klebanov, Polyakov
Witten

and type IIB string theory on $AdS_5 \times S^5$

$$ds^2 = \underbrace{\frac{R^2}{z^2}\left(dx^2 + dz^2\right)}_{AdS_5} + \underbrace{R^2\, d\Omega_5^2}_{S_5}$$

(this is a solution to Einstein's equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}$$

$$F_{\mu}{}^{\alpha_1\alpha_2\alpha_3\alpha_4} F_{\nu\alpha_1\alpha_2\alpha_3\alpha_4} \quad)$$

Large 't Hooft limit $\longleftrightarrow$ small curvature

$$g^2 N_c \to \infty \qquad \longleftrightarrow \qquad \sqrt{\alpha'} R \gg 1$$

R/ℓ_s $=$

correlation functions are computable

string theory $\to$ supergravity

The dictionary of gauge/gravity duality:

gauge theory	gravity
operator $\hat{\mathcal{O}}$	field ϕ
$T_{\mu\nu}$	graviton $h_{\mu\nu}$
dimension of operator	mass of field
global symmetry	gauge symmetry
conserved current	gauge field
anomaly	Chern-Simons term
......	....

$$\int e^{iS_{4D} + \phi_0 \mathcal{O}} = \int_{\substack{\phi \to \phi_0 \\ z \to 0}} e^{iS_{5D}}$$

$\lim_{z \to 0} \phi(x,z) = \phi_0(x)$

partition fn of 4D theory with source

partition fn of 5D theory with nontrivial boundary condition

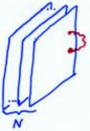
ORIGIN OF THE IDEA :

Consider type IIB string theory in (9+1)D

contains : strings (including graviton, dilaton...)

Dp-branes $p = 1, 3, 5, 7$

Stack N D3 branes on top of each others



Fluctuations of the branes are described by $\mathcal{N}=4$ super Yang-Mills theory fields = open strings

$N \gg 1$: space is curved and the metric is known

gravitons, dilatons....

= closed strings

The two pictures are two different description of the same object

Type II B string theory on $AdS_5 \times S^5$ $\Leftrightarrow$ $\mathcal{N} = 4$ SYM in flat spacetime

a conformal field theory

Mathematically:

$$Z[J] = \int \mathcal{D}\phi \dots \; e^{iS[\phi] + i\int J\phi} \quad = e^{iW[J]}$$

$$W[J] = S_{cl}[\varphi_{cl}] \leftarrow \text{classical action}$$

φ_{cl} solves eq. of motion $\quad \varphi_{cl}\big|_{z\to 0} \to J(x)$

Example: current-current correlator (R-charge current)

corresponds to gauge field on 5D

$$\partial_z\left(\frac{\partial_z A_\perp(z,q)}{z}\right) - \frac{q^2}{z}A_\perp = 0 \;\Rightarrow \qquad A_z = 0$$

$$A_\perp(z,q) = Qz\, K_1(Qz)\, \underbrace{A_\perp(0,q)}_{\text{Source.}}$$

$$\langle j^\mu j^\nu \rangle \sim \frac{\delta^2 S_{\text{Maxwell}}}{\delta^2 A_\perp(0)}$$

$$= \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\frac{1}{z}\frac{\partial}{\partial z}\Big[\underbrace{Qz\,K_1(Qz)}_{1 + Q^2z^2\ln(Qz)}\Big]\Big|_{z\to 0}$$

$$= (g^{\mu\nu}q^2 - q^\mu q^\nu)\ln Q^2 \qquad \text{conformal field theory}$$

PLASMA THERMODYNAMICS

AND BLACK HOLES

A reminder on GR and black holes

Metric $\quad ds^2 = g_{\mu\nu}(x)\, dx^\mu dx^\nu$

Einstein equation $\quad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}$

Ricci tensor
$\sim$ curvature $\partial^2 g_{\mu\nu}$

stress-energy
of matter

Example: Schwarzschild black hole
is a solution with $T_{\mu\nu} = 0$

$$ds^2 = -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 \underbrace{\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)}_{d\Omega_2^2}$$

Properties:

$r \to \infty$ flat space

$r = r_0 = 2GM$ metric is singular, but curvature is finite

this is the black hole horizon

Behavior of the metric near horizon :

$$r - r_0 = \frac{\rho^2}{4r_0} \;\Rightarrow\; ds^2 = -\frac{\rho^2}{4r_0^2}\,dt^2 + d\rho^2 + r_0^2 d\Omega_2^2$$

this is simply a Minkowski version of polar coordinates

$$ds^2 = d\rho^2 + \rho^2 d\varphi^2 \qquad \varphi = \frac{it}{2r_0}$$

has no curvature singularity.

Hawking temperature :

$$\varphi \sim \varphi + 2\pi$$

Euclidean time $\quad it \sim it + \underbrace{4\pi r_0}_{\beta = \frac{1}{T}}$

$$T_H = \frac{1}{4\pi r_0}$$

Black hole entropy

$$E = M = \frac{r_0}{2G}$$

$$dS = \frac{dE}{T} = 4\pi r_0 \cdot \frac{dr_0}{2G}$$

$$S = \frac{\pi r_0^2}{G} = \frac{A}{4G}$$

Finite-temperature AdS/CFT correspondence:

black 3-brane solution:

$$ds^2 = \frac{r^2}{R^2}\left(-f(r)dt^2 + d\vec{x}^2\right) + \frac{R^2}{r^2 f(r)}dr^2 + R^2 d\Omega_5^2$$

$$f(r) = 1 - \frac{r_0^4}{r^4}$$

corresponds to

$\mathcal{N}=4$ SYM at temperature

$$T = T_H = \frac{r_0}{\pi R^2}$$

$$\text{Entropy} = \frac{A}{4G}$$

A: area of event horizon $r = r_0$

G: 10d Newton constant

$$A = \int dx\, dy\, dz \sqrt{g_{xx} g_{yy} g_{zz}} \times \pi^3 R^5$$

$\pi^3 R^5$: area of S^5

$$= V_{3D}\, \pi^3 r_0^3 R^2 \sim T^3$$

$$G \sim \frac{R^8}{N^2}$$

$$\Rightarrow S = \# N^2 T^3$$

$\#$: coefficent $= \frac{3}{4} \times$ (free gas)

HYDRODYNAMICS

FROM BLACK HOLE PHYSICS

Idea : use gauge/gravity duality to investigate the hydrodynamic regime of field theory

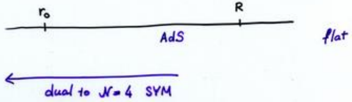
finite-T QFT $\Longleftrightarrow$ black hole with translationally invariant horizon

"black brane"

Example :

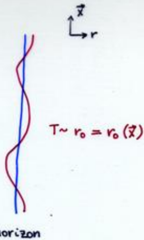
$$ds^2 = H^{-1/2}\left(-f\,dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{1}{H^{1/2}}\left(\frac{dr^2}{f} + r^2 d\Omega_5^2\right)$$

$H = 1 + \frac{R^4}{r^4}$ $\qquad f = 1 - \frac{r_0^4}{r^4}$ $\qquad r_0 \ll R$



Hawking temperature $\quad T = \frac{r_0}{\pi R^2}$

Dynamics of flat horizons:



Generalizing black hole thermodynamics $M, Q \ldots$

to black brane <u>hydrodynamics</u>

$$T = T_H(\vec{x}) \;, \quad \mu = \mu(\vec{x}) \quad \ldots$$

Event horizons behave as vicous fluids

$$S = \frac{\text{Area of horizon}}{4G}$$

Bekenstein
Hawking

$\mathcal{N}=4$ SYM : $S(g^2N \to \infty) = \frac{3}{4} S(g^2N \to 0)$

What is viscosity from the point of view of gravity?

Viscosity : Kubo's formula

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d\vec{x}\, e^{i\omega t} \langle [T_{xy}(t,\vec{x}),\ T_{xy}(0,\vec{0})] \rangle$$

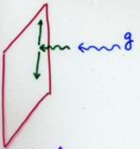
$$= -\lim_{\omega \to 0} \lim_{\vec{q} \to 0} \frac{1}{\omega} \mathrm{Im}\, G^{R}_{xy,xy}(\omega, \vec{q})$$

retarded Green's function of T_{xy}

Similar relations exist for other kinetic coefficients (diffusion constants, conductivities ...)

Gravity counterpart of Kubo's formula:

AdS/CFT "dictionary"



Coupling: $h_{\mu\nu} \, T_{\mu\nu}$

bulk graviton

boundary stress energy

1997 Klebanov: absorption of a graviton falling at right angle to the black brane

$\kappa = \sqrt{8\pi G}$

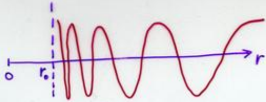
$$\sigma_{abs} = -\frac{2\kappa^2}{\omega} \operatorname{Im} G^R(\omega)$$

$$= \frac{\kappa^2}{\omega} \int d^4x \; e^{i\omega t} \langle [T_{xy}(x), T_{xy}(0)] \rangle$$

Viscosity = absorption cross section of low-energy gravitons

$$\eta = \frac{\sigma_{abs}(0)}{2\kappa^2} = \frac{\sigma_{abs}(0)}{16\pi G}$$

Absorption cross section can be found classically



$\leftarrow$ incoming waves

$$``\Box h_{xy}" = 0$$

$$h''_{xy} + \frac{5r^4 - r_0^4}{r(r^4 - r_0^4)} h'_{xy} + \omega^2 \frac{r^4(r^4 + R^4)}{(r^4 - r_0^4)^2} h_{xy} = 0$$

The computation of σ_{abs} is made easy by 2 theorems, valid for a wide class of back ground:

- Equation for h_{xy} is the same as of a minimally coupled scalar
- For a minimally coupled scalar

$$\lim_{\omega \to 0} \sigma_{abs}(0) = \text{Area of event horizon}$$

Das, Gibbons, Mathur

Consequences of 2 theorems:

$$\eta = \frac{\sigma_{abs}(\omega \to 0)}{16\pi G} = \frac{A}{16\pi G}$$

$$S = \frac{A}{4G}$$

$$\Rightarrow \quad \frac{\eta}{S} = \frac{1}{4\pi}$$

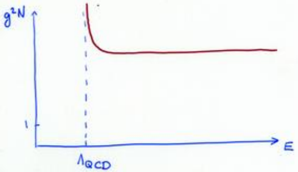
An attempt to build a model of hadrons

So far we have consider $\mathcal{N}=4$ SYM theory which is a conformal field theory but ....

Many more examples of gauge/gravity duality have been discovered

including theories with confinement and chiral symmetry breaking

To be able to compute using string theory: strong gauge coupling



consequence : 2 different scales

$$\Lambda_{QCD} \ll \frac{1}{\sqrt{\alpha'}}$$

masses of lowest hadrons

Spin ≤ 2

string excitations

arbitrary spin

Approach to QCD

- top-down : start from $\mathcal{N}=4$ SYM

  adding perturbations to break supersymmetry, conformal invariance ...

  ⇒ theories which sometimes have features similar to QCD

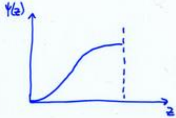
- Bottom-up :

  Start from what we know from phenomenology

  Build a model based on the qualitative features

  We will see that the simplest model works better than one has any right to expect.

"HADRONIC PHYSICS"

4D	5D
hadron	normalizable mode $\psi(z)$
	
hadron mass^2	eigenvalue of a 5D operator
decay constant $\langle 0 \vert J_\mu \vert \rho \rangle$	$\lim_{z \to 0} \frac{\psi'(z)}{z^{\#}}$
meson coupling g_{ABC}	overlap integral $\int \psi_A(z) \psi_B(z) \psi_C(z)$

We want a holographic model which exhibits some properties of QCD

For lack of imagination, we take the metric to be AdS_5

$$ds^2 = \frac{R^2}{z^2}\left(dt^2 - d\vec{x}^2 - dz^2\right)$$

will set $R = 1$.

"conformal window"

Confinement: spacetime ends at some radius

will truncate AdS_5 at some $z = z_m$

and impose some boundary conditions (e.g., Neuman) at $z = z_m$

- Chiral symmetry: two sets of conserved currents in QCD

$$J^\mu_L = \bar{q}\gamma^\mu \frac{1-\gamma^5}{2} q, \qquad J^\mu_R = \bar{q}\gamma^\mu \frac{1+\gamma^5}{2} q$$

$\Rightarrow$ two bulk 5D gauge field.

$$A^L_\mu \qquad A^R_\mu$$

(recall AdS/CFT: operator in 4D theory corresponds to field in 5D theory)

conserved currents $\longleftrightarrow$ massless gauge fields

- Chiral symmetry breaking: in QCD characterized by the chiral condensate $\langle \bar{q}_R q_L \rangle$

$$\langle \bar{q}_R^{\alpha} q_L^{\beta} \rangle \sim \delta^{\alpha\beta} \qquad \alpha, \beta = 1 \dots N_f$$

$\Rightarrow$ in 5D we introduce bulk scalar $X^{\alpha\beta}$ bifundamental with respect to $SU(N_f) \times SU(N_f)$

Mass of X: $\quad \Delta(\Delta - 4) = m_{5D}^2 \qquad R = 1$

In QCD dimension of $\bar{q}q$ is $\Delta = 3$

ignore anomalous dimension

$$\Rightarrow m_X^2 = -3$$

$\chi SB \Rightarrow$ non zero vev for X. $\quad \langle X \rangle = X_0(z)$

Small z: $\quad X_0^{\alpha\beta}(z) = \frac{1}{2}\left(m_q z + \sigma z^3\right) \delta^{\alpha\beta}$

explicit symmetry breaking by quark masses

spontaneous χSB by chiral condensate

The model:

$$S = \int d^5x \, \sqrt{g} \, \mathrm{Tr}\left[|D_\mu X|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right]$$

$$D_\mu X = \partial_\mu X - i A_\mu^L X + i X A_\mu^R \qquad \begin{array}{l} F_L = dA_L + A_L^2 \\ F_R = dA_R + A_R^2 \end{array}$$

on truncated AdS $\quad ds^2 = \frac{1}{z}\left(dt^2 - d\vec{x}^2 - dz^2\right)$

$$0 \leq z \leq z_m$$

The X background should arise dynamically, but we simply take $\quad X = \frac{1}{2}(m_q z + \sigma z^3)\,\mathbb{1}$

Impose b.c. on $z = z_m$ ("infrared brane")

$$F_{z\mu} = 0 \qquad z = z_m$$

$D_z X = 0$ (in practice $X = X_0 \Sigma \quad \Sigma \in SU(2)$
$D_z \Sigma = 0$)

4 free parameters $\quad m_q \quad \sigma \quad z_m \quad g_5$

cf. 3 in QCD $\quad m_q, \Lambda_{QCD}, N_c$

Using rules of AdS/CFT correspondence we can compute current-current correlator and match the large $Q^2 \gg m_\rho^2$ behavior with QCD $\ln Q^2$

$\Rightarrow$ get rid of one free parameter

$$g_5^2 = \frac{12\pi^2}{N_c}$$

3 free parameters:

- m_q
- confinement scale
- χSB scale

Result:

	Exp	Model A	Model B
m_π	140	140^*	140
m_ρ	776	776^*	800
m_{a_1}	1230	1363	1223
f_π	92.4	92.4^*	85
$F_\rho^{1/2}$	345	329	340
$F_{a_1}^{1/2}$	433	452	437
$g_{\rho\pi\pi}$	6.0	4.5	5.1

$z_m = (333\ \text{MeV})^{-1}$

$6 = 301\ \text{MeV}$