Resonances and Lattice QCD

David Richards (Jefferson Laboratory/LHPC)

- Motivation
- Review
- Recipe
  - Variational Method
  - Group-theoretical methods
  - Numerical implementation
- Exploratory Tests
- Conclusions and Future Plans

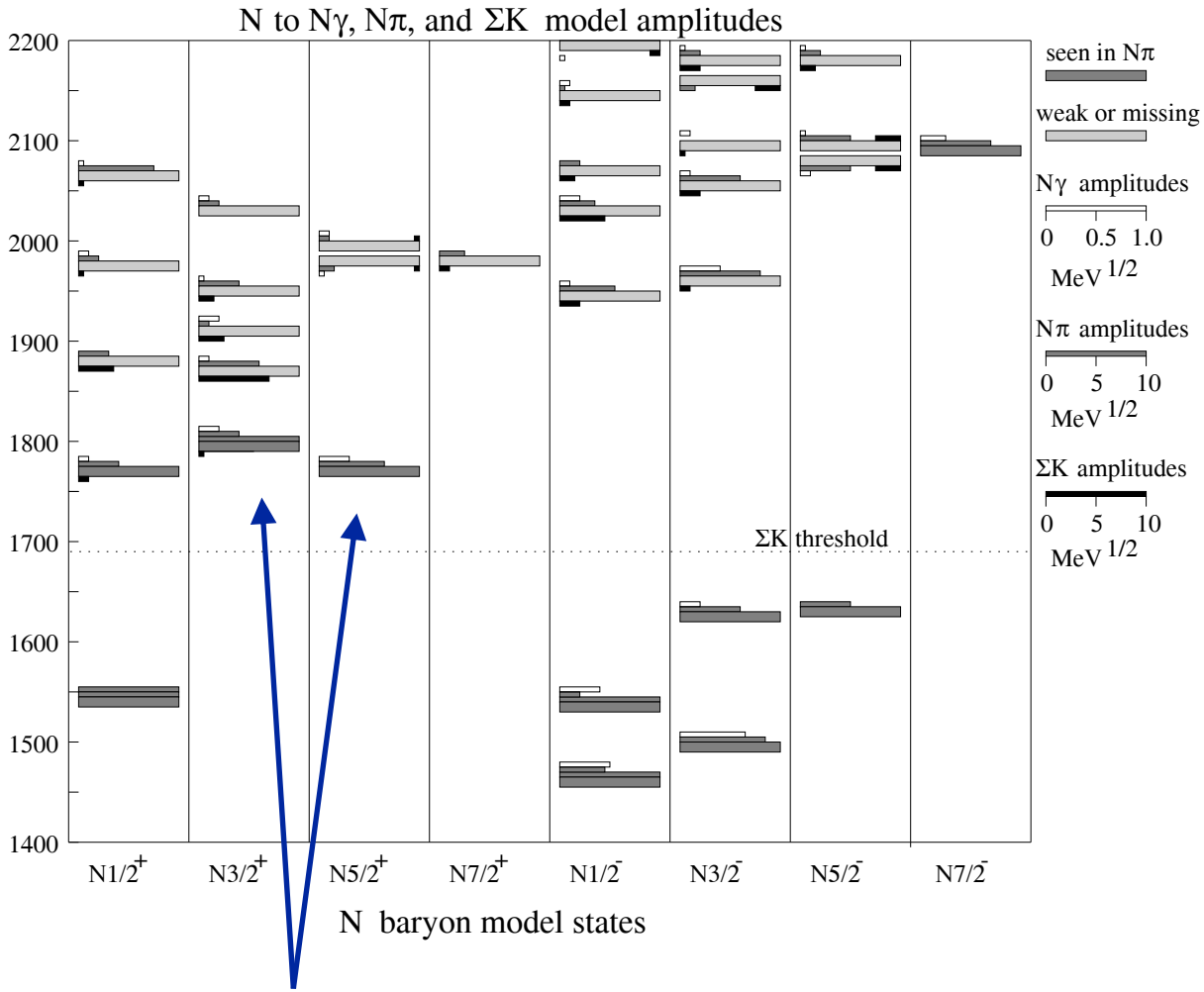
LHP Collaboration

- *S. Basak, J. Dudek, R. Edwards, G. Fleming, U. Heller, J. Juge, A. Lichtl, N. Mathur, C. Morningstar, DGR, I. Sato, S. Wallace*

Spectroscopy and QCD

- **Spectroscopy is classic tool for gleaning information about structure of theory**
- **Both experimental and ab initio N* and Hybrid programs aim at discovering effective degrees of freedom of QCD, and resolving competing low-energy models:**
  - **Valence quark models with harmonic-oscillator potential cannot easily accommodate Roper and $\Lambda(1405)^-$**
  - **Quark models predict a richer spectrum of missing resonances than observed experimentally**
  - **Diquark-quark picture of baryons – *fewer degress of freedom***
  - **Chiral soliton models**

Isospin-1/2 N* Sector

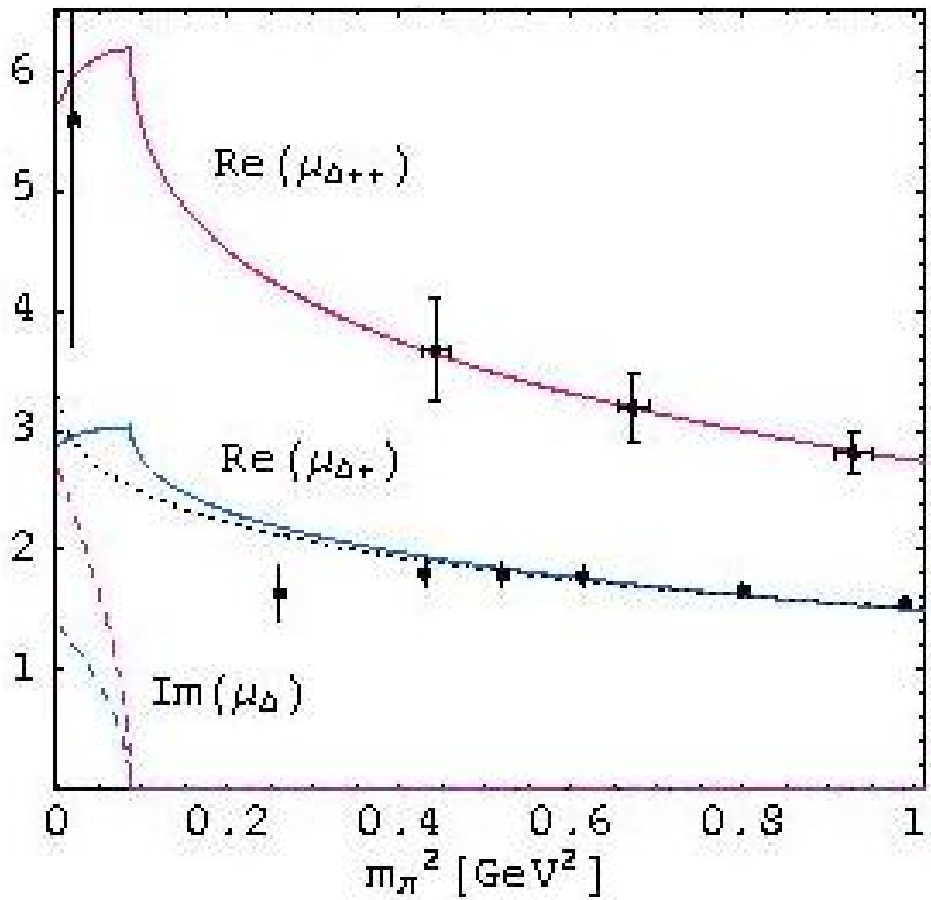


Capstick and Roberts, PRD58 (1998) 074011

Note ordering….

Spectroscopy review - I

One baryon resonance studied for many years…..

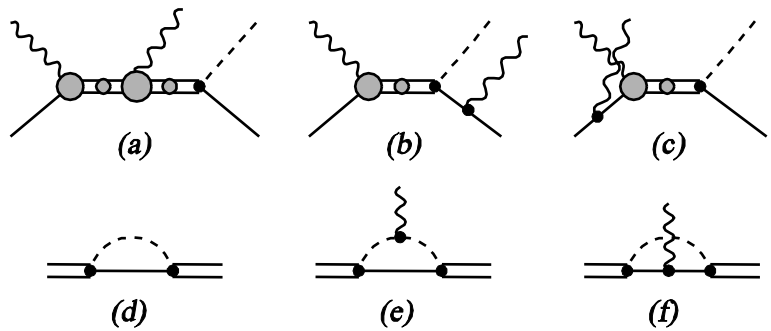


Lattice points from

Leinweber (1992)

Cloet,Leinweber,Thomas (2003)

Lee *et.al*. (2004)



Chiral calculations

Pascalutsa, Vanderhaeghen (2004)

Thomas, Young (…)

Spectroscopy Review - II

- **First generation calculations – largely for quarks masses around that of *strange quark;***
- **Computations performed in quenched approximation to QCD.**
- **Masses of lowest lying baryon of each parity for J = ½, 3/2 for N, Δ, Σ, Λ**
- **Computations performed using a variety of fermion actions – different O(a) discretisation errors. But only limited investigation of systematic errors.**
- **Computations use simple three-quark interpolating fields - *insensitive to excited glue, multiquark states***

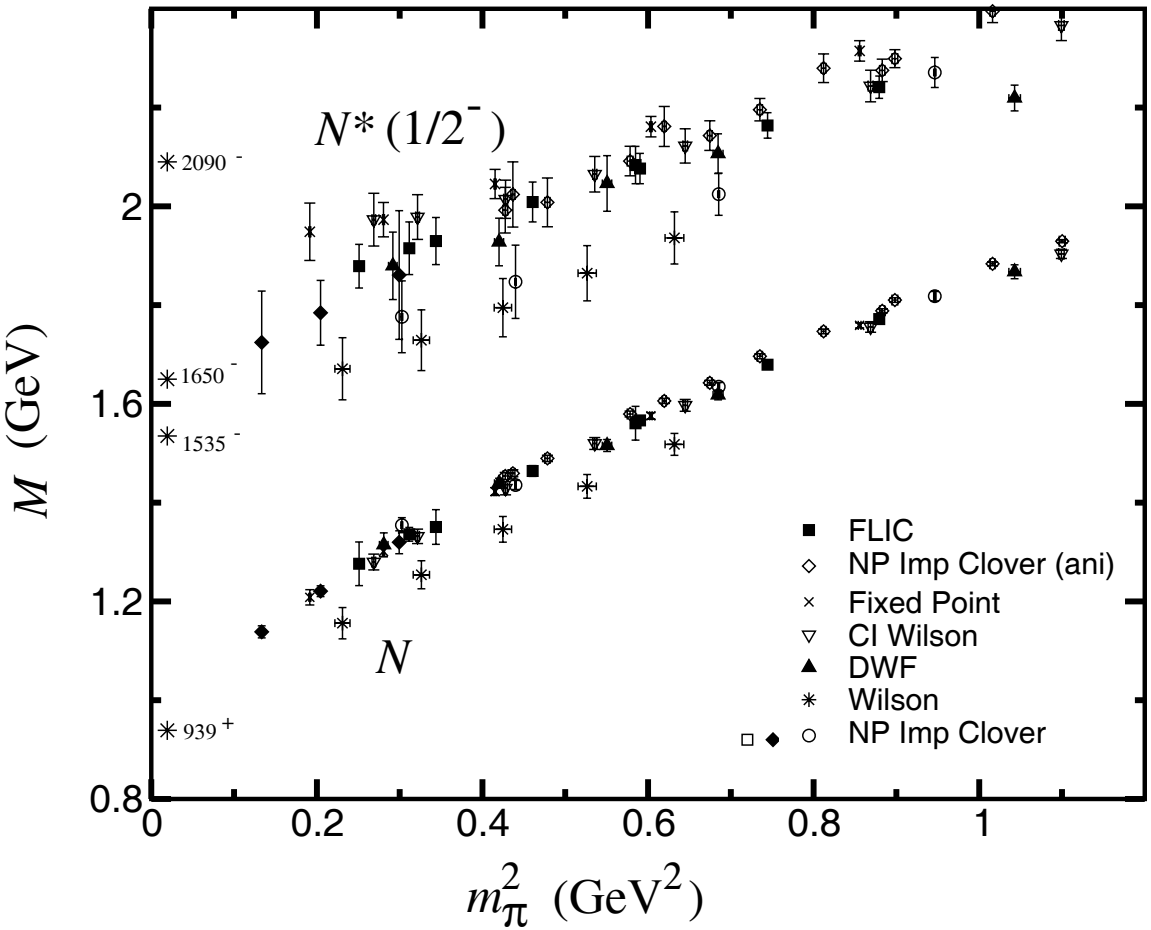
Parity Partner - I

Different symbols

↕

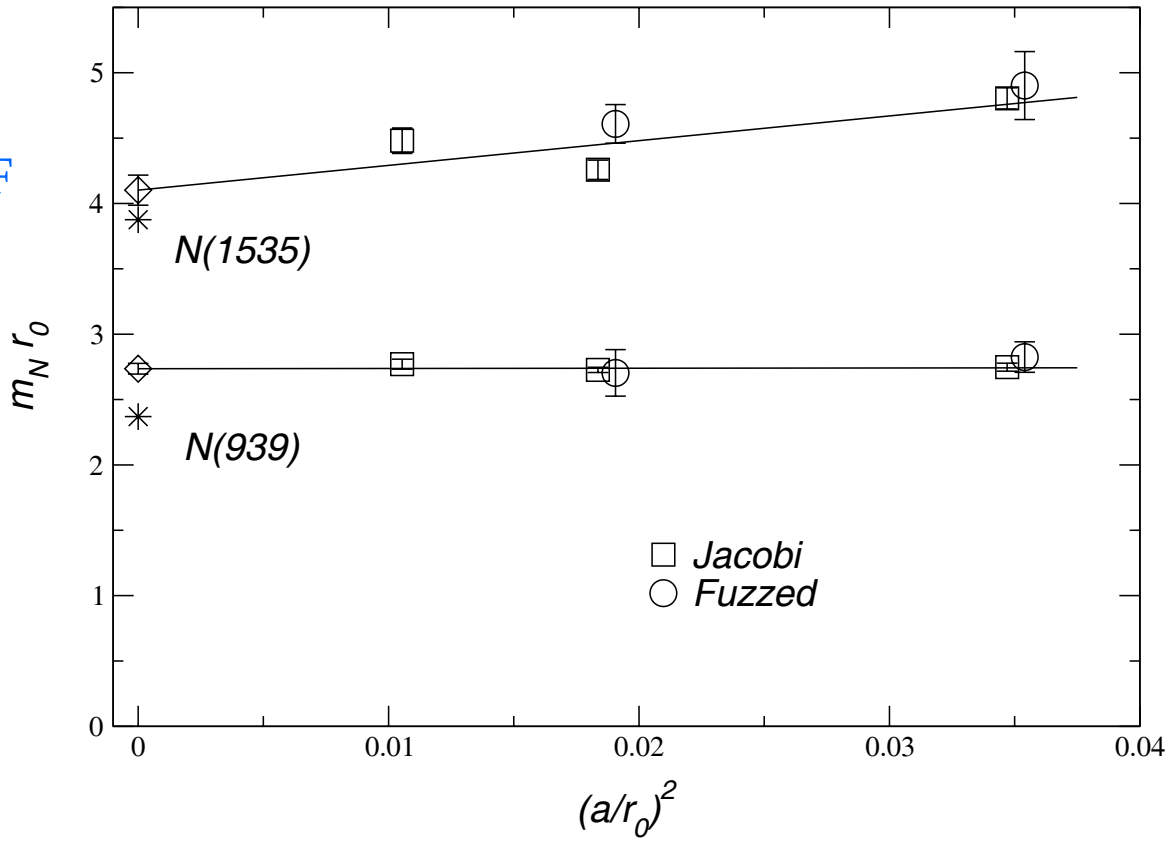
Different actions

Nucl-th/0406032



Parity partner - II

LHPC/UKQCD/QCDSF
PLB532, 63 (2002)

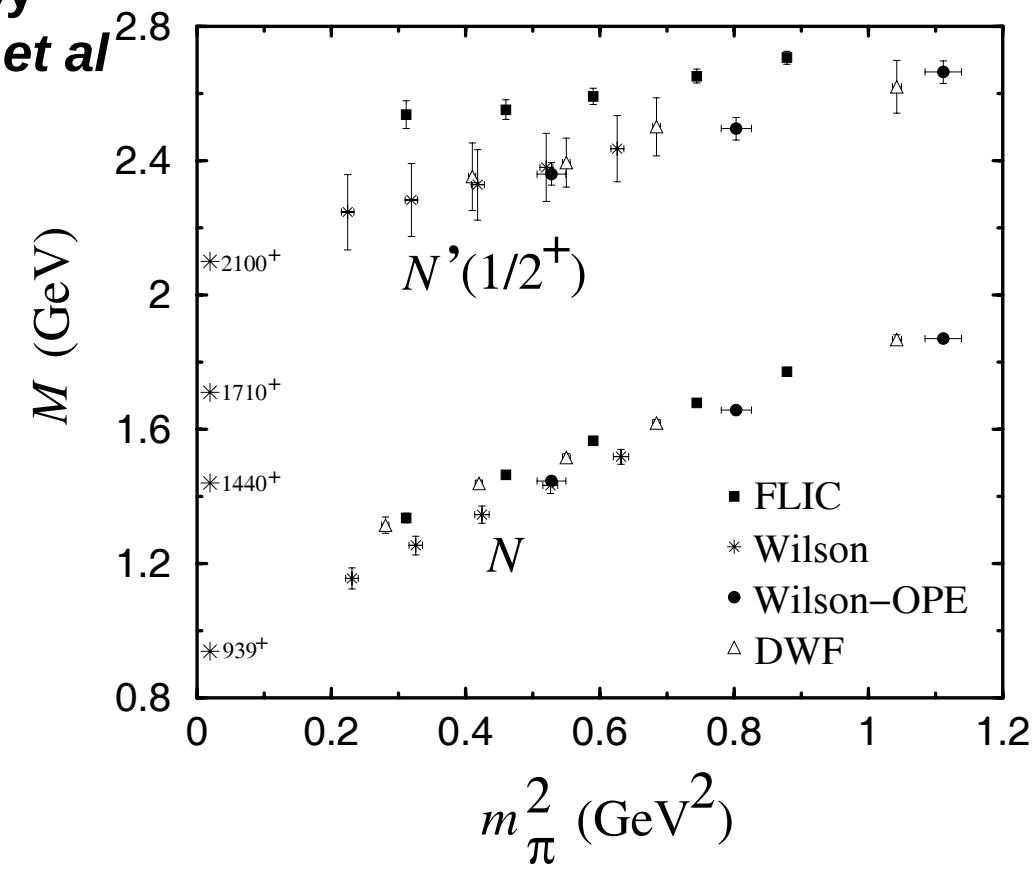


Nucleon Radial Excitation

Comparison of results by Adelaide group, Zanotti *et al*
PRD67, 114506 (2003).

$$O_{N'} = (uCd)\gamma_5 u$$

"Bad" nucleon operator – vanishes in NR limit.



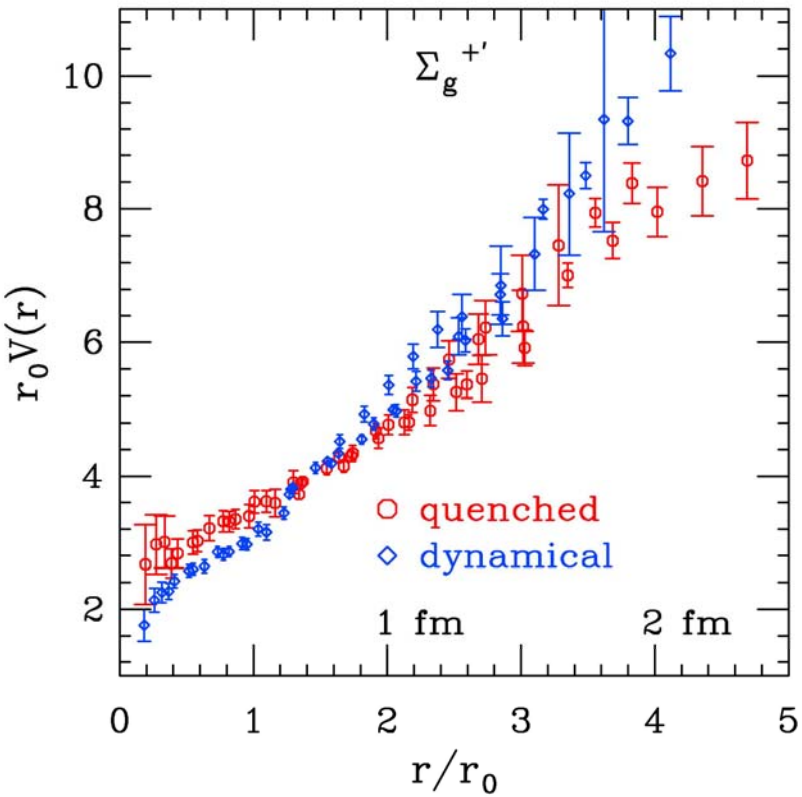
Need to extract radial and higher spin excitations

Hybrid Resonance Spectrum - I

- **Spectrum of hybrids composed of heavy quarks has been important lattice focus**
  - **Adiabatic Potentials**
  - **Spectrum of charmonium**

- **1 GeV Excitation Energy associated with excited string**
  - **1^{-+} lightest hybrid**

MILC Collaboration

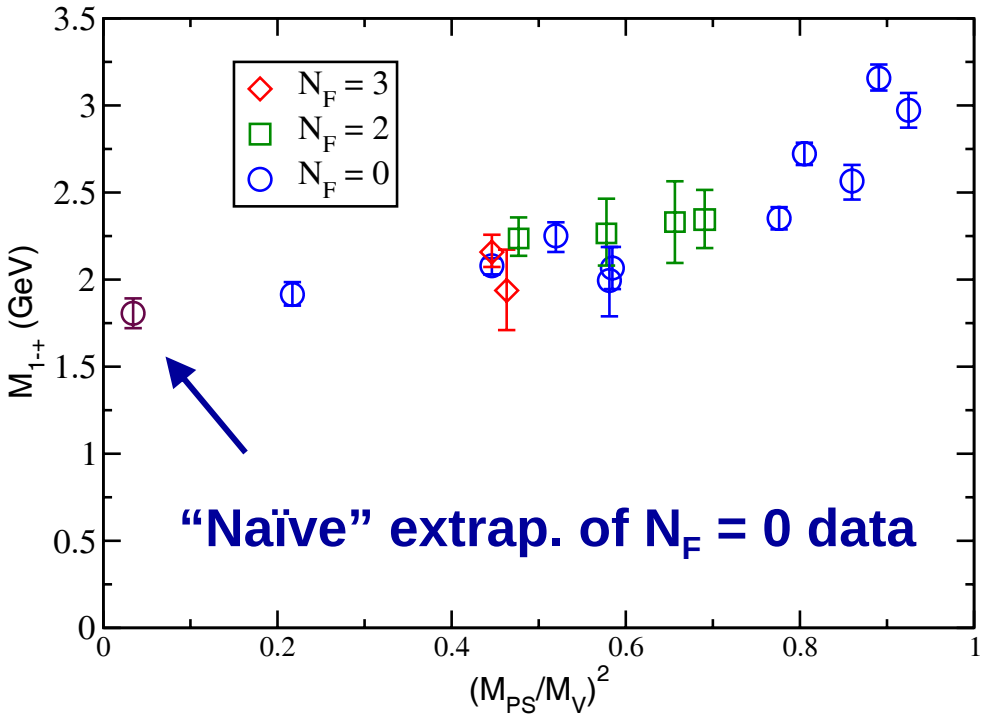


Hybrid Resonance Spectrum - II

- Does picture of hybrids obtained in charmonium sector persist to light-quark sector?

Compilation of data for mass of 1^{-+} from *MILC Collaboration*

Scale set by string tension



Energies from correlation functions

- **Stationary state energies can be extracted from asymptotic decay rate of temporal correlations of the fields (in the imaginary time formalism)**
- **Spectral representation of a simple correlation function**
  - **assume transfer matrix, ignore temporal boundary conditions**
  - **focus only on one time ordering**

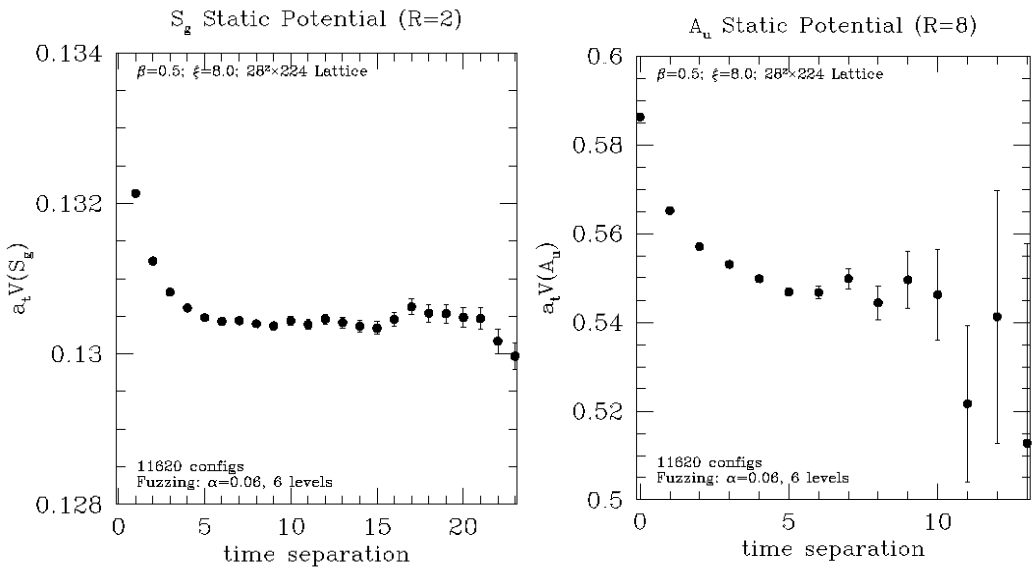
insert complete set of energy eigenstates

$$C(t) = \langle 0 \mid \phi(t)\phi(0) \mid 0 \rangle = \sum_n \langle 0 \mid e^{Ht}\phi(0)e^{-Ht} \mid n \rangle \langle n \mid \phi(0) \mid 0 \rangle$$
$$= \mid \langle n \mid \phi(0) \mid 0 \rangle \mid^2 e^{-E_n t} = \sum_n A_n e^{-E_n t}$$

- ***Extract lowest energy and amplitude as $t \to \infty$***

Effective mass

- The "effective mass" is given by $m_{\text{eff}}(t) = \ln\left(\frac{C(t)}{C(t+1)}\right)$
- The effective mass tends to the energy) asymptotically
- Effective mass plot is convenient visual tool to see signal extraction
  - seen as a plateau
- Plateau sets in quickly for good operator
- Excited-state contamination before plateau



Principal correlators

- Extracting excited-state energies described in C. Michael, NPB 259, 58 (1985) and Luscher and Wolff, NPB 339, 222 (1990)
- Can be viewed as exploiting the *variational method*
- Given $N \times N$ correlator matrix $C_{\alpha\beta}(t) = \langle\, 0 \mid O_\alpha(t)\, O_\beta(0) \mid 0 \,\rangle$, one defines the N *principal correlators* $\lambda_i(t,t_0)$ as the eigenvalues of

$$C^{-1/2}(t_0)C(t)C^{-1/2}(t_0)$$

where t_0 (the time defining the "metric") is small

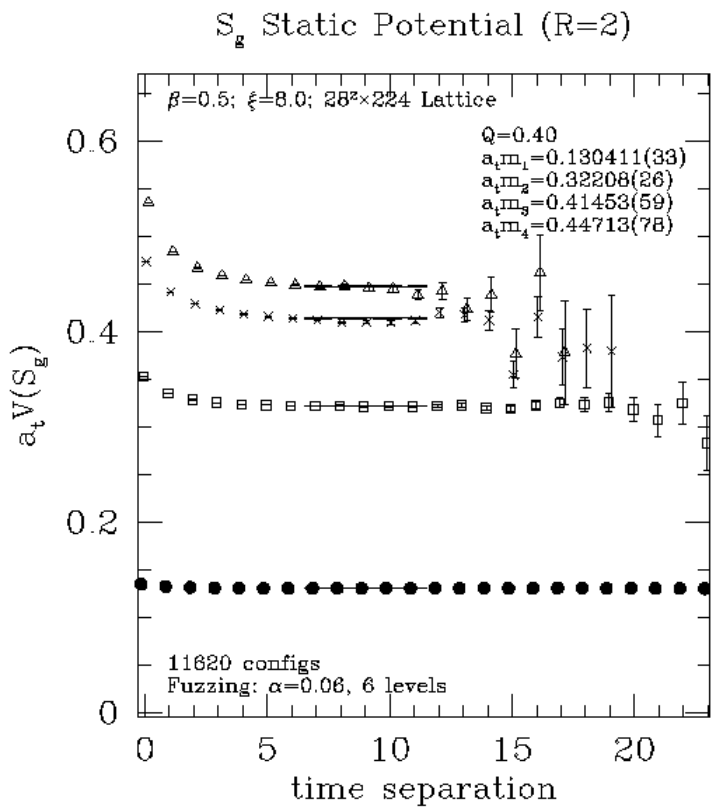
- Principal effective masses defined from correlators plateau to lowest-lying energies

$$\lambda_i(t,t_0) \to e^{-E_i(t-t_0)}\left(1 + O(e^{-\Delta E(t-t_0)})\right)$$

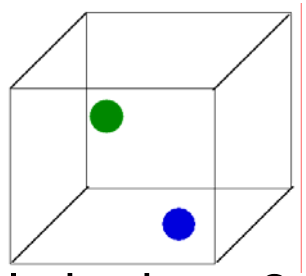
min ($E_n - E_i$)

Principal effective masses

- **Perform single-exponential fit to each principal correlator to extract spectrum! *Can use multi-exponentials to minimize sensitivity to excited states***
- **Note that principal effective masses (as functions of time) can cross, approach asymptotic behaviour from below**
- **Final results are *independent of t_0*, but choosing larger values of this reference time can introdu larger errors**

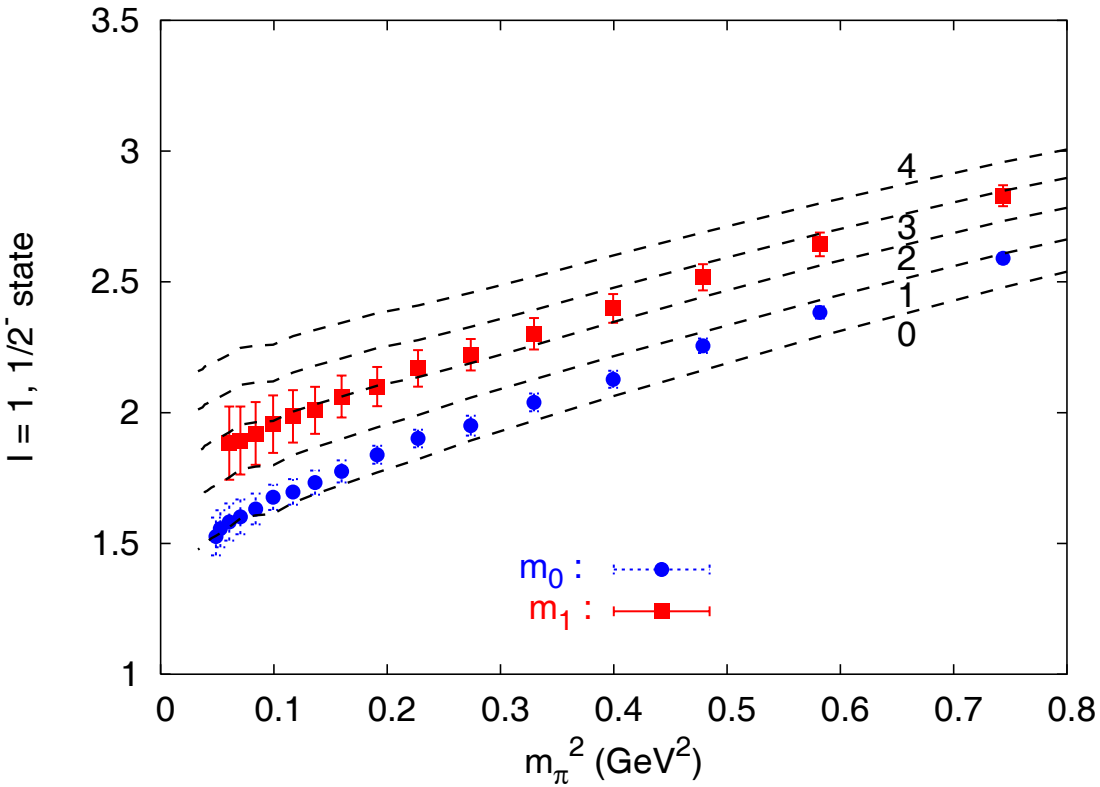


Unstable Particles



- computations done in a periodic box
  - momenta quantized
  - discrete energy spectrum of stationary states → single hadron, 2 hadron,…
- scattering phase shifts → resonance masses, widths (in principle) deduced from finite-box spectrum
  - B. DeWitt, PR **103**, 1565 (1956) (sphere)
  - M. Luscher, NPB**364**, 237 (1991) (cube)
- Two-particle states and resonances identified by examining behaviour of energies in finite volume
  - **Resonances with milder volume dependence**

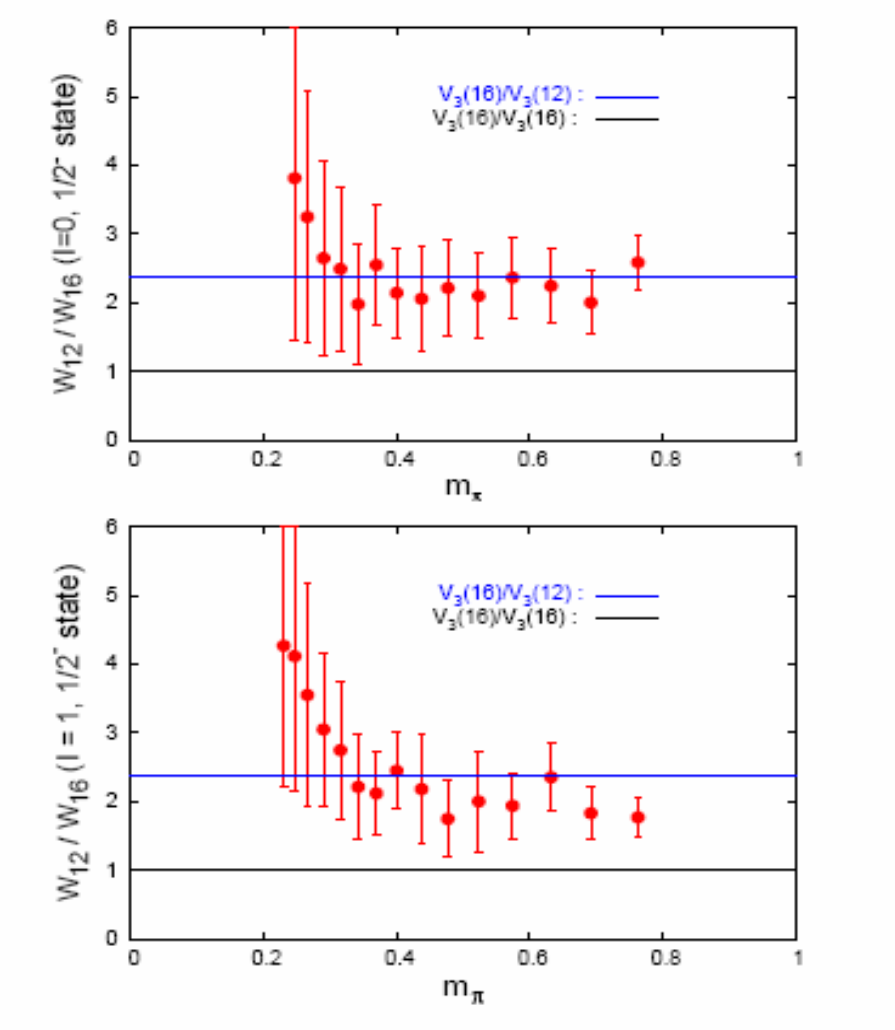
Resonance or scattering state - I



Mathur *et al*, Phys.Rev. D70 (2004) 074508

Fit to ground and first-excited states of I=1 pentaquark correlator, together with scattering energies

Resonance or scattering state - II

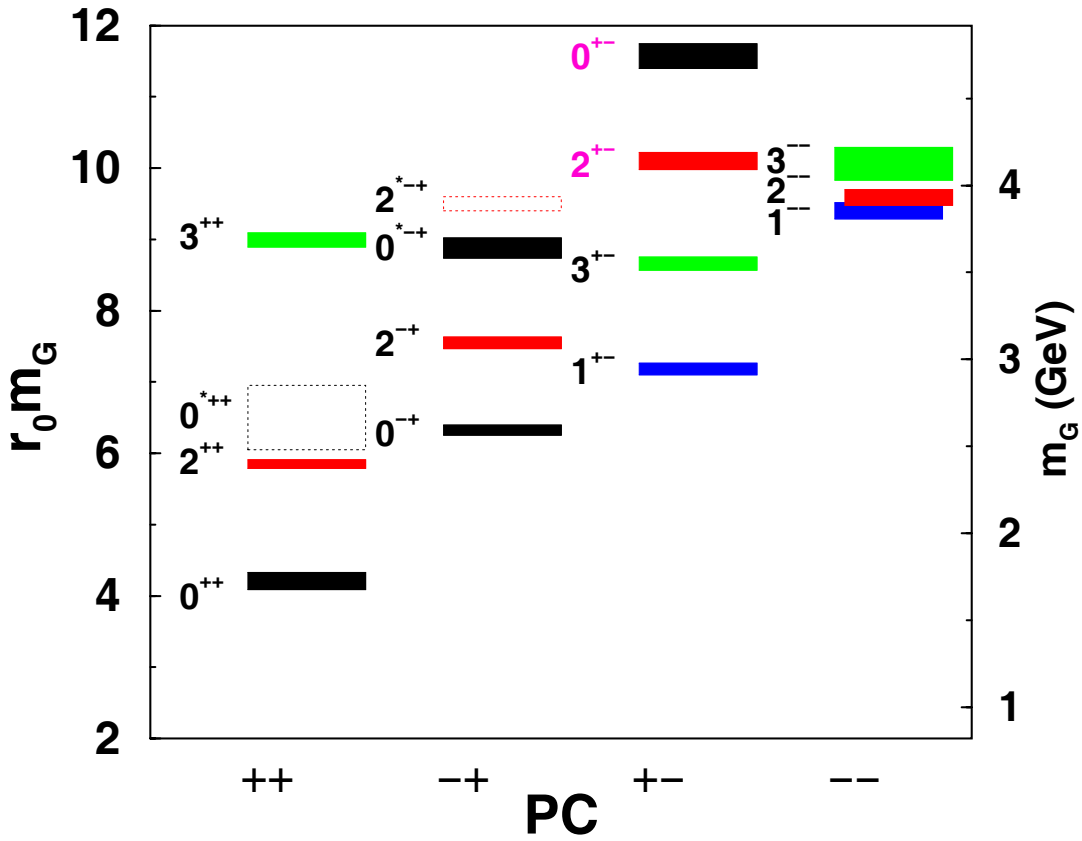


Two-particle amplitude expected to have different volume dependence to single-particle amplitude – ***Amplitudes more consistent with two-particle state***

Variational methods: glueballs

Morningstar and Peardon PRD60, 034509

- **Quenched glueball calculations provide road-map**
- **Variational methods**
- **Large matrix of correlators**
- **Method more demanding for particles containing quarks**



Baryons: Cubic Group of Lattice

- **Rotations restricted to isotropic cubic lattice form *Octohedral group O***
  - **O has 24 elements**
  - **O has five conjugacy classes, hence five single-valued representations, $A_1(1)$, $A_2(1)$, $E(2)$, $T_1(3)$, $T_2(3)$**
  - **Under elements R of O, operator lying in Λ irreducible representation transforms as (λ is row)**

$$U(R)O_{\lambda}^{(\Lambda)}U(R)^{\dagger} = \sum_{\lambda'} O_{\lambda'}^{(\Lambda)} D_{\lambda'\lambda}^{(\Lambda)}(R)$$

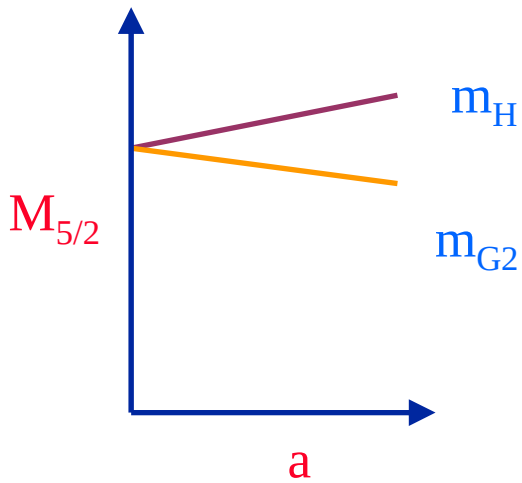
  - ***Spatial inversions*** **$I_s \rightarrow O_h$, with 48 elements; parities labelled g or u**

Double Octohedral Group

- **Has 48 elements**
- **Contains irreducible representations of O, together with 3 spinor irreps G_1, G_2, H: R.C.Johnson, PLB114, 147 (82)**

J	$n^J_{G_1}$	$n^J_{G_2}$	n^J_H
$\frac{1}{2}$	1	0	0
$\frac{3}{2}$	0	0	1
$\frac{5}{2}$	0	1	1
$\frac{7}{2}$	1	1	1
$\frac{9}{2}$	1	0	2
$\frac{11}{2}$	1	1	2
$\frac{13}{2}$	1	2	2
$\frac{15}{2}$	1	1	3
$\frac{17}{2}$	2	1	3

Note that states with J >= 5/2 lie in representations with lower spins.



Spins identified from degeneracies in contiuum limit

Baryon Fields and operators: design

- States are classified according to
  - Flavor structure, F
  - Parity
  - Total Spin, *J*, and Helicity, or third component of spin, J_z.

S. Basak *et al.*, *PRD72:074501,2005* *PRD72:094506,2005*

- In lattice computation, need to construct operators $B_i^{\Lambda\,\lambda\,F}(t, x)$ transforming irreducibly under lattice symmetries Λ, λ and form correlation matrix

$$C_{ij}^{\Lambda\lambda F}(t) = \sum_{\vec{x}} \langle 0|B_i^{\Lambda\lambda F}(t,\vec{x})\bar{B}_j^{\Lambda\lambda F}(0)|0\rangle$$

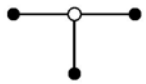
- Begin by considering flavor and orbital radial structure

Minimize Propagator Inversions

Flavour, Orbital and Radial Structure

- **Classify states according to *SU(2) Flavour***
- **Build up *radial structure* using displacement operators D**
- **Provide our elemental $B_i^F(t,x)$ operators having correct *flavour and colour structure***

Quark-diquark

Illustration	Name	Explicit form ($\|i\| \neq \|j\| \neq \|k\|$)
	single-site	$\phi^F_{ABC}\,\varepsilon_{abc}\,\tilde{\psi}_{Aa\alpha}\,\tilde{\psi}_{Bb\beta}\,\tilde{\psi}_{Cc\gamma}$
	singly-displaced	$\phi^F_{ABC}\,\varepsilon_{abc}\,\tilde{\psi}_{Aa\alpha}\,\tilde{\psi}_{Bb\beta}\,\left(\tilde{D}^{(p)}_j\tilde{\psi}\right)_{Cc\gamma}$
	doubly-displaced-I	$\phi^F_{ABC}\,\varepsilon_{abc}\,\tilde{\psi}_{Aa\alpha}\,\left(\tilde{D}^{(p)}_{-j}\tilde{\psi}\right)_{Bb\beta}\,\left(\tilde{D}^{(p)}_j\tilde{\psi}\right)_{Cc\gamma}$
	doubly-displaced-L	$\phi^F_{ABC}\,\varepsilon_{abc}\,\tilde{\psi}_{Aa\alpha}\,\left(\tilde{D}^{(p)}_j\tilde{\psi}\right)_{Bb\beta}\,\left(\tilde{D}^{(p)}_k\tilde{\psi}\right)_{Cc\gamma}$
	triply-displaced-T	$\phi^F_{ABC}\,\varepsilon_{abc}\,\left(\tilde{D}^{(p)}_{-j}\tilde{\psi}\right)_{Aa\alpha}\left(\tilde{D}^{(p)}_j\tilde{\psi}\right)_{Bb\beta}\left(\tilde{D}^{(p)}_k\tilde{\psi}\right)_{Cc\gamma}$
	triply-displaced-O	$\phi^F_{ABC}\,\varepsilon_{abc}\,\left(\tilde{D}^{(p)}_i\tilde{\psi}\right)_{Aa\alpha}\left(\tilde{D}^{(p)}_j\tilde{\psi}\right)_{Bb\beta}\left(\tilde{D}^{(p)}_k\tilde{\psi}\right)_{Cc\gamma}$

Spatial structure of operators

Final step: projection formula

- **Central formula**

$$B_i^{\Lambda\lambda F}(t,\vec{x}) = \frac{d_\Lambda}{g_{O_h^D}} \sum_{R\in O_h^D} D^{(\Lambda)*}_{\lambda\lambda}(R) U_R B_i^F(t,\vec{x}) U_R^\dagger$$

- **Group theory performed in Maple/C++ – requires explicit Dirac basis.**

Irrep	Δ, Ω	N	Σ, Ξ	Λ
G_{1g}	221	443	664	656
G_{1u}	221	443	664	656
G_{2g}	188	376	564	556
G_{2u}	188	376	564	556
H_g	418	809	1227	1209
H_u	418	809	1227	1209

Total numbers of operators assuming two different displacement lengths

Several weeks on 16-node cluster

- **Include multi-hadron operators**

Simple example: local nucleon operators…

- **Elemental operators:**

$$\Phi_{\alpha\beta\gamma} = \epsilon^{abc}(u^a_\alpha u^b_\beta d^c_\gamma - u^a_\gamma u^b_\beta d^c_\alpha)$$

- **Constraints:**

$$\Phi_{\alpha\beta\gamma} + \Phi_{\gamma\beta\alpha} = 0$$
$$\Phi_{\alpha\beta\gamma} + \Phi_{\beta\gamma\alpha} + \Phi_{\gamma\alpha\beta} = 0$$

- **20 linearly-independent operators**
- **3 embeddings of G_{1g}, G_{1u} , 1 embedding each of H_g, H_u; NO G_2**
- ***Three linearly independent nucleon operators, and a single $N^{*3/2}$ operator***

(u C γ_5 d) u, (u C d) γ_5 u, Rarita-Schwinger projection

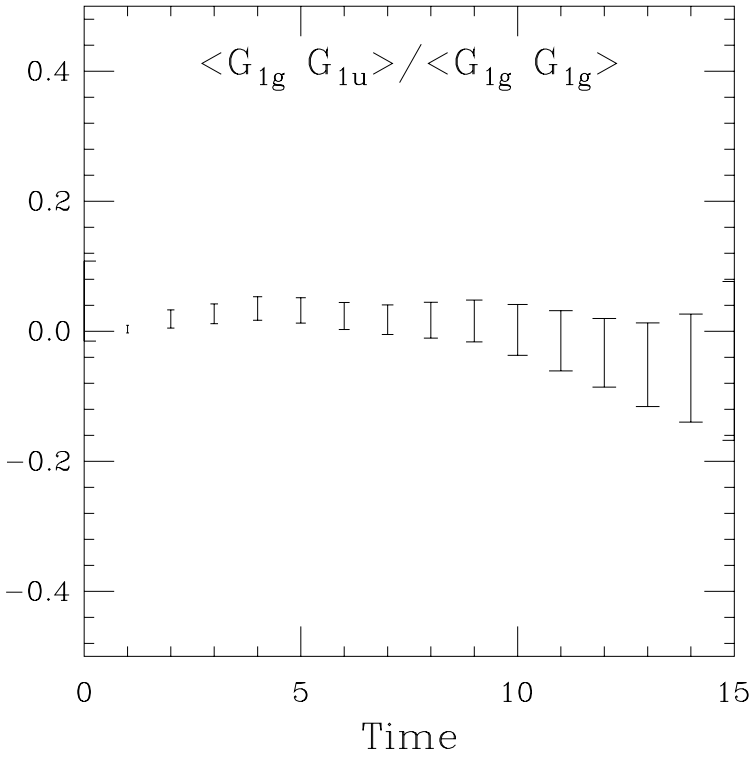
Implementation…

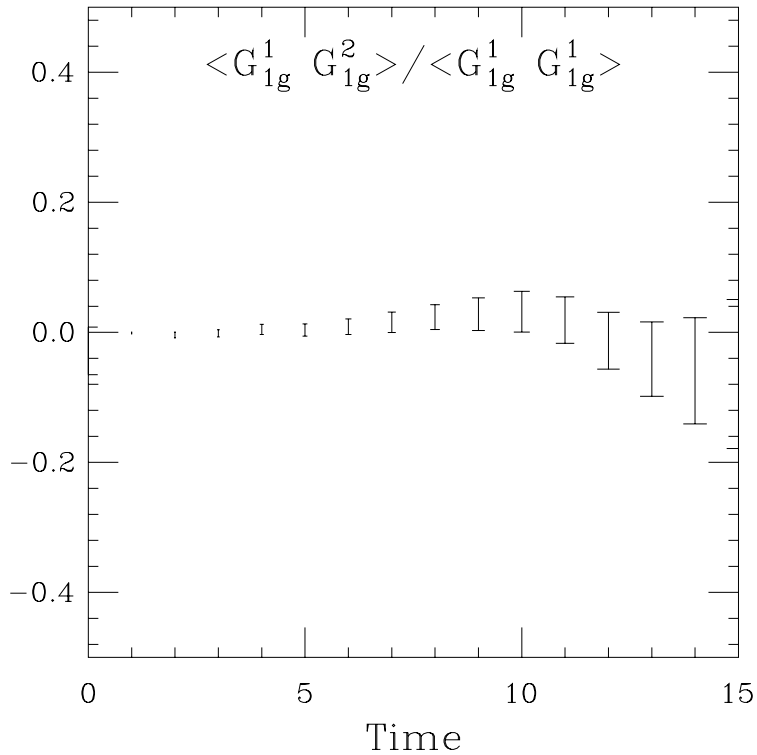
- **On each configuration, form generalised baryon correlator**

$$B_{\alpha\beta\gamma,\bar{\alpha}\bar{\beta}\bar{\gamma}} = \sum_{\vec{x}} \epsilon^{abc}\epsilon^{\bar{a}\bar{b}\bar{c}} A^{a\bar{a}}_{\alpha\bar{\alpha}}(x,0) B^{b\bar{b}}_{\beta\bar{\beta}}(x,0) C^{c\bar{c}}_{\gamma\bar{\gamma}}(x,0)$$

- ***Antisymmetrise*** **over α,γ - I = 1/2.**
- ***Symmetrise*** **over α and β – *Wick contractions***
- **Form correlators according to table.**
- **Illustration: DWF valence propagators, on MILC Asqtad lattices: m_π ' 600 MeV**
- ***All-to-all propagators***

Orthogonality of States

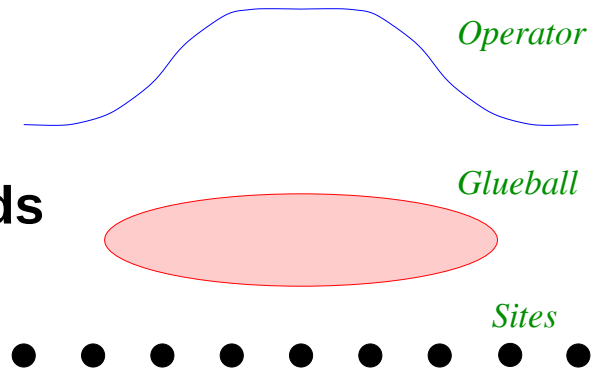




- Different rows orthogonal
- Different irreps. orthogonal
- Different *embeddings* of *same irrep* not orthogonal

Smearing - I

- Statistical noise generally increases with temporal separation
- Effective masses associated with correlation functions of simple local fields do not reach a plateau before noise swamps the signal
  - need better operators
  - better operators have reduced couplings with higher-lying states
- Recipe for making better operators
  - Construct operators using *smeared* fields
    - link variable smearing
    - quark field smearing



Implementation: Quark Smearing - I

- **The quark smearing integral transform can be effected by the application of the smearing operator:**

Adam Lichtl, Dublin '05

$$\tilde{\Psi}(x) = Q_\sigma \Psi(x)$$

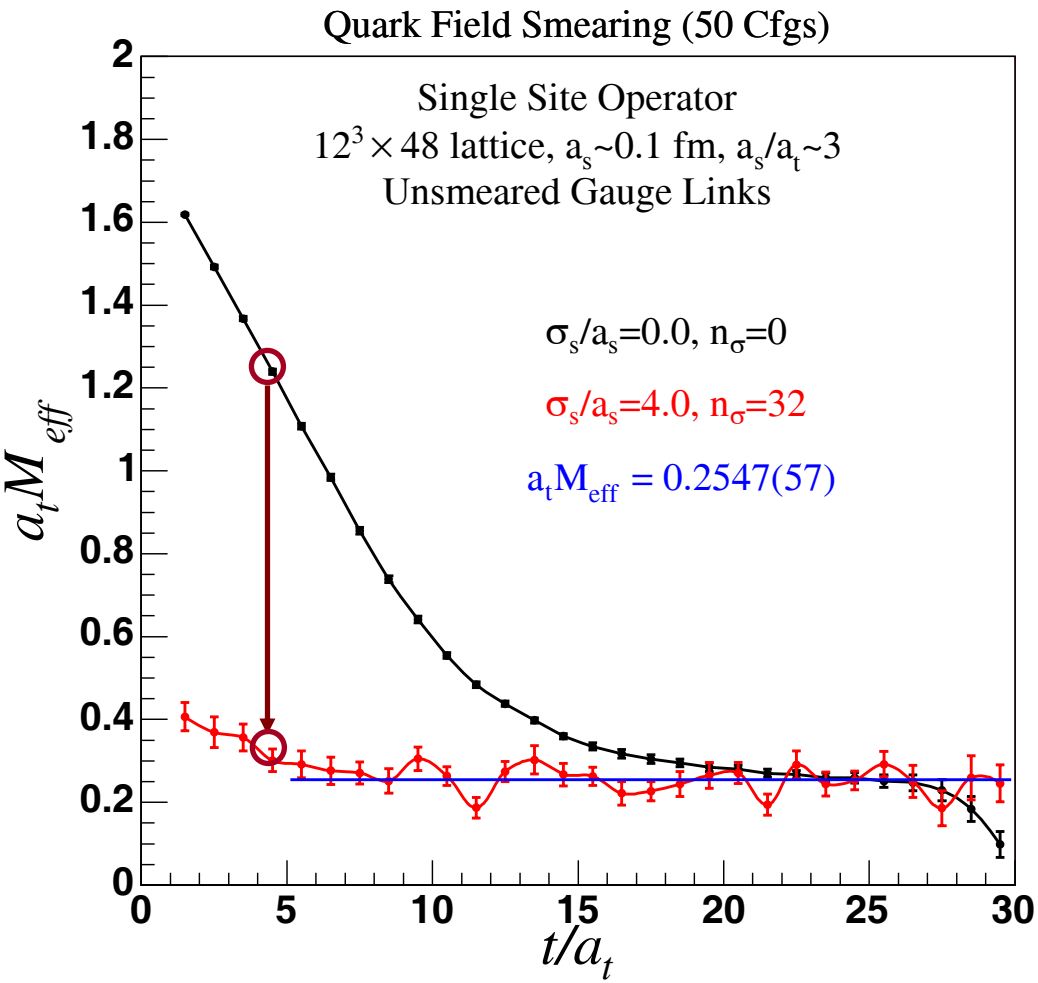
$$Q_\sigma = e^{\sigma^2 \nabla^2 / 2}$$

$$Q_\sigma = lim_{n_\sigma \to \infty}(Q_\sigma^{n_\sigma}) = lim_{n_\sigma \to \infty}(1 + \frac{\sigma^2 \Delta}{2n_\sigma})^{n_\sigma}$$

- **This attenuates the high momentum modes of the quark field → reduced contamination**

The Chroma library uses: $Q(\sigma_s, n_\sigma) = \boxed{(1+\frac{\sigma_s^2 \Delta}{4n_\sigma})^{n_\sigma}}$

Implementation: Quark Smearing - II



Implementation: Link Smearing - I

- **We see that link smearing plays a crucial role when we construct displaced operators**
- ***Stout link smearing* combines neighboring links in an analytic manner**
  - **Morningstar and Peardon, hep-lat/0311018**

$$U_\mu^{(n+1)}(x) = e^{i\rho\Theta_\mu^{(n)}(x)} U_\mu^{(n)}(x)$$

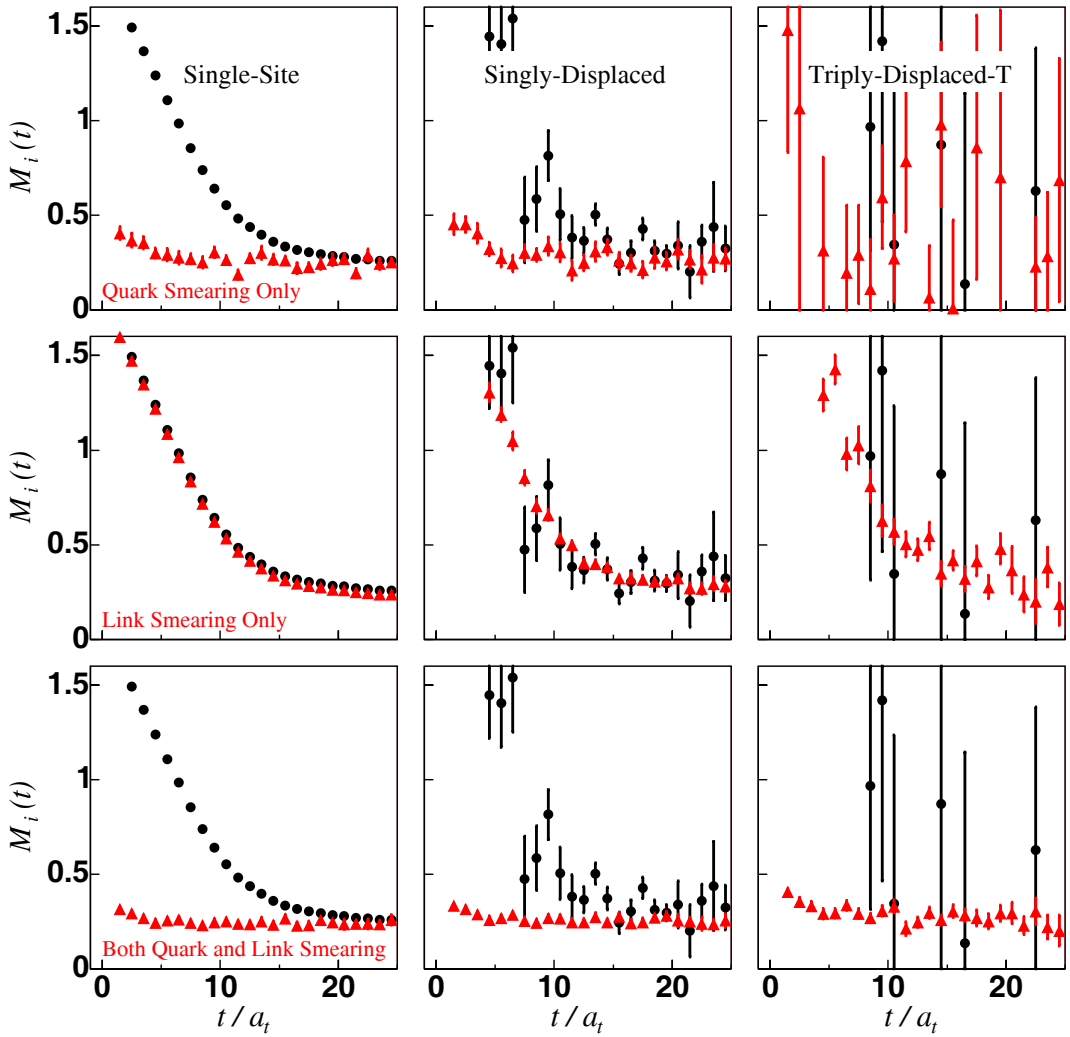
$$\Theta_\mu^{(n)} = \frac{i}{2}(\Omega_\mu^\dagger(x) - \Omega_\mu(x)) - \frac{i}{2N} Tr(\Omega_\mu^\dagger(x) - \Omega_\mu(x))$$

$$\Omega_\mu(x) = C_\mu(x) U_\mu^\dagger(x)$$

$$C_\mu(x) = \text{Sum of spatial staples from } x \text{ to } x + \hat{\mu}$$

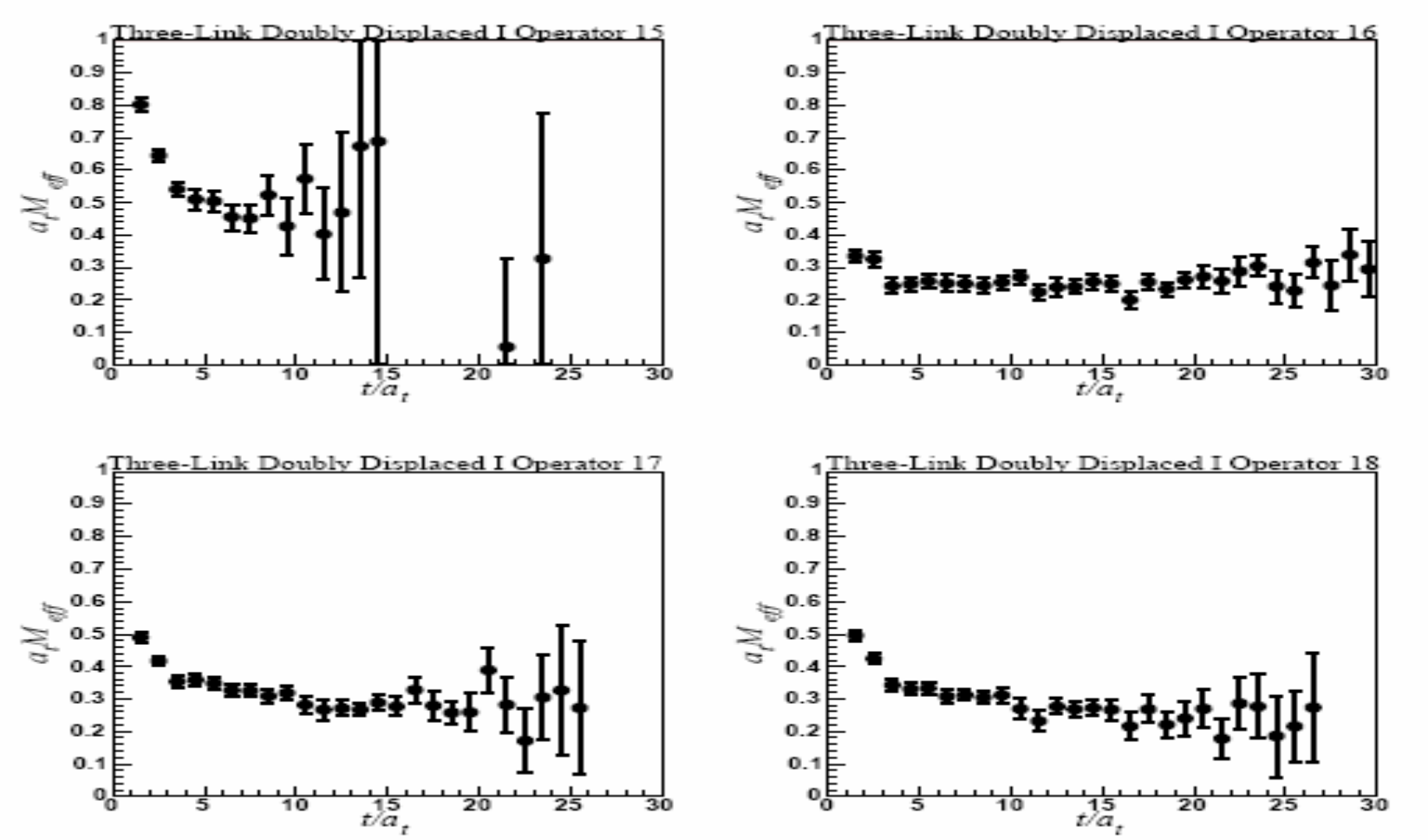
Implementation: Link Smearing - II

- ***Quark field smearing***: **reduces coupling to high-lying states.**
- ***Link smearing***: **reduces variance in correlator.**



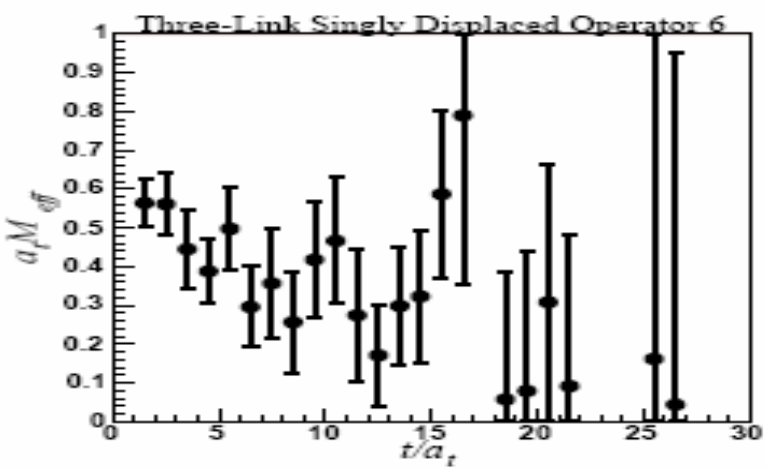
Implementation: Pruning - I

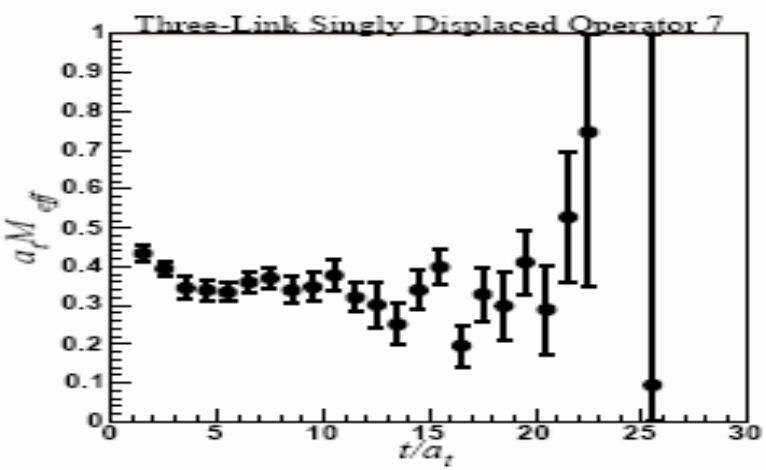
G_{1g}

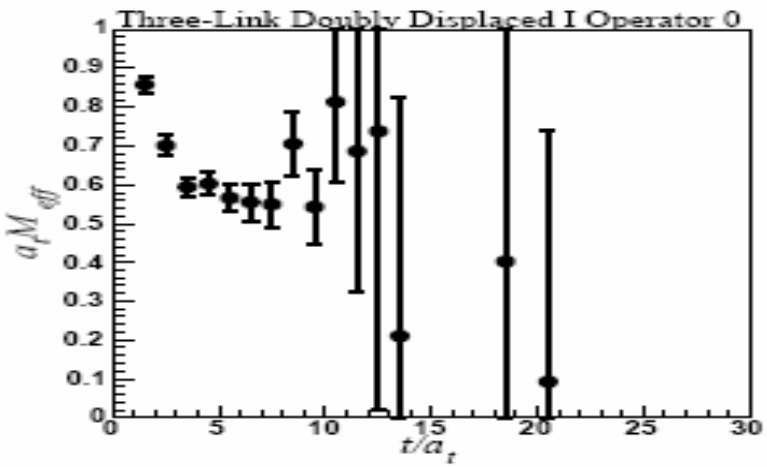


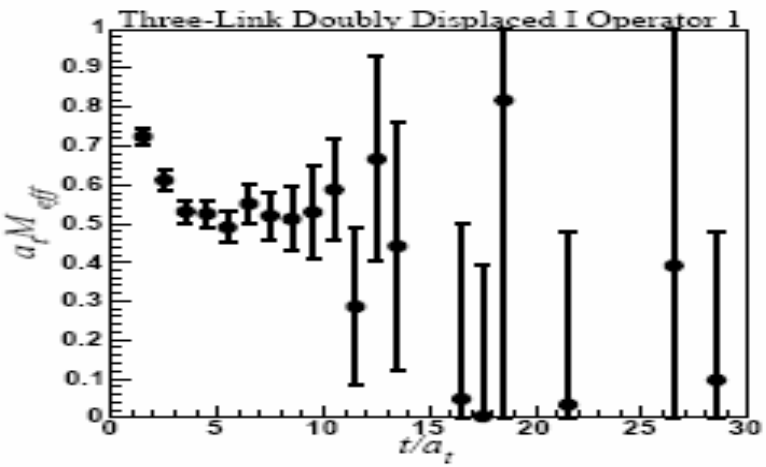
Implementation - II

G_{2u}







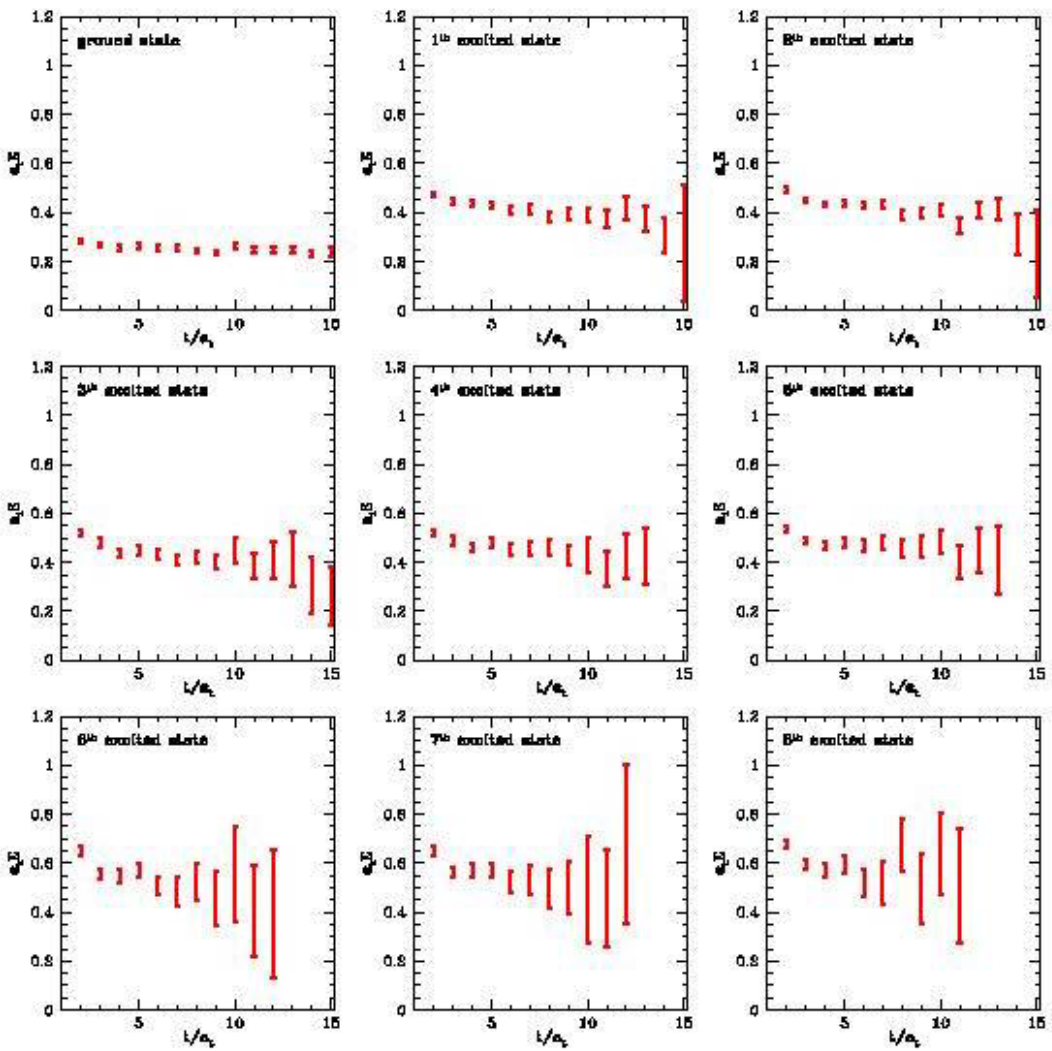


***A priori*, we do not know the "good" interpolating fields**

Principle Eigenvalues

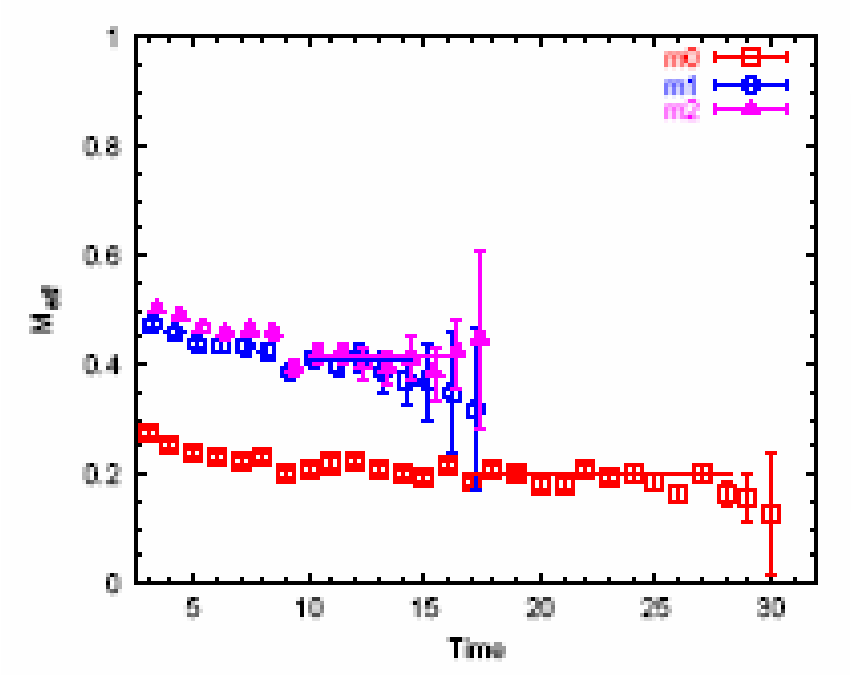
Extract up to 8 or 9 principle correlators

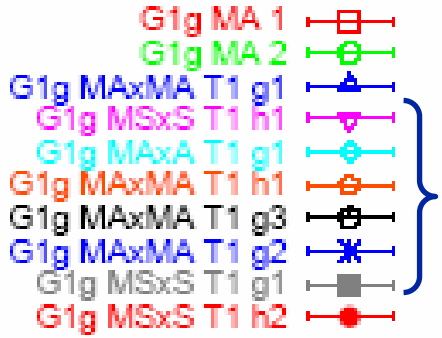
cf BGR Collaboration: Gaussians of different widths



Principle eigenvalues

- principal effective masses for small set of 10 operators





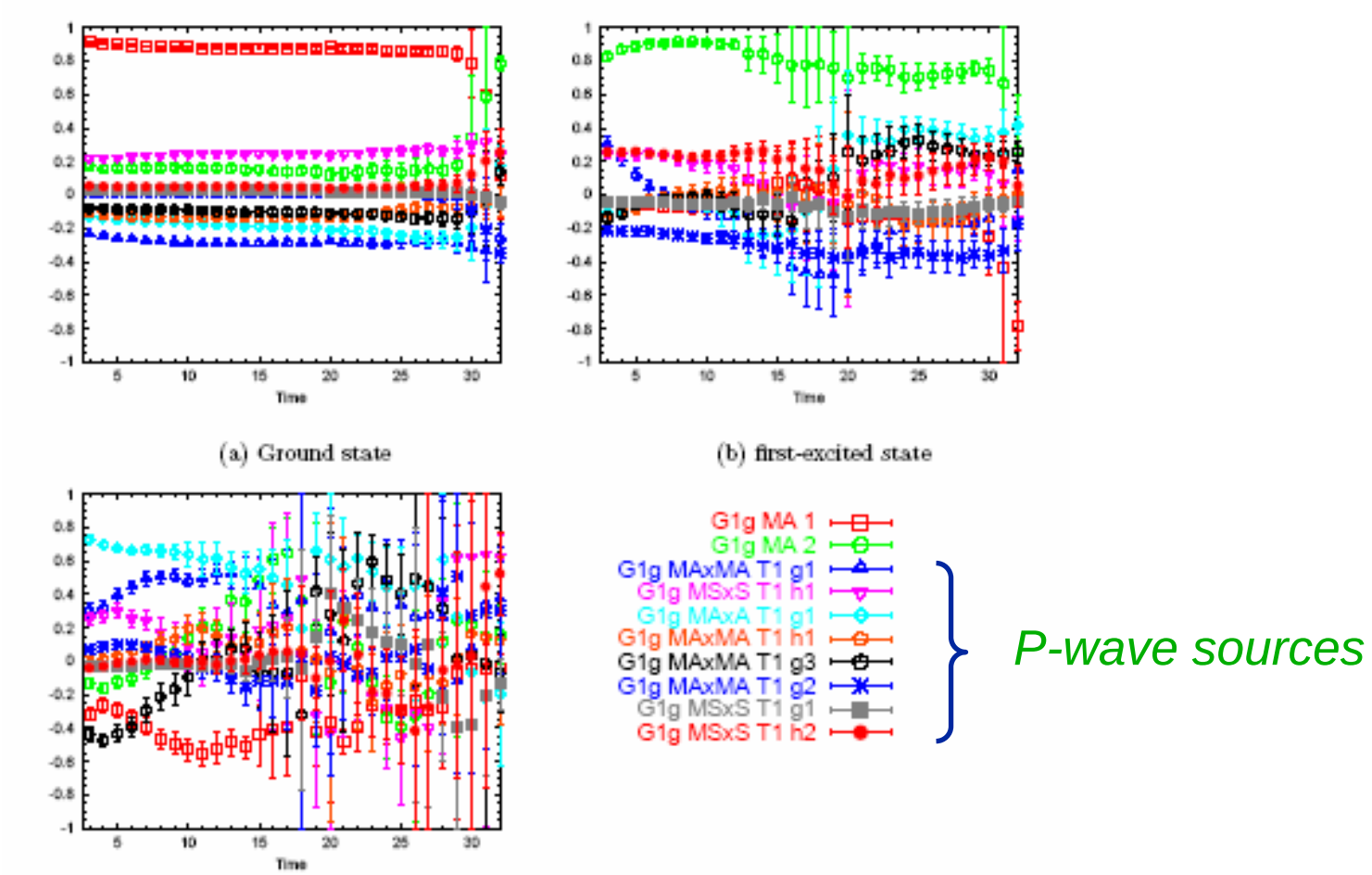
P-wave sources

Ikuro Sato, PhD, UMD (2005)

$16^3 \times 64$ lattice, anisotropic Wilson with $\xi = 3.0$; $m_\pi = 500$ MeV

Eigenvectors….

- Eigenvectors provide information regarding basis functions



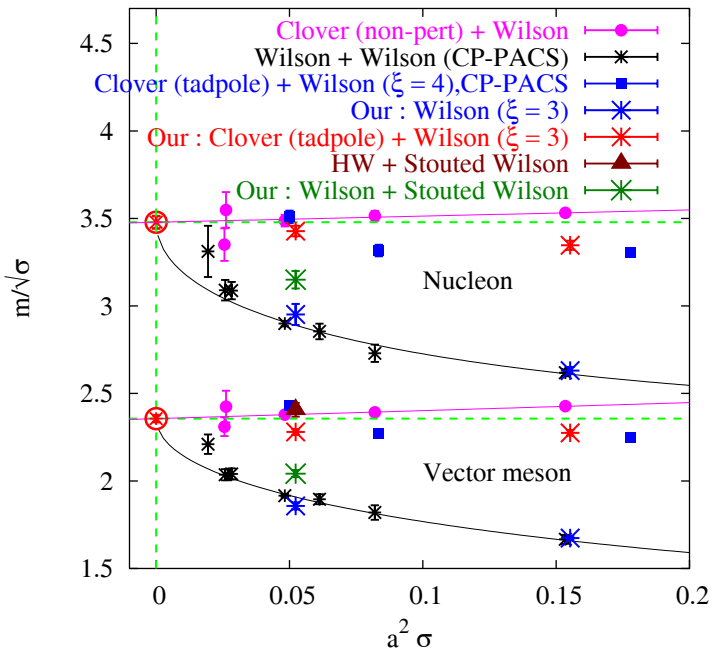
(a) Ground state

(b) first-excited state

P-wave sources

Future Plans

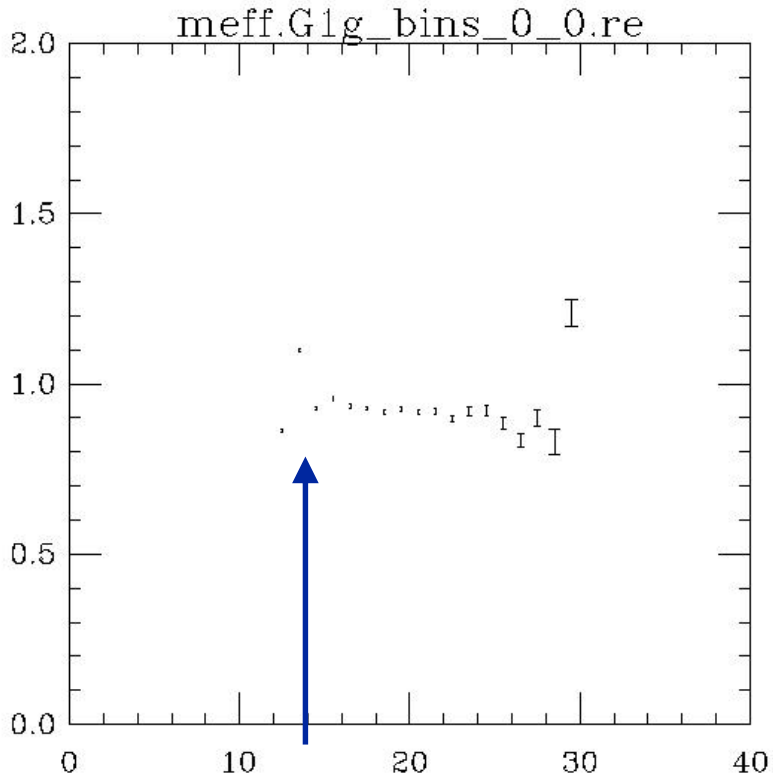
- Generate dynamical anisotropic lattices
- Two lattice spacings – *spin identification*
- Three volumes – *Isolate two-particle contributions*
- Choice of action:
  - Require well-defined transfer matrix
  - ~~DWF or overlap~~
  - Anisotropic clover action



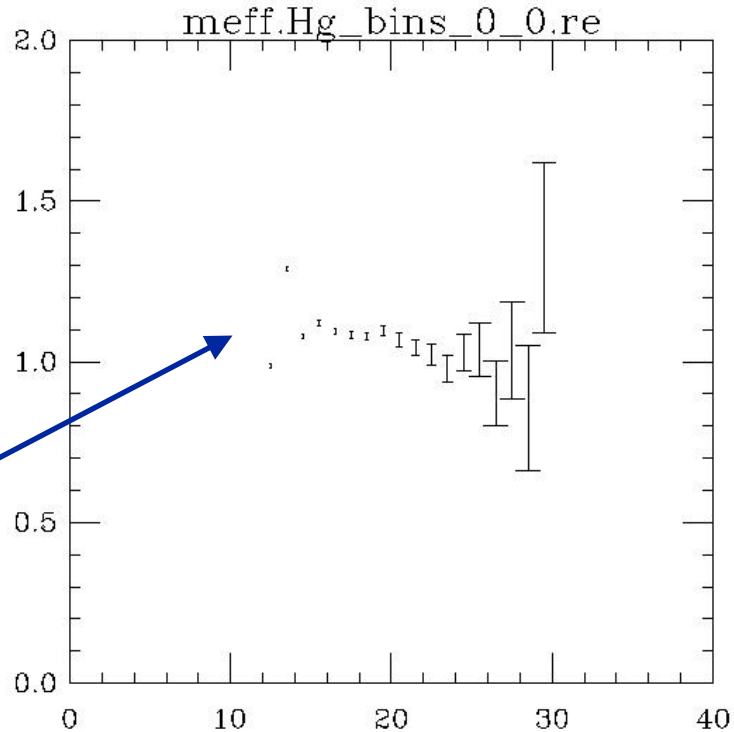
Choice of Action

- **LHPC hadron structure studies employ DWF fermions on asqtad gauge background.**
- **DWF is *O(a)*-improved – *crucial for study of matrix elements*.**
- **DWF is non-local in time and obscures interpretation of transfer matrix – *extremely problematic for resonance spectroscopy*.**
- ***Study of baryon resonance spectrum using all possible local operators***

DWF - I

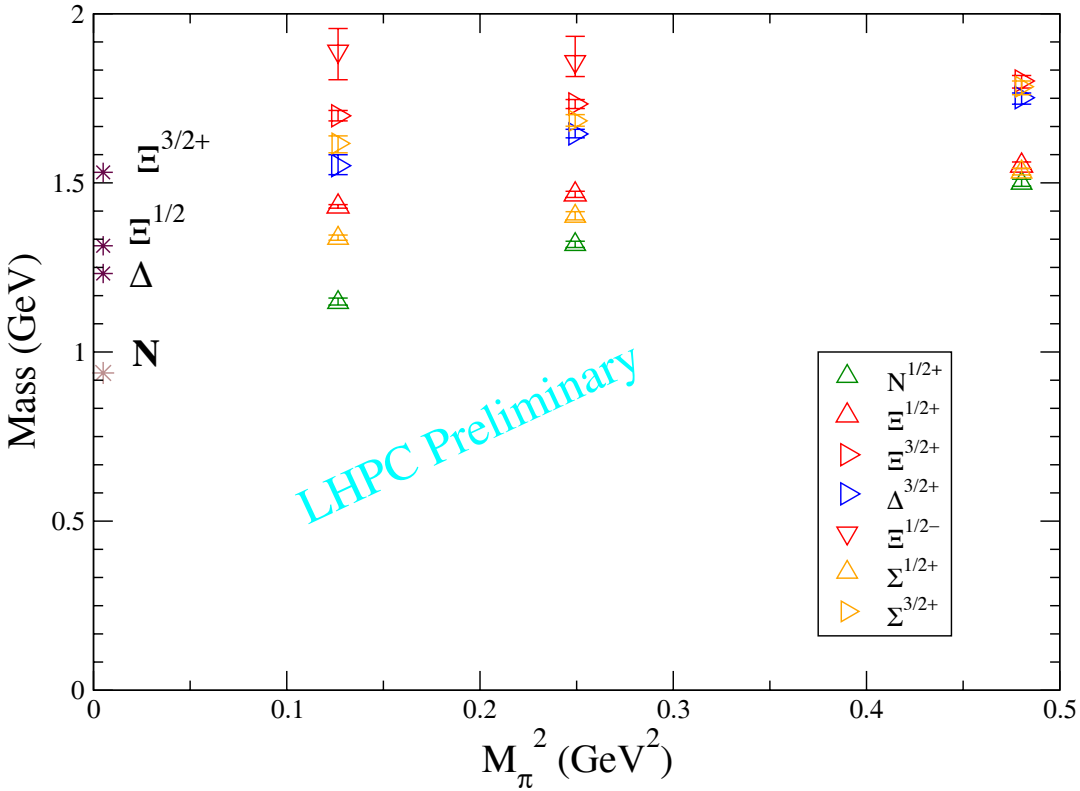


NPLQCD + LHPC Propagatos



Non-locality at short distance

DWF- II



Focus on cascades: *narrow width*

Conclusions and Perspective

- **Exploration of resonance spectrum requires use of variational methods and development of suitable basis of operators**
- **Developed baryon operators lying in IR's of cubic group of the lattice – *essential to identify spin and parity***
- **Operator construction is being extended to include excited glue, and to hybrids**
- **Proposal to generate dynamical, anisotropic clover lattices**
- **Aiming to establish efficacy of all-to-all methods – do not require choice of operator *Juge***
- **For "well-established" resonances, can straightforwardly study radiative transitions –**
  - **$\gamma N \to \Delta$,**
  - **Radiative transitions in charmonium – *Dudek, Edwards, Mathur, DGR***