GDH sum rule and magnetic moments in chiral EFT

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Topics

- GDH sum rule and its derivatives
- Magnetic moment of the nucleon (chiral behavior)
- Magnetic moment of the Delta-resonance (experimental determination and chiral behavior)

Gerasimov-Drell-Hearn (GDH) sum rule

$$\frac{e^2}{2M^2}\kappa^2 = \frac{1}{\pi}\int_0^\infty \frac{d\omega}{\omega}\Delta\sigma(\omega)$$

$\kappa = (g-2)\,s$ anomalous magnetic moment

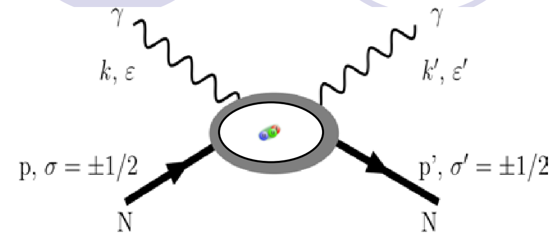
$\Delta\sigma = \sigma_{1+s} - \sigma_{1-s}$ doubly-polarized total photoabsorption cross section (photon circular polarized parallel or anti-parallel to the target's spin)

Principles/Assumptions:

- **Low-energy theorem for Compton scattering** (gauge-invariance, crossing symmetry,…)
- **Analyticity** (forward Compton amplitude obeys disp. relations along the production cut)
- **Unitarity** (optical theorem: Im forward Compton amplitude = total phtotoabsorption)
- **Stability??** (neglect the target's decay)

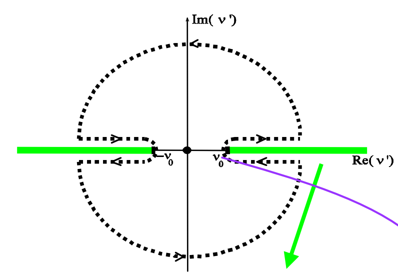
Derivation steps

Forward Compton scattering amplitude:



$$\mathrm{Amp}_{t=0} = \underbrace{f(\omega)\,\vec{\varepsilon}\,' \cdot \vec{\varepsilon} + g(\omega)\, i\vec{S} \cdot (\vec{\varepsilon}\,' \times \vec{\varepsilon}) + \ldots}_{2s+1 \text{ terms} \;\leftrightarrow\; \# \text{ of e.m. moments}}$$

Analyticity: $\mathrm{Re}\, g(\omega) = \frac{2\omega}{\pi} \int\limits_{\omega_{th}}^{\infty} d\omega' \frac{\mathrm{Im}\, g(\omega')}{\omega'^2 - \omega^2}$



Unitarity: $\mathrm{Im}\, g(\omega) = \frac{\omega}{2} \Delta\sigma(\omega)$

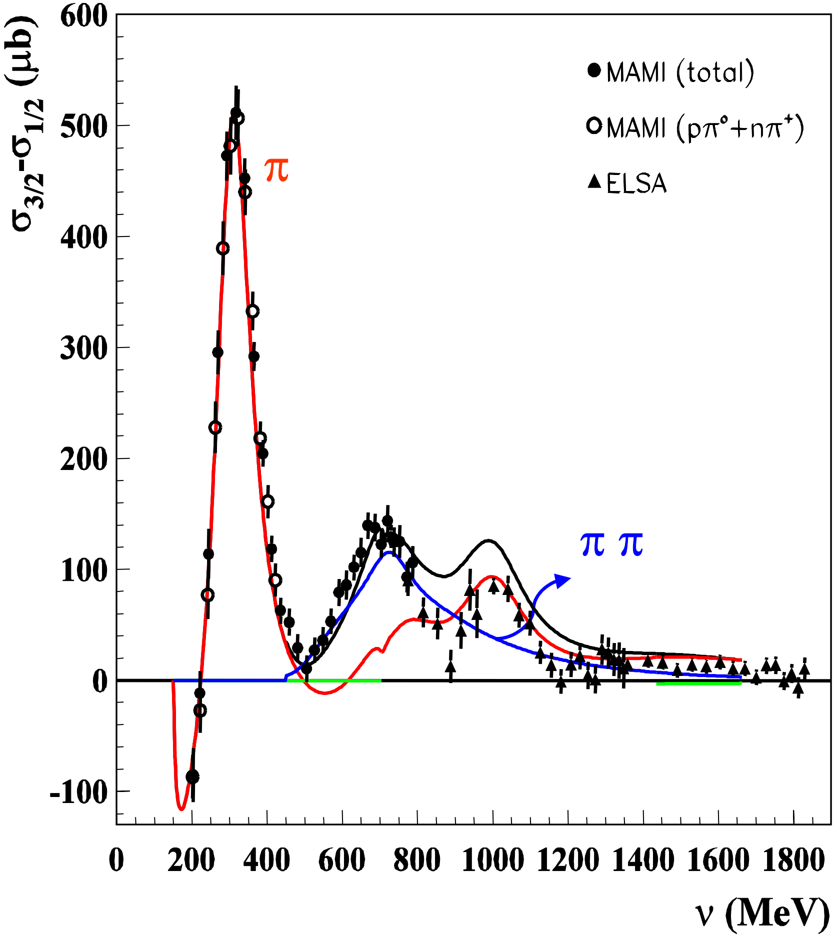
Low-energy theorem: $g(\omega) = \frac{e^2 \kappa^2}{4 s M^2} \omega + O(\omega^3)$

$\omega_{th} \geq 0$

Stability condition

Emperical verification of the GDH sum rule for the proton

$$\frac{e^2}{2M^2}\kappa^2 = \frac{1}{\pi}\int_0^\infty \frac{d\omega}{\omega}\Delta\sigma(\omega)$$



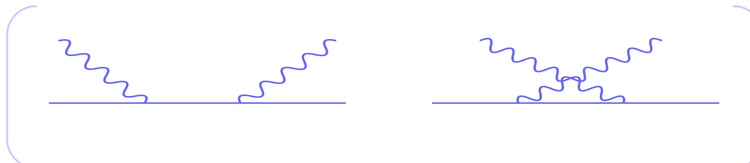
PROTON	Eγ [GeV]	GDH [μb]
MAID2000 estimate	< 0.2	-28.5 ± 2
MAMI experiment	0.2 - 0.8	226 ± 5 ± 12
ELSA experiment	0.8 - 2.9	27.5 ± 2.0 ± 1.2
Bianchi-Thomas Simula et al. : estimate	> 2.9	-14 ± 2
Total		211 ± 15
GDH sum rule		205

QED

$$\frac{e^2}{2M^2}\kappa^2 = \frac{1}{\pi}\int_0^\infty \frac{d\omega}{\omega}\Delta\sigma(\omega)$$

Compute both sides of the SR in perturbation theory…

O(e⁴) 0

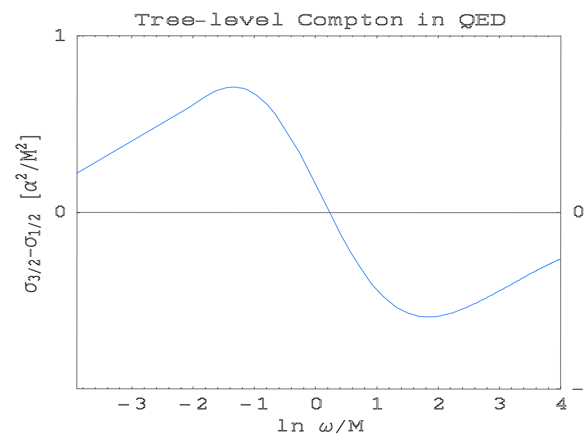


^{2}

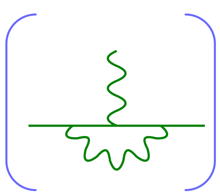
$$\Delta\sigma^{(4)} = -\frac{2\pi\alpha^2}{M^2 x}\left[(1+\frac{1}{x})\log(1+2x) - 2\left(1+\frac{x^2}{(1+2x)^2}\right)\right],$$

$x = \omega/M.$

[Altarelli, Cabibbo, Maiani, PLB (72)]



O(e⁶)



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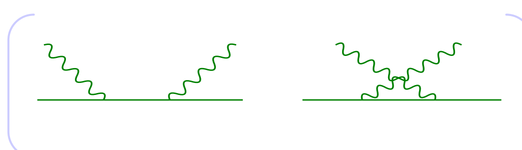
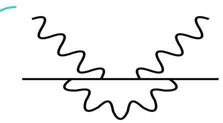




+ other 1-loop graphs

The l.h.s is the Schwinger's correction, the r.h.s. (much harder) done numerically by Dicus and Vega, PLB (2001).

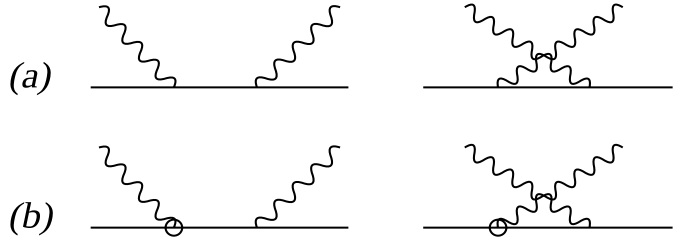
Derivatives of the GDH sum rule

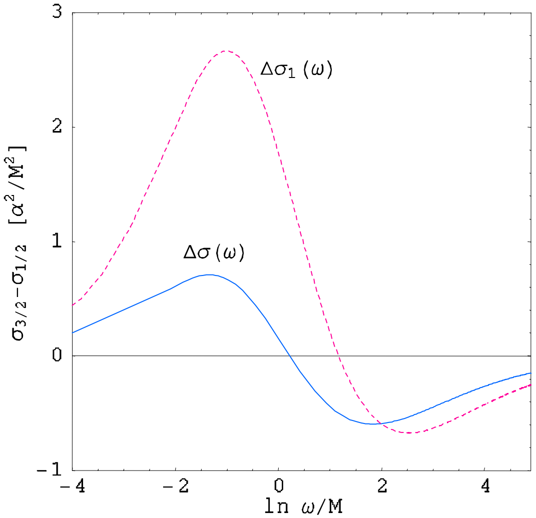
[VP, Holstein &Vanderhaeghen, PLB(2004); PRD(2005)]

- Introduce a trial a.m.m.: $\mathcal{L} = \mathcal{L}_{QED} + \frac{\kappa_0}{4M}\,\bar{\psi}\, i\sigma_{\mu\nu}\,\psi\, F^{\mu\nu}$

- The total value of amm and the cross-sections are dependent on κ_0 .
- By taking derivatives of the GDH SR w.r.t. κ_0 at $\kappa_0=0$ we obtain a new set of SRs, e.g., 1st derivative:

$$\frac{e^2}{M^2}\kappa = \frac{1}{\pi}\int_0^\infty \frac{d\omega}{\omega}\,\Delta\sigma'(\omega)\Big|_{\kappa_0=0}$$





$$\Delta\sigma'(\omega)\big|_{\kappa_0=0} = \frac{2\pi\alpha_{em}^2}{M\omega}\left[6 - \frac{2M\omega}{(M+2\omega)^2} - \left(2+\frac{3M}{\omega}\right)\ln\left(1+\frac{2\omega}{M}\right)\right]$$

$$\kappa = \frac{\alpha_{em}}{2\pi}$$

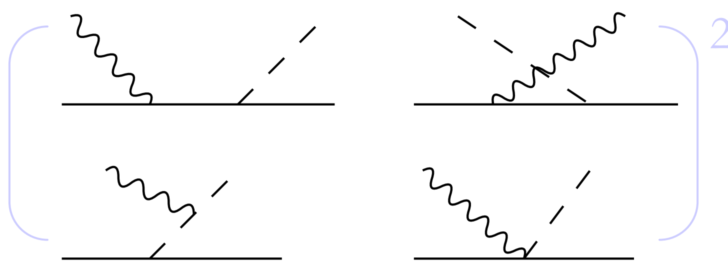
GDH and GDH' in Chiral Effective Field Theory

To lowest order in $g_{\pi NN}$ $(=g_A M/f_\pi)$

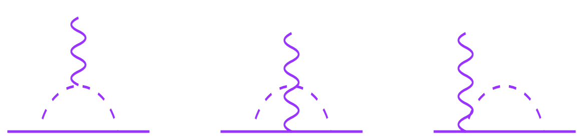
O(g²): $$\frac{e^2}{M^2}\,\kappa_0\,\delta\kappa = \frac{1}{\pi}\int\frac{d\omega}{\omega}\,(\Delta\sigma_E + \kappa_0\,\Delta\sigma_\kappa)$$

$$0 = \int\frac{d\omega}{\omega}\,\Delta\sigma_E$$

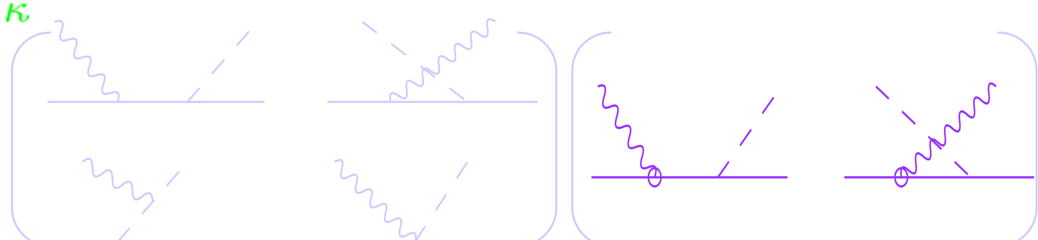
$0 \quad = \quad ?$

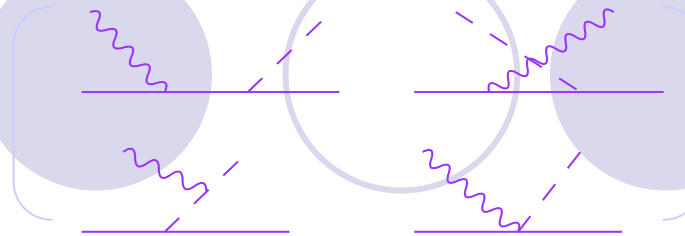


$$\delta\kappa = \frac{M^2}{\pi e^2}\int\frac{d\omega}{\omega}\,\Delta\sigma_\kappa$$



$= \quad ?$

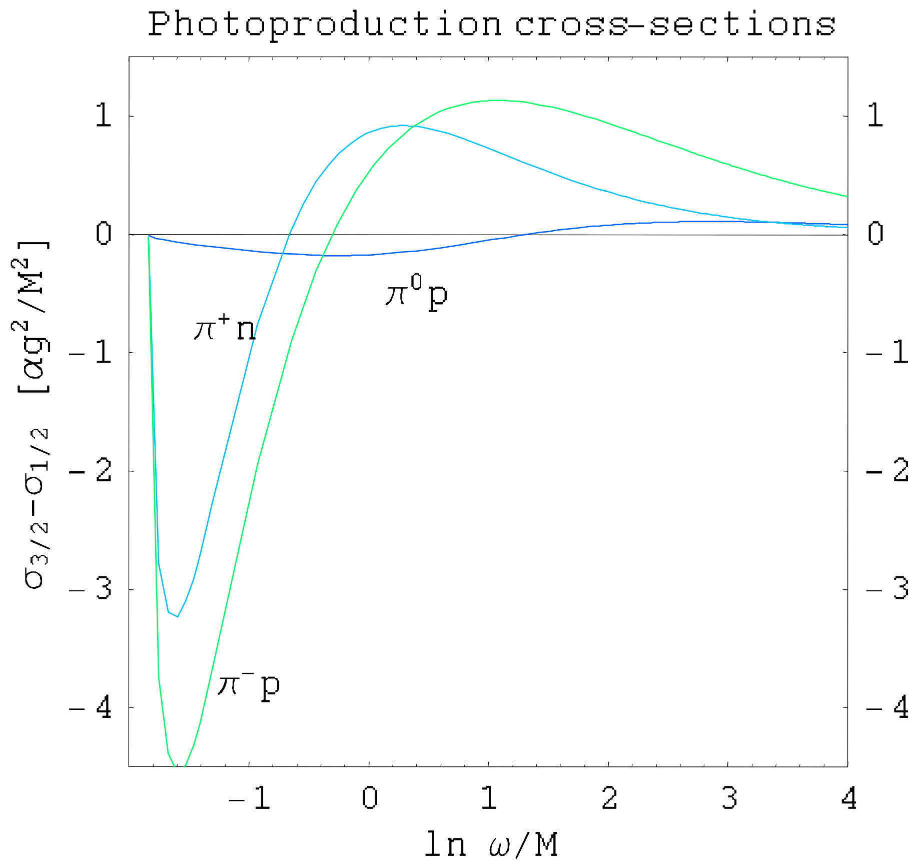




$$0 = \int \frac{d\omega}{\omega}\ \Delta\sigma_E$$

Verified

only in the fully relativistic calculation. Any “heavy-baryon” type of expansion does not do it.



$$\delta\kappa_p = \frac{M^2}{\pi e^2}\int_{\omega_{\rm th}}^{\infty}\frac{d\omega}{\omega}\left(\Delta\sigma_\kappa^{(\pi^0 p)} + \Delta\sigma_\kappa^{(\pi^+ n)}\right)$$

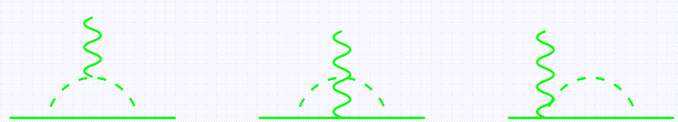
$$= \frac{g^2}{(4\pi)^2}\left\{1 - \frac{\mu\,(4 - 11\mu^2 + 3\mu^4)}{\sqrt{1-\frac{1}{4}\mu^2}}\arccos\frac{\mu}{2} - 6\mu^2 + 2\mu^2\,(-5 + 3\,\mu^2)\ln\mu\right\}$$

$$\delta\kappa_n = \frac{M^2}{\pi e^2}\int_{\omega_{\rm th}}^{\infty}\frac{d\omega}{\omega}\left(\Delta\sigma_\kappa^{(\pi^0 n)} + \Delta\sigma_\kappa^{(\pi^- p)}\right)$$

$$= \frac{-2g^2}{(4\pi)^2}\left\{1 - \frac{\mu\,(2 - \mu^2)}{\sqrt{1-\frac{1}{4}\mu^2}}\arccos\frac{\mu}{2} - 2\mu^2\ln\mu\right\}$$

where $\mu = m_\pi / M_N$.

Agrees

with direct  calculation

Chiral behavior of the nucleon magnetic moment

For *small* μ:

HB LO   Rel. corr.

$$\delta\kappa_p = \frac{g^2}{(4\pi)^2}\left\{1 - 2\pi\mu - 2\,(2+5\ln\mu)\,\mu^2 + \frac{21\pi}{4}\,\mu^3 + O(\mu^4)\right\}$$

$$\delta\kappa_n = \frac{g^2}{(4\pi)^2}\left\{-2 + 2\pi\mu - 2\,(1-2\ln\mu)\,\mu^2 - \frac{3\pi}{4}\,\mu^3 + O(\mu^4)\right\}$$

For *large* μ:

$$\delta\kappa_p = \frac{g^2}{(4\pi)^2}\,(5-4\ln\mu)\frac{1}{\mu^2} + O(\mu^{-4})$$

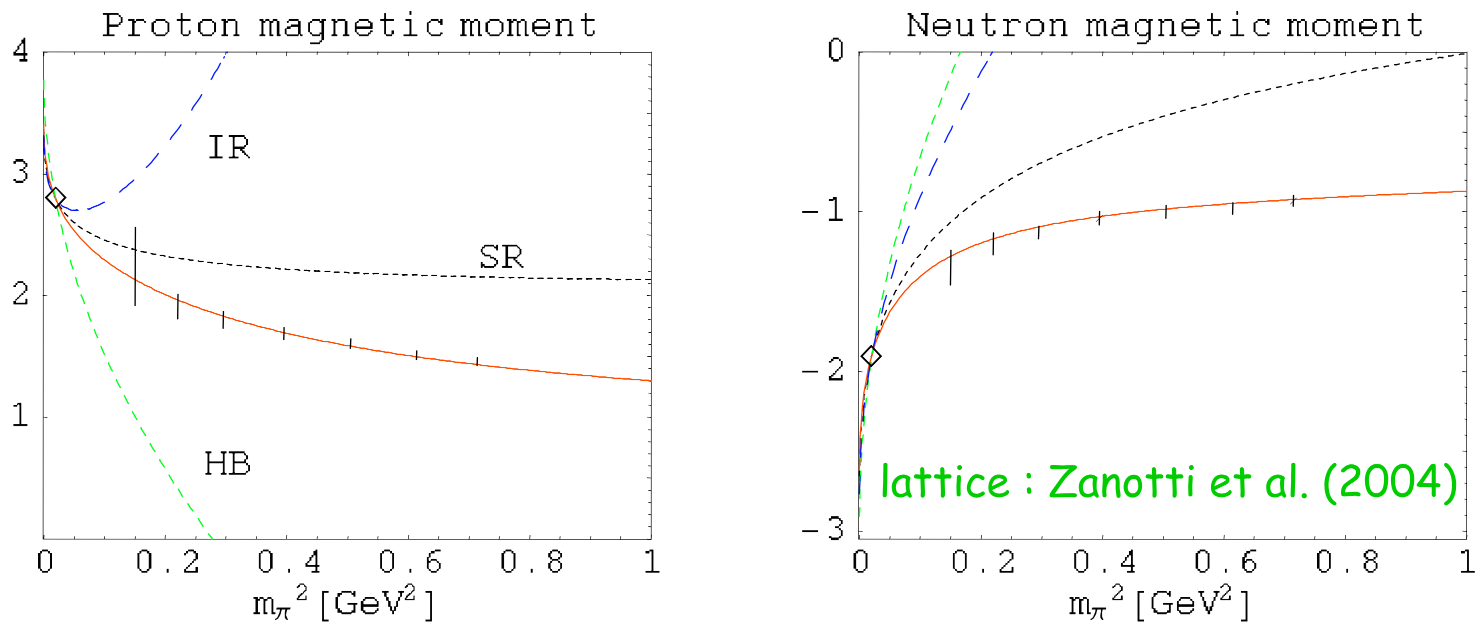
$$\delta\kappa_n = \frac{g^2}{(4\pi)^2}\,2(3-4\ln\mu)\frac{1}{\mu^2} + O(\mu^{-4})$$

Goes as $1/m_q$ $\quad (m_\pi^2 \sim m_q)$

Exactly as expected from the quark model.

Relativistic effects in chiral behavior of magnetic moments

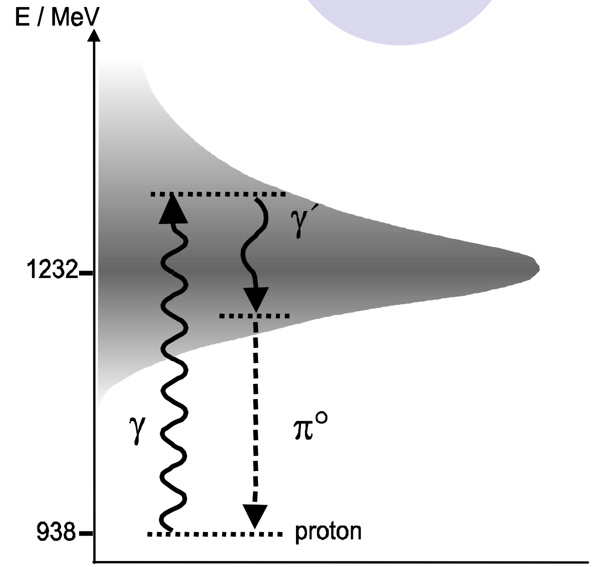
Relativistic chiral loops (SR) compared with heavy-baryon expansion (HB) or Infrared-Regularized ChPT (IR)



Red curve is the single-parameter fit to lattice data
The parametrization is based on SR result

$$\mu = \frac{\mu_0}{1 + a\, m_\pi^2} + \delta\kappa$$

Motivation for μ_Δ



- Observation of the magnetic moment of a strongly unstable particle.
- Δ-resonance is the best-studied example of such a particle.
- Excitation energy of the Δ-resonance, M_Δ-M_N ´ Δ ' 300 MeV is relatively low, which allows to treat it as a low-energy scale in an EFT expansion, the Δ can be incorporated in ChPT.

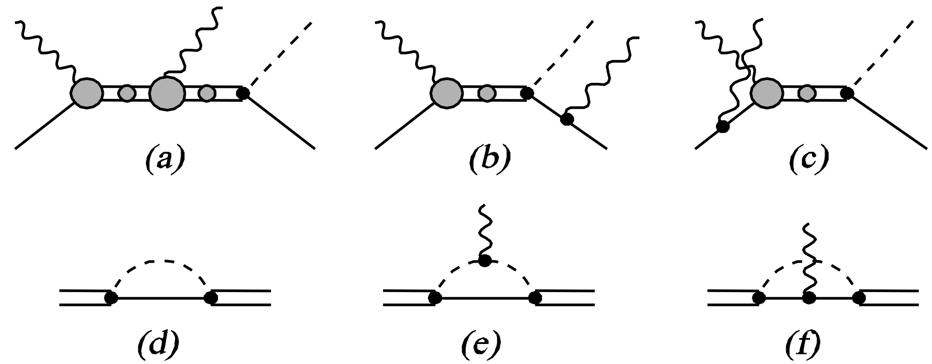
$\gamma p \longrightarrow \gamma\, \pi^0\, p$: ChEFT calculation

[VP & Vanderhaeghen, PRL 94 (2005)]

Power counting [VP&Phillips, PRC(2003)]: $\delta=(M_\Delta - M_N)/\Lambda_{\chi PT}$, $m_\pi/\Lambda_{\chi PT} \gg \delta^2$

To next-to-leading order in the resonance region

(counting $\omega' \gg m_\pi$, $\omega \gg M_\Delta - M_N$)



$$S_{\mu\nu}(p) = \frac{-\mathcal{P}^{(3/2)}_{\mu\nu}(p)}{(p\cdot\gamma - M_\Delta)[1 - i\mathrm{Im}\,\Sigma'(M_\Delta)] - i\mathrm{Im}\,\Sigma(M_\Delta)},$$

$$\bar{u}_\alpha(p')\,\Gamma^{\mu\alpha\beta}_{\gamma\Delta\Delta}(p',p)\,u_\beta(p)\,\epsilon_\mu$$
$$= e\,\bar{u}_\alpha(p')\left[\epsilon\cdot\gamma\,F(q^2) + \frac{(p'+p)\cdot\epsilon}{2M_\Delta}\,G(q^2)\right]u^\alpha(p)\,,\quad \mu_\Delta = F(0),\;\; Z_\Delta = 1 - \Sigma'(M_\Delta)$$

The Ward-Takahashi identity,

$$q_\mu \Gamma^{\mu\alpha\beta}(p',p) = e\left[(S^{-1})^{\alpha\beta}(p') - (S^{-1})^{\alpha\beta}(p)\right],\;\; \text{demands}\;\; F(0)+G(0) = 1-\Sigma'(M_\Delta).$$

$\gamma p \longrightarrow \gamma \pi^0 p$: observables

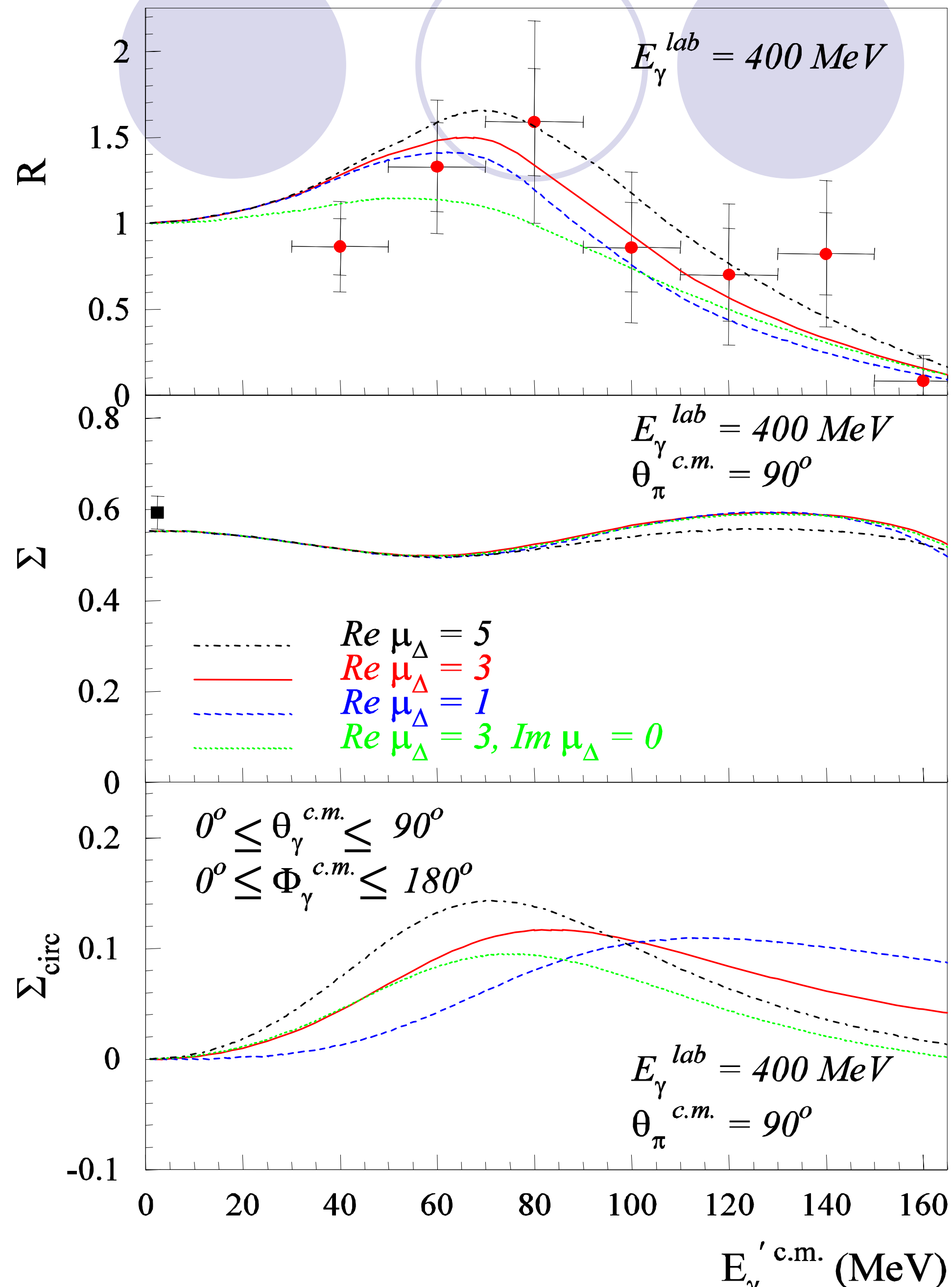
Exp. data points from TAPS@MAMI 2002

1. Ratio of the angle-integrated cross-section to the soft-photon limit:

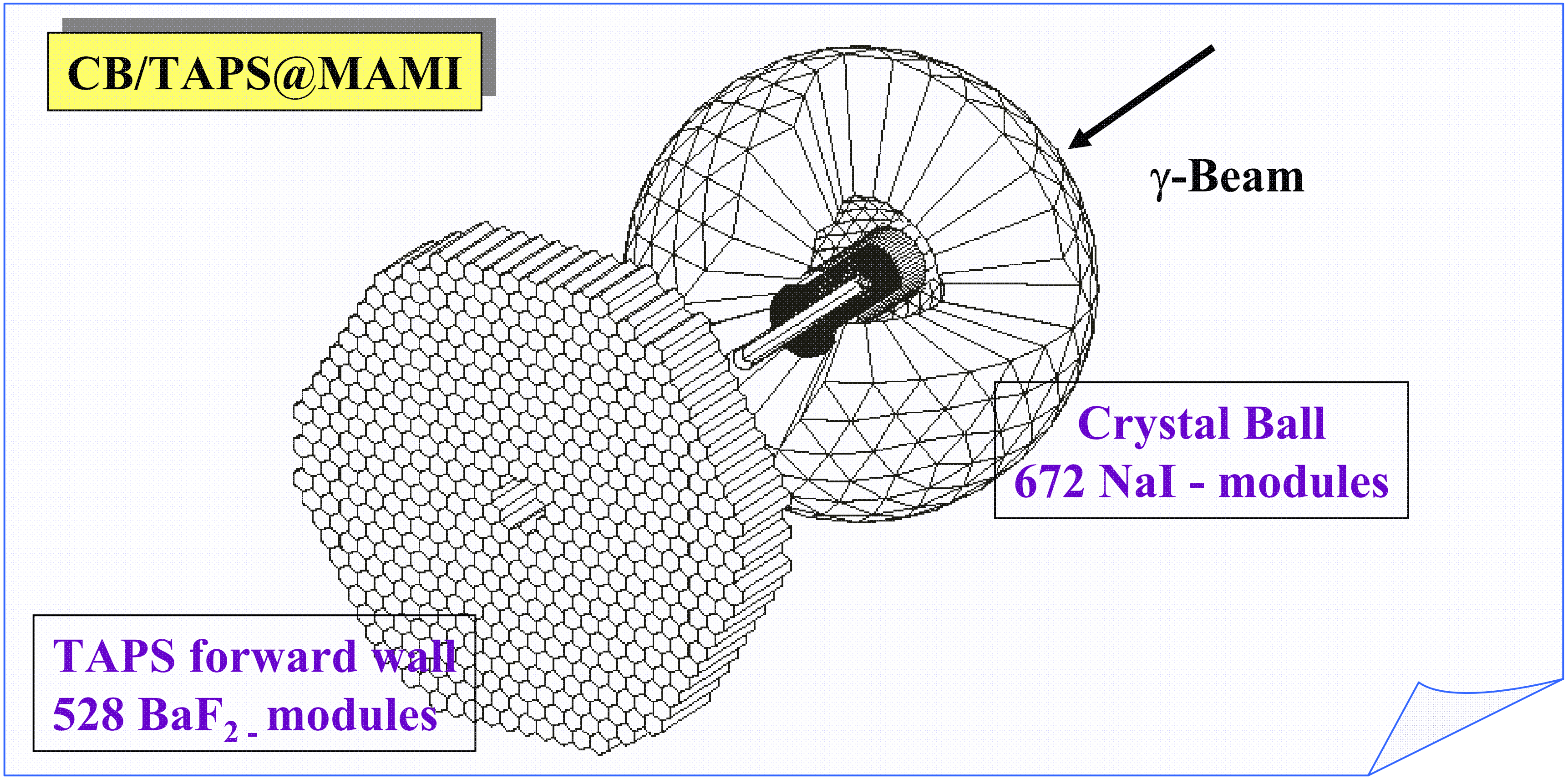
$$R \equiv \frac{1}{\sigma_\pi} \cdot E'_\gamma \frac{d\sigma}{dE'_\gamma}$$

2. Σ – linear-pol. photon beam asymmetry.

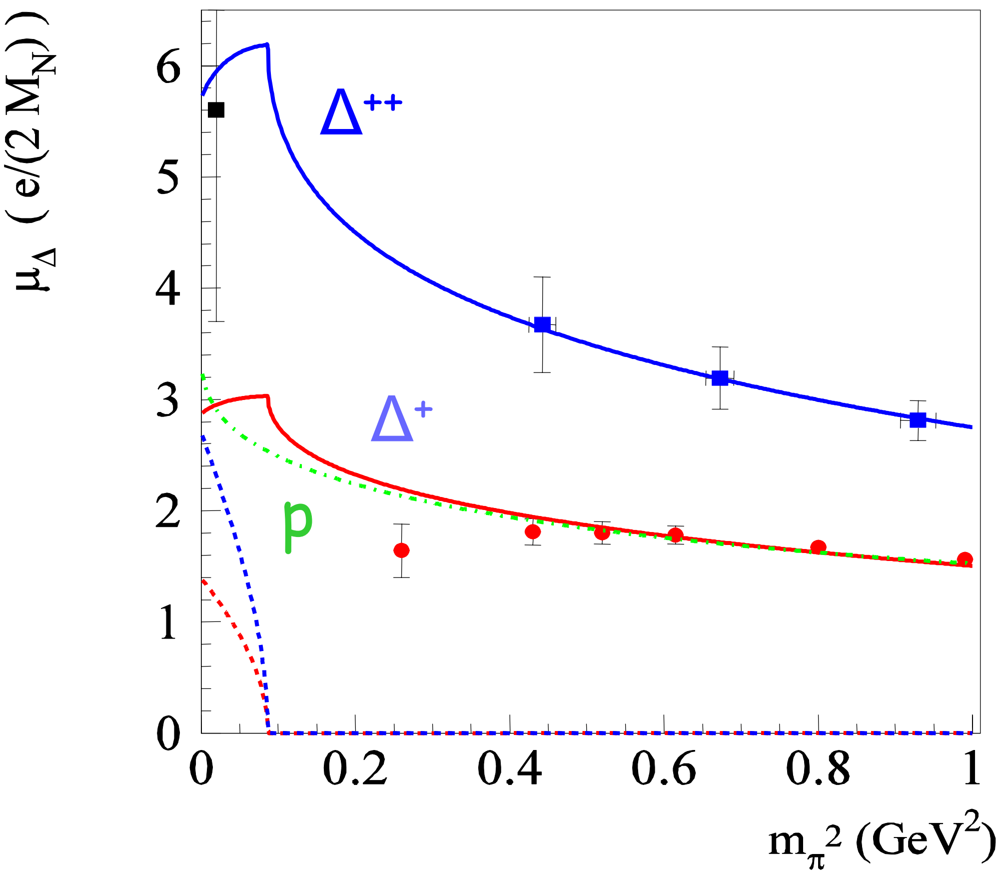
3. Σ_{circ} – circular-pol. photon beam asymmetry.



2005: New Crystal Ball Collaboration dedicated expt to improve statistics by two orders of magnitude.



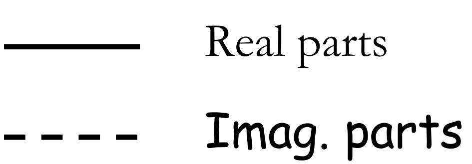
Chiral behavior of the Δ^{++} and Δ^{+} magnetic moments



Lattice data points from

[1] D.B. Leinweber, Phys. Rev. D (1992); I.C. Cloet, D.B. Leinweber and A.W.Thomas, Phys. Lett. B563 (2003).

[2] F.X. Lee *et.al.*, hep-lat/0410037



Summary

- **GDH** $\frac{e^2}{2M^2}\kappa^2 = \frac{1}{\pi}\int_0^\infty \frac{d\omega}{\omega}\Delta\sigma(\omega)$ **, and GDH'** $\frac{e^2}{M^2}\kappa = \frac{1}{\pi}\int_0^\infty \frac{d\omega}{\omega}\,\Delta\sigma'(\omega)|_{\kappa_0=0}$

analyticity constraint

- **Magnetic moment of the nucleon (chiral behavior)**

Exact analyticity extends the limit of applicability of ChEFT

- **Magnetic moment of the Delta-resonance: experimental determination and chiral behavior**

ChEFT provides framework for both

Power counting for the Δ(1232) (δ-expansion)

- ✓ The excitation energy of the Δ resonance, $\Delta = M_\Delta - M_N$¼290 MeV resonance can also be treated as small: $\delta = \Delta/\Lambda_\chi$¿ 1.
- ✓ Feature of the "δ-expansion": scale hierarchy m_π¿ Δ ¿ Λ_χ. => $\delta = \Delta/\Lambda$, $m_\pi/\Lambda = \delta^2$.
- ✓ This distinguishes the *low-energy* (p » m_π) and the *resonance* (p » Δ) regions.
- ✓ Crucial for *correct* counting of the One-Δ-reducible (OΔR) graphs:

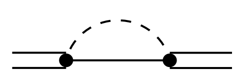
OΔR propagator

$$\frac{1}{p-\Delta}$$

»

p » m_π, $1/\Delta = O(1/\delta)$ [c.f., S_N» $1/p = O(1/\delta^2)$]

p » Δ , $1/(p-\Delta-\Sigma) = O(1/\delta^3)$

$\Sigma =$  $+ \ldots = O(p^3) = O(\delta^3)$