Nucleon structure calculations are described using a hybrid combination of improved staggered sea quarks and domain wall valence quarks at a range of pion masses extending down to 350 MeV. Results for the nucleon axial coupling are presented including chiral perturbation theory fits to describe both the pion mass and volume dependence of the axial coupling.
Chiral and Volume Extrapolations of $g_A$: Part I

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for LHPC, in particular

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J. Negele    W. Schroers

http://talks.drubryantrenner.org/int_3-16-06.pdf
Outline

- hybrid lattice calculation: domain wall fermions on staggered lattices with $m_\pi$ down to 350 MeV and $L$ up to 3.5 fm

- lattice calculation of nucleon matrix elements: $g_A$ as an example

- chiral perturbation theory: $m_\pi$ and $L$ dependence of $g_A$
Hybrid Lattice Calculation

- asqtad staggered sea quarks (MILC) with $a = 0.124$ fm

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<th>$L$</th>
<th>$m_{\pi}^{\text{asqtad}}$</th>
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- domain wall valence quarks with HYP smearing
Tuning the Domain Wall Quark Mass

- determine the domain wall quark mass so that the domain wall and the lightest asqtad pion masses match

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<th>$a m_{u/d}^{\text{DWF}}$</th>
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• at finite $L_5$ there is an explicit chiral symmetry breaking which can be characterized by the residual quark mass

\[ \Delta^{\mu} J^{a}_{5\mu} = 2m_q J^{a}_5 + 2J^{a}_{5\text{mid}} \quad \text{and} \quad J^{a}_{5\text{mid}} \approx m_{\text{res}} J^{a}_5 \]

• we determine $L_5$ so that the residual quark mass is less than 10% of the quark mass

![Graph showing the residual quark mass as a function of $L_5$.](image-url)
we also investigate the $L_5$ dependence of other observables
Nucleon Axial Coupling

- accurately measured in neutron $\beta$ decay: $g_A = 1.2695(29)$
- it is a measure of spontaneous chiral symmetry breaking
- it probes the spin content of the nucleon: $g_A = \langle 1 \rangle_u - \langle 1 \rangle_d$
- it can be renormalized non-pertubatively by using the Ward identity
- it can be used as a laboratory for finite size effects on the lattice
- it is a non-singlet so there are no disconnected diagrams
Example Plateau
Volume Dependence (RBCK-Quenched)

- $V = 2.4 \text{ (fm)}^3$ (DBW2 glue)

- $V = 1.2 \text{ (fm)}^3$ (DBW2 glue), $V = 1.6 \text{ (fm)}^3$ (Wilson glue)

graph from RBCK hep-lat/0306007
Volume Dependence (QCDSF)

- curve is leading order chiral perturbation theory
- $m_\pi = 717$ (MeV)

graph from QCDSF hep-lat/0409161
Axial Charge $g_A$

- curve is one loop chiral perturbation theory (including $\Delta$) \[1\]
- parameters: $f_\pi$, $m_\Delta - m_N$, $g_{NN}^A$, $g_{N\Delta}^A$, $g_{\Delta\Delta}^A$, $C(\lambda)$
- $f_\pi$, $m_\Delta - m_N$ and $g_{N\Delta}^A$ are taken from experiment

\[1\] Beane and Savage hep-ph/0404131
Axial Charge $g_A$

- all full QCD calculations of $g_A$

- our final extrapolated value is $g_A = 1.212(84)$
Conclusions

• light quark calculations using domain wall fermions on staggered lattices with $m_\pi$ down to 350 MeV and $L = 2.5$ and 3.5 fm

• this allows for a calculation of $g_A$ at $m_\pi = 354$ MeV, lighter than any other full QCD calculations, with an accuracy of 5%

• calculations with $L = 2.5$ fm and $L = 3.5$ fm allow for control over the finite size effects which plague other calculations

• we are entering the regime of sufficiently light quark masses to apply chiral perturbation theory to describe both the $m_\pi$ and $L$ dependence of $g_A$

• our extrapolated result is $g_A = 1.212(84)$ which is a 7% error band