Toward the theory of strongly coupled Quark-Gluon Plasma (sQGP)

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outlook

- Why do we think we have to understand strongly coupled quark-gluon plasma (sQGP)?
- **New spectroscopy** at T>Tc: mesons, colored pairs, baryons, polymers, or multiparticle clusters?
- **Topology and confinement:**
  - instantons => Nc dyons => flux tubes => large potentials at T > Tc?
- classical strongly coupled non-Abelian plasma and its first Mol. Dynamics
- Summary
Why do we think that QGP is strongly coupled at RHIC?

• 1a: Because hydro works well, viscosity is very low: $\eta/s = 0.1 - 0.2 \ll (pQCD \text{ pred.})$ (Teaney, ES, Heinz, Kolb…01-…)

• 1b: Because parton cascade requires very large cross sections $\gg (pQCD \text{ pred})$ (Molnar-Gyulassy)

  (a comment: they are not the same $\Rightarrow$ a cascade makes no sense in a strongly coupled regime, while hydro only works better)

• 1c: charm diffusion: $D_c \ll pQCD$ also (from $R_{AA}$,v2 of electrons at RHIC, Moore+Teaney, Molnar…)

• 1d: very strong jet quenching, including charm, again well beyond $pQCD$, no Casimir scaling…
Why do we think that QGP is strongly coupled at RHIC?

• 2a: Because at $3T_c > T > T_c$ the interaction is strong enough (Karsch et al, 04) to make multiple bound states (ES+Zahed, 03), most of them colored. Hadron-like states lead to hadron-size cross sections.

• 2b: Marginal states with small binding may lead to resonance cross sections (ES+Zahed, 03), a la Feshbach resonances => trapped atoms make a near perfect liquid as well (sorry no time on that).

• 2c: Baryons survive till about $1.6T_c$ and seem to get very heavy rapidly: important for dense cold QGP.

• 2d: Polymeric chains or possibly local quasi-crystalline clusters?

• (even if their fraction is small, the effect on transport can be large => kevlar etc)
3: N=4 SUSY YM theory at strong coupling at finite T is very similar to sQGP at RHIC

• 3a: p,e=O( N^2 T^4) and even the famous coefficient .8 is better reproduced by the large-g series (not small-g) (Klebanov...96,02)
• 3b: viscosity is small: \( \eta/s = 1/4\pi \) (Son et al,03)
• 3d: quasiparticles are heavy M>>T while their lightest bound states have M=O(T) and can be excited (ES+Zahed,03 recent: charmonium at strong coupling)
• (3e: maybe one can work out a complete gravity dual to RHIC, ES,Sin+Zahed, in progress)
Flows and transport properties at RHIC
Elliptic flow with ultracold trapped Li$^6$ atoms, $a \to \infty$ regime

The system is extremely dilute, but can be put into a hydro regime, with an elliptic flow, if it is specially tuned into a strong coupling regime via the so called Feshbach resonance.

Similar mechanism was proposed (Zahed and myself) for QGP, in which a pair of quasiparticles is in resonance with their bound state at the “zero binding lines”.

The coolest thing on Earth, $T=10$ nK or $10^{-12}$ eV can actually produce a Micro-Bang! (O’Hara et al, Duke)
2001-2004: hydro describes radial and elliptic flows for all secondaries (pt<2GeV) because QGP is a nearly ideal liquid $v_2 = <\cos(2\phi)>$

**PHENIX,** Nucl-ex/0410003

red lines are for ES+Lauret+Teaney done before RHIC data, never changed or fitted, describes SPS data as well! It does so because of the correct hadronic matter /freezout via (RQMD)
Sonic boom from quenched jets

- The energy deposited by jets into liquid-like strongly coupled QGP must go into conical shock waves.
- We solved relativistic hydrodynamics and got the flow picture.
- If there are start and end points, there are two spheres and a cone tangent to both.

Wake effect or “sonic boom”
PHENIX jet pair distribution

Note: it is only projection of a cone on phi

Note 2: more recent data from STAR find also a minimum in \( \langle p_t(\phi) \rangle \) at 180 degr., with a value Consistent with background
Summary on flows

- Radial and elliptic flow systematics $v_2(m,s,y,pt,A)$ is explained by hydro
- Energy from quenched jets goes to conical flow (the Mach cone direction)
- The magnitude of the effect provides bounds on viscosity $\eta/s < .2$ or so
  ($>>1$ in weakly coupled plasma)
Binary and Multi-body states in sQGP
2003: lines of zero meson binding appeared on the QCD Phase Diagram

Are regions of meson binding the divider between wQGP and sQGP?
(ES+I.Zahed, "rethinking" paper PRC 2003, the beginnig of sQGP...):
Potential energy of a static dipole at $T=(1-2)T_c$ and $r=0.5-1$ fm is even larger than for $T=0$ (lattice $N_f=2$, from O. Kaczmarek’s talk)
How good actually is deconfinement?
Potential energy at large distance (lattice Nf=2, from O.Kaczmarek’s talk)

Simple messages:
• $|U|/T = O(10) >> 1$
• Do not even think about expanding $\exp(-U/T)$ ...

• as 2 objects are produced at large $r$, $U(r=\infty) = 2M_{\text{eff}}$
• $M_{\text{eff}}$ larger than const. quark mass
Solving for the bound states
ES+I.Zahed, hep-ph/0403127

- In QGP there is no confinement => Hundreds of colored channels SHOULD have bound states as well!
- From \((16+2*2*Nf*3)^2 \sim 1000\) binary channels about 1/3 the strongest

<table>
<thead>
<tr>
<th>channel</th>
<th>rep.</th>
<th>charge factor</th>
<th>no. of states</th>
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<tr>
<td>(gg)</td>
<td>1</td>
<td>9/4</td>
<td>(8_s)</td>
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<tr>
<td>(gg)</td>
<td>8</td>
<td>9/8</td>
<td>(9_s \times 16)</td>
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<tr>
<td>(qg + \bar{q}q)</td>
<td>3</td>
<td>9/8</td>
<td>(3_c \times 6_s \times 2 \times N_f)</td>
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<tr>
<td>(qg + \bar{q}g)</td>
<td>6</td>
<td>3/8</td>
<td>(6_c \times 6_s \times 2 \times N_f)</td>
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<tr>
<td>(\bar{q}q)</td>
<td>1</td>
<td>1</td>
<td>(8_s \times N_f^2)</td>
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<tr>
<td>(qq + \bar{q}q)</td>
<td>3</td>
<td>1/2</td>
<td>(4_s \times 3_c \times 2 \times N_f^2)</td>
</tr>
</tbody>
</table>

- \(gg\) color 8*8=64=27+2*10+2*8+1: only the 2 color octets \((gg)_8\) have \((16\times3_s \times 3_s = 144)\) states.
2004: With such potentials not only usual mesons but many colored binary states should also exist! ES+Zahed, PRD 2004

First attempt to resolve the long-standing pressure puzzle! (quasiparticles are too heavy to get the right pressure)

2\textit{M}_q(T), 2\textit{M}_g(T) fitted to (Karsch et al.) quasiparticle masses, as well as example of “old” \textit{M}_\pi(T) and “new” octet \textit{M}^8_{gg}(T)

The QGP pressure: crosses are lattice thermodynamics for $N_f = 2$ (Bielefeld, 2000), the lines represent the contributions of $q + g$ quasiparticles, “mesons” $\pi - \rho\ldots$, colored exotics ($gg_8, gg_3$) and total (the upper curve).
Baryons at $T>T_c$ ? Polymer chains?
J.Liao+ES,05

- Lattice favors "potential-like" (b) behavior of the potentials for baryons.
  - $(a)= V(1j)+V(2j)+V(3j)$ (j=junction)
  - $(b)=(V(12)+V(13)+V(23))(1/2)$ Casimir
  - Fortunately $\langle V \rangle (a)$ and $\langle V \rangle (b)$ do not differ by more than 15%!

- polymeric chains $\bar{q} \ g \ldots \ gq$, with color changing

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**FIG. 1.** The interaction in baryons for "string-like" interaction (a) versus the "potential-like" interaction (b). The double circles with different colors (online) in (c) represent gluons, and it is an example of 4-chain $\bar{q}ggq$. 
Bindings from variational calculation for baryons and ggg chain (J.Liao+ES,05)

<table>
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<tr>
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<th>$T_m$</th>
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<tr>
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<tr>
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<td>1/2</td>
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<td>1.4</td>
</tr>
<tr>
<td>$ggq$ (closed chain)</td>
<td>3</td>
<td>1/2</td>
<td>-1.10</td>
<td>1.6</td>
</tr>
</tbody>
</table>
higher baryonic susceptibilities show a ``peak” and a ``wiggle”

Baryons: Fading or Gaining weight?


\[ d_n(T) = \frac{\partial^n (p/T^4)}{\partial (\mu/T)^n} \bigg|_{\mu=0} = n! c_n(T) \]
Baryons, not quarks dominate \( d_4 \) and \( d_6 \)

\[
\hat{\partial}^2 \mu I / \hat{\partial}^2 \mu = ((2I_3)/B)^2
\]

Derivatives work like this:
- For quarks \( d_\text{In}/d_\text{n}=1 \)
- For \( N \) and \( \Delta^+ \), \( \Delta^0=1/9 \)
- For \( \Delta^{++} \) and \( \Delta^{--}=1 \)
- For 4 \( N \) and 16 \( \Delta \) = .466

(Liao+ES, hep-ph/05....)

FIG. 6. The susceptibilities ratios \( d_4^I/d_4 \) (the thin solid) and \( d_6^I/d_6 \) (the thick solid). The dashed lines correspond to ideal quark gas (upper) and ideal baryonic gas (lower).
Unlike colored objects, such as q, qg, qq etc, baryons (N...) should evolve through the QCD phase transition continuously. Their mass must grow into the sQGP side to 3 Meff. This generates large T and mu derivatives!

FIG. 5. Masses of various states studied in this work. The thin solid line is for quark and the dashed line is twice quark mass which is roughly for quark-gluon and diquark. The lower thick solid line is for nucleon states and the upper one for Δ states. These masses are used for calculation of Fig.7.

NOTE the inflection point at which M'' changes sign.
If so, the contribution of Baryons (N, Delta) reproduce the peak in $d_4$ and a wiggle in $d_6$.

The wiggle due to baryon mass dependence with a inflection point $M''$ changes sign.

Of course, direct study of the baryon correlators would be better than any susceptibility….
Entropy associated with static dipole, at large $r$ vs $T/T_c$ (Kaczmarek’s talk)

- $S/\text{charge} = O(10)$
- $N(\text{states}) = \exp(10)$

What those states may be?

- B-type mesons: $2 \times N_f = 6$
- Qg states: another 6
- (string picture) $\Rightarrow$ qbar g g g.. gQ polymer
  - $6^n$ states or $n = 5-6$
- mini-crystal of 6 closest cubic cell? $(2 \times N_f)^6$ is about $e^{(10)}$
Confinement and topology at $T > T_c$
Instanton liquid
(see review T. Schafer and ES, RMP 98)

• Soft hadronic physics and chiral dynamics are dominated by the lowest Dirac eigenstates =>
• Those are treated as being composed out of (linear combination of) instanton zero modes
• Predicted/reproduced many mesonic/baryonic correlation functions calculated on the lattice
• Among many predictions, large scalar diquark binding about 350 MeV (new lattice data!)
• Among many other applications, m(quark) dependence for its small values: one of the objectives of this program. => Long quark loops in QCD vacuum are due to them
What is the role for topology at $T>T_c$, above chiral restoration/deconfinement?

⇒ Transition from instanton liquid at $T<T_c$ to $Q=0$ clusters, such as instanton-antiinstanton molecules (Ilgenfritz, ES, 93 and later works)

⇒ Why is chiral symmetry restored, even in quenched QCD?

⇒ any relation to confinement/deconfinement?
Caloron = baryon made of $N_c$ "instanton quarks"

KvBLL solutions:
T. C. Kraan and P. van Baal, K. Lee and C. Lu [1998]

New solutions with the top. charge $Q=1$, but
With the nonzero holonomy
(A_0 or Higgs vev)
=>
(long suspected because there are 4 $N_c$ collective coordinates and monopoles have 4)

Now it has brane derivation
Those are seen in lattice configurations
(From Ilgenfritz’ talk here)

Non-trivial $Q = 1$ caloron: Spectral flow (upper left), IPR of zero-mode (breathing, upper right), action density (bottom left) and zero-mode density (bottom right), for three $z$’s corresponding to maximal localization.
QGP with monopoles

- Dual superconductivity as a confinement mechanism (‘tHooft, Mandelstam 1980’s) require monopole condensation $v=\langle \phi \rangle$
- But maybe we better look at $T>T_c$ and study dyon dynamics without their condensation
- Lorentz force on monopoles makes them reflect from a region with E, or even rotate around the E flux $\Rightarrow$ compresses E into flux tubes, even in classical plasma!
Can a flux tube exist without a dual superconductor?

- Here are magnetic flux tubes at the Sun,
- where classical electrons rotate around it
- B: about 1 kG,
- Lifetime: few months
More about monopoles

\[ Z = \prod dx_a s \sqrt{\det g_{ab}(x)} \exp(-\int mg_{ab}\dot{x}_a\dot{x}_b dt)^2) \]

- **metric of moduli space** for monopoles is known
- It leads to very nontrivial geometry/classical dynamics (Atiya, Hitchin, Taub…)
- which is consistent with Lorentz O(velocity^2)- terms in S at large r (Giddens, Manton) one can get from Maxwell eqns.
- Electric/magnetic Coulomb forces are in the volume
- One can attempt to do quantum dynamics now: \``monopolium\" and \``baryons\", more… (Ilgenfritz showed a chain of 6-chain \``cooled\" from su(2) lattice)
- Fermionic 3d zero modes \=> a light quark can travel through space \``on the back of a monopole\" for free!
More on monopoles

• \( g(\text{el}) g(\text{mag}) = \text{const (Dirac)} \)
• Thus at high \( T \) \( g(\text{el}) \ll 1 \) and \( g(\text{mag}) \gg 1 \) while near \( T_c \) they gets interchanged and monopoles get "liberated"
• This phenomenon studied in detail in N=2 SUSY (Seiberg-Witten)
• Electric-magnetic duality including solutions got very nice brane-based explanation
New approach:
Classical QGP and its Molecular Dynamics
(B.Gelman, ES, I. Zahed, in progress)
• For $SU(2)$ charge $Q$ is a unit vector, $\vec{Q} = (Q^1, Q^2, Q^3)$

\[
\begin{align*}
\frac{dx_i}{dt} &= \frac{p_i}{m}, \\
\frac{dp_i}{dt} &= \left(\frac{g^2}{4\pi}\right) \sum \vec{Q}_i \cdot \vec{Q}_j / r_{ij}^2, \\
\frac{d\vec{Q}_i}{dt} &= \left(\frac{g^2}{4\pi}\right) \sum \vec{Q}_i \times \vec{Q}_j / |r_{ij}|
\end{align*}
\]

• Note: $d\vec{Q}_i^2 / dt = 0$

**Wong eqn** can be rewritten as x-p canonical pairs, 1 pair for $SU(2)$, 3 for $SU(3)$, (as a so called Darboux variables).

We do $su(2) \Rightarrow C$ is a unit vector on a sphere $O(3)$
If $<\text{kin}> \ll |\text{pot}|$ we expect quarks (gluons) to freeze into the minimal energy state

- This is of course for $\pm$ Abelian charges,
- But "green" and "anti-green" quarks do the same!

And such local order would be preserved in a liquid also, as it is in molten solts (strongly coupled TCP with $<\text{pot}>/ <\text{kin}> = O(60)$)
Structure factor for cQGP

- $G_d$ correlation function for $\Gamma = 0.83, 31.3, 131$, respectively; red circles correspond to $t^* = 0$, and blue squares correspond to $t^* = 6$
- $\Gamma = 0.83$ is a weak correlation between the particles; relaxes rather quickly with time
- The correlation is more robust for $\Gamma = 31.3$ (liquid)
- For $\Gamma = 131$ correlation is very stable (solid)
cQGP made of 64 colored particles, projection of a cube on x-y plane, red is the path of particle #1.

strong coupling, Gamma is about 100 close to freezing

Color $\rightarrow$ red
Transport in a strongly coupled plasma in one slide

- Gamma = \( <\text{pot.energy}> / <\text{kin.energy}> \)
- Diffusion constant decreases with interaction strength – less mobility
- Viscosity first decreases then grows (in glass/solid) due to "transverse phonons" transport

![](chart.png)
Self-diffusion

\[ D(\tau) = \frac{1}{3N} \left\langle \sum_{i=1}^{N} \vec{v}_i(\tau) \cdot \vec{v}_i(0) \right\rangle \]

\[ D = \int_{0}^{\infty} D(\tau) \, d\tau \]

\[ D \approx \frac{0.4}{\Gamma^{4/5}} \]
Shear viscosity

- Green-Kubo relation for viscosity

\[ \eta = \int_0^\infty \eta(\tau) \, d\tau \]

\[ \eta(\tau) = \frac{1}{3 \, TV} \left\langle \sum_{x<y} \sigma_{xy}(\tau) \, \sigma_{xy}(0) \right\rangle \]

\( \sum_{x<y} \) — a sum over the three pairs of distinct tensor components \((xy, yz\) and \(zx)\); the stress-energy tensor are given by

\[ \sigma_{xy} = \sum_{i=1}^N m_i v_{ix} v_{iy} + \frac{1}{2} \sum_{i \neq j} r_{ij, x} F_{ij, y} \]

\( F_{ij} \) is the force on particle \( i \) due to particle \( j \)
First results on viscosity:

QGP (blue arrow) is about the best liquid one can possibly make

- Stress-tensor autocorrelation correlation function $\eta(t)$ for $\Gamma = 0.83, 3.13, 13.1$

$$\eta \approx 0.001 \Gamma + \frac{0.242}{\Gamma^{0.3}} + \frac{0.072}{\Gamma^2}$$
Translating it all to QGP

Comparison with sQGP in $T = 1.5 - 3 T_c$

$$V_{\text{eff}} = \frac{\hbar^2}{2mr^2} - \frac{C \alpha_s}{r}$$

$$r_0 = \frac{\hbar^2}{mC\alpha_s} = \lambda$$

$$\langle \alpha_s C \rangle = 1$$

$$\lambda = r_0 \approx \frac{1}{3T}$$

$$n \approx (0.244 T^3) (8 + 6 N_f) \approx 6.3 T^3$$

$$a_{WS} = \left(\frac{3}{4\pi n}\right)^{1/3} \approx \frac{1}{3T} \approx \lambda$$

$$\tau_0 = \omega_p^{-1} = \left(\frac{4\pi n \langle \alpha_s C \rangle}{m}\right)^{-1} \approx \frac{1}{5.1 T}$$

$$\lambda \approx \frac{1}{3T}$$

$$\tau_0 \approx \frac{1}{5.1 T}$$

$$m \approx 3 T; \Gamma \approx 3$$

$$\eta \approx 7.8 T^3, s \approx 20 T^3; \eta/s \approx 0.34$$

$$D \approx 0.1/T$$
Conclusions

- Strongly coupled QGP (not a gas of weakly coupled partons) has been produced at SPS/RHIC
- Robust collective flows, even for charm =>
- Strong jet quenching and even sonic boom from quenched jets
- “New spectroscopy”: Strong potentials, large entropy at $T=(1-2)T_c$
- Many binary bound states, mesonic, colored, s-wave baryons, polymeric chains
- "QGP with monopoles"
- What role do they play in pre-confinement?
- Fermionic modes?

- Classical QGP and its MD:
  - a strongly coupled liquid with local color and crystalline order can be studied in real time
The first look at \(<\text{pot}>/\text{<kin}>\) in sQGP

\[
\Gamma = \frac{\langle \text{pot} \rangle}{\langle \text{kin} \rangle} = \langle \frac{(\text{Casimir})\alpha_s}{r} \rangle / T
\]

\[
\Gamma = (\text{Casimir})\alpha_s[N(\text{d.o.f.})]^{(1/3)}N(\text{corr})
\]

- Casimir=\((4/3)\) for qq, 3 for gg
- \(N(\text{d.o.f.})=2(N_c^2-1)+2*2*N_c*N_f\)
- \# of quasiparticles, about \(50>>1\)
- \(N(\text{corr})=1\) (pair), 2 in a polymeric chain, 12 in a fcc cubic crystal to be discussed

\[
\Rightarrow \quad \Gamma \approx 10 \quad \text{even for } \alpha_s=O(1)
\]
Theoretical sQGP in N=4 SUSY YM and AdS/CFT
A gift by the string theorists: AdS/CFT correspondence, the only way to calculate at VERY strong coupling

- The $\mathcal{N}=4$ SUSY Yang Mills gauge theory is conformal (CFT) (the coupling does not run). At finite $T$ it is a QGP phase at ANY coupling. If it is weak it is like high-T QCD $\Rightarrow$ gas of quasiparticles. What is it like when the coupling gets strong $\lambda = g^2 N_c \gg 1$?
- AdS/CFT correspondence by Maldacena turned the strongly coupled gauge theories to a classical problem of gravity in 10 dimensions
- Example: a modified Coulomb's law (by Maldacena)
  \[ V(L) = -\frac{4\pi^2}{\Gamma(1/4)^4} \frac{\sqrt{\lambda}}{L} \]
  becomes a screened potential at finite $T$
Bound states in AdS/CFT
(ES and Zahed, PRD 2004)

• The quasiparticles are heavy
  \( M_q = O(\sqrt{\lambda} \, T) \gg T \)
• But, while levels fall on a center, there are light binary
  bound states with the mass = \( O(M_q/\sqrt{\lambda}) \approx O(T) \) with the orbital momentum
  \( l = O(\lambda) \). They are colored \( N_c^2 \) and out of them
  the matter is made of!
• New development (e.g. Strassler et al., "charmonium
  from the 5-th dimension"): there is no falling to
  the center. The s-wave \( (l=0) \) states exist
  and have exactly \( M = O(M_q/\sqrt{\lambda}) \)
QCD vs CFT: let us start with EoS (The famous .8 explained!)

- CFT free energy at large $\lambda$ is $F = (3/4 + O(1/\lambda^{3/2}))F_{\text{free}}$ (I. Klebanov et al 1996...)

- Lattice results (Bielefeld group) for QCD thermodynamics: pressure normalized to Stephan-Boltzmann value

- Weak (5 terms) vs. strong $(3/4 + \text{const}/\lambda^{3/2})$ coupling for the CFT: the ratio of the pressure to Stephan-Boltzmann value vs the 't Hooft coupling $\lambda = g^2 N$. 
D. Son et al calculated viscosity via Kubo formula from $\langle T_{ij}(x)T_{ij}(y)\rangle$

$$\eta/s = 1/(4\pi)$$

- D3 is the brane (our space), x, y are on it
- T is provided by "black brane" via Howking radiation (Witten 98)
- Correlator is a graviton propagator,
- Blue g does not contribute to Im G, but the red g does.
A complete "gravity dual" for RHIC from 10-d GR? (ES, Sin, Zahed, in progress)

- Black Holes + Howking rad. Is used to mimic the finite T (Witten, 98)
- How black hole is produced can be calculated from GR (tHooft … Nastase)
- Entropy production => black hole formation, falling into it is viscosity
- Moving b.h. => hydro expansion
Additional slides
Effective $\alpha_s = O(1)$ (not < .3 as in in all $T=0$ applications)