INT Seattle, April 2006

Results for dynamical chirally improved fermions

Christian B. Lang
Inst. F. Physik – FB Theoretische Physik
Universität Graz

In collaboration with P. Majumdar and W. Ortner

Phys. Rev. D 73 (2006) 034507
(= hep-lat/0512014)
hep-lat/0512045

BernGrazRegensburg
QCD collaboration

UNI GRAZ

1. **Action, parameters and updating**
2. Monitoring equilibration and quality
3. Some results

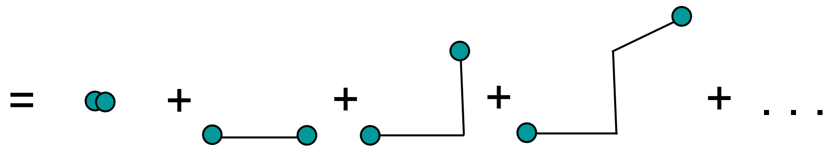
GW-type Dirac operators

- Overlap (Neuberger) ← Exact but very expensive
- „perfect“ (Hasenfratz et al.)
- Domain Wall (Kaplan)
- „Chirally Improved“ fermions

Approximate

= systematic (truncated) expansion

$$D=\sum_{\alpha=1}^{16}\Gamma_\alpha\sum_{p\in P_\alpha}c_p^\alpha\left\langle l_1 l_2 \ldots . l_{|p|}\right\rangle$$



Gattringer PRD 63 (2001) 114501;
Gattringer et al. Nucl. Phys. B697 (2001) 451

…obey the Ginsparg-Wilson relations approximately and have similar circular shaped spectrum (still some fluctuation!)

Dynamical study: parameters

- Lüscher-Weisz gauge action
- Chirally improved fermions
- Stout smearing

- 8^3x16 and 12^3x24
- lattice spacing 0.11 … 0.14 fm
- n_f=2 light quarks

- Hybrid Monte Carlo simulation:
  Each „trajectory“ (one unit HMC time):
  - Pseudofermion formulation
  - Molecular dynamics trajectory (100 steps)
  - Monte Carlo accept/reject step with P=det(D'/D) estimated with one pseudofermion

  Several 100 units HMC time

- Observables:
  - M_π, M_ρ, f_π, eigenvalue spectrum, top. charge

MD fermion force term

- Dirac operator contains longer paths: the force term is technically more complicated

$$\phi^\dagger \frac{d}{dt}(M^\dagger M)^{-1}\phi \longrightarrow$$

with $\dot{U}_{i,\mu} = i p_{i,\mu} U_{i,\mu}$

$$\frac{dM}{dt} = \sum_{j,\mu,k} c_{j,\mu,k} W^{(1)}_{j,\mu,k} (\pm i p_{j,\mu}) W^{(2)}_{j,\mu,k}$$

coefficients products of links

- Cyclically reorder terms such that p comes to the front

Run parameters

#	$L^3 \times T$	β_1	am	Δt	steps	acc.	HMC	a_S[fm]
a	$12^3 \times 24$	5.2	0.02	0.008	120	0.82(2)	463	0.115(6)
b	$12^3 \times 24$	5.2	0.03	0.01	100	0.94(2)	363	0.125(6)
c	$12^3 \times 24$	5.3	0.04	0.01	100	0.93(1)	438	0.120(4)
d	$12^3 \times 24$	5.3	0.05	0.01	100	0.92(2)	302	0.129(1)
e	$8^3 \times 16$	5.3	0.05	0.015	50	0.93(1)	1245	0.135(3)
f	$8^3 \times 16$	5.4	0.05	0.015	50	0.93(2)	649	0.114(3)
g	$8^3 \times 16$	5.4	0.08	0.015	50	0.94(1)	776	0.138(3)

1. Action, parameters and updating

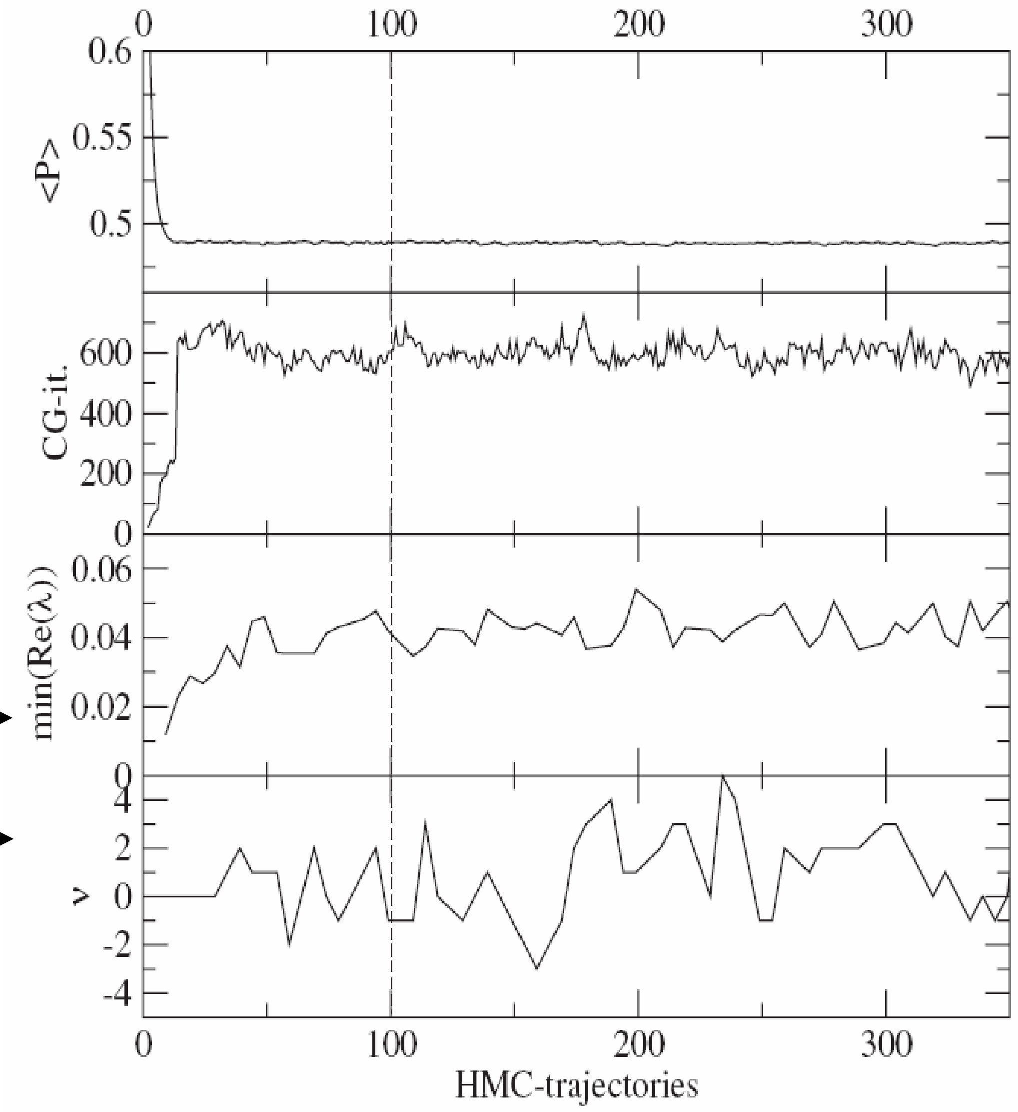
2. Monitoring equilibration and quality

3. Some results

Equilibration signals

$12^3 x24$
$\beta=5.2$
$a\ m=0.03$
$a_S=0.125(6)$

measured every 5th configuration



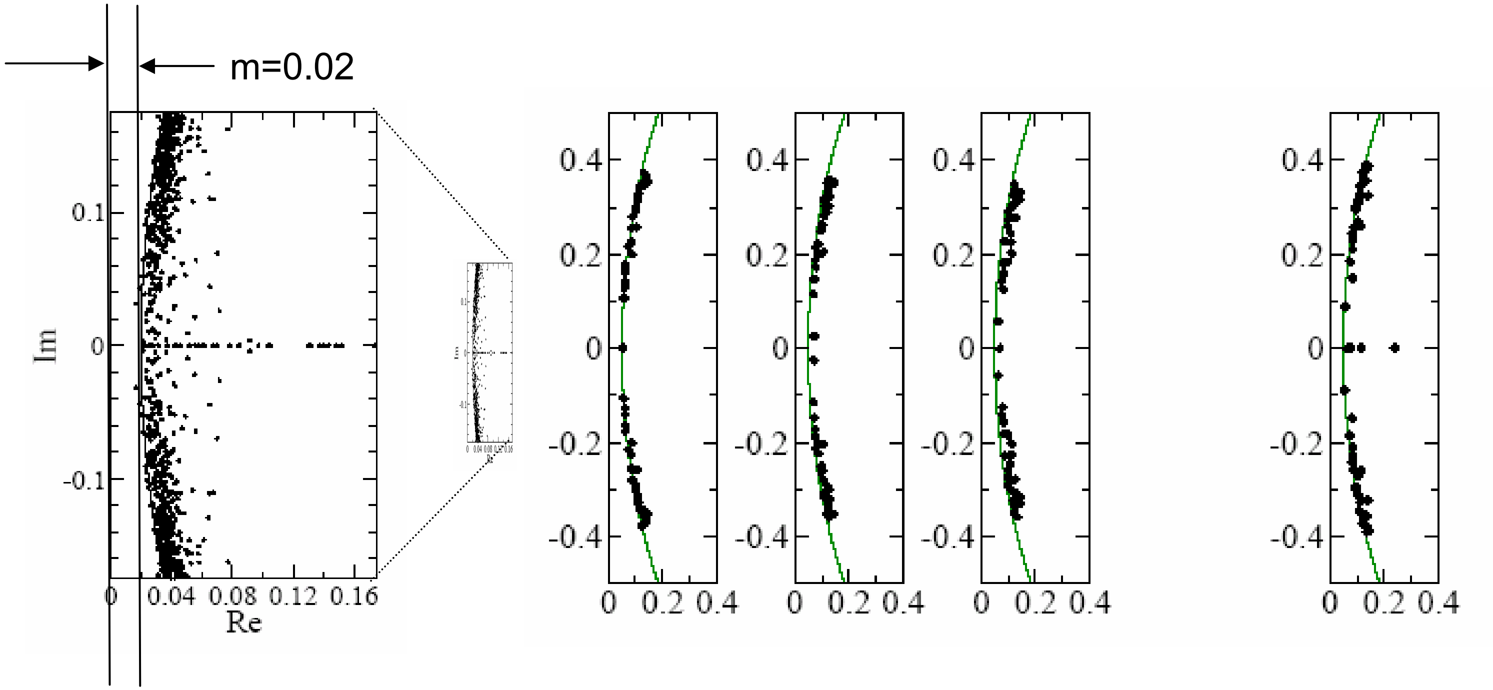
plaquette

CG it's for a/r step

smallest eigenvalue of D_{CI}

top. charge

Spectral properties of D_{CI}



12^3x24
(21 configurations)
(dynamical)

8^3x16
(single configurations)
(dynamical)

8^3x16
(quenched)

Topological lumps

$$\rho_0(x) \equiv \sum_{c,\alpha} \overline{\psi}_{c\alpha}(x)\psi_{c\alpha}(x)$$

$$\rho_5(x) \equiv \sum_{c,\alpha,\beta} \overline{\psi}_{c\alpha}(x)(\gamma_5)_{\alpha\beta}\psi_{c\beta}(x)$$

$$I_k = V \sum_{x} \rho_k(x)^2$$

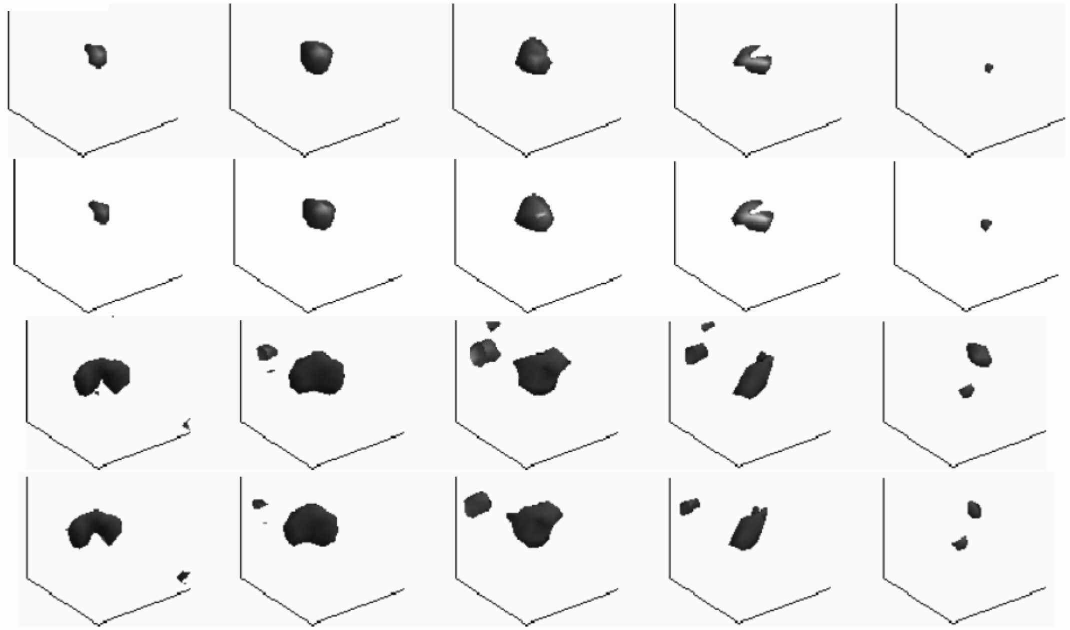
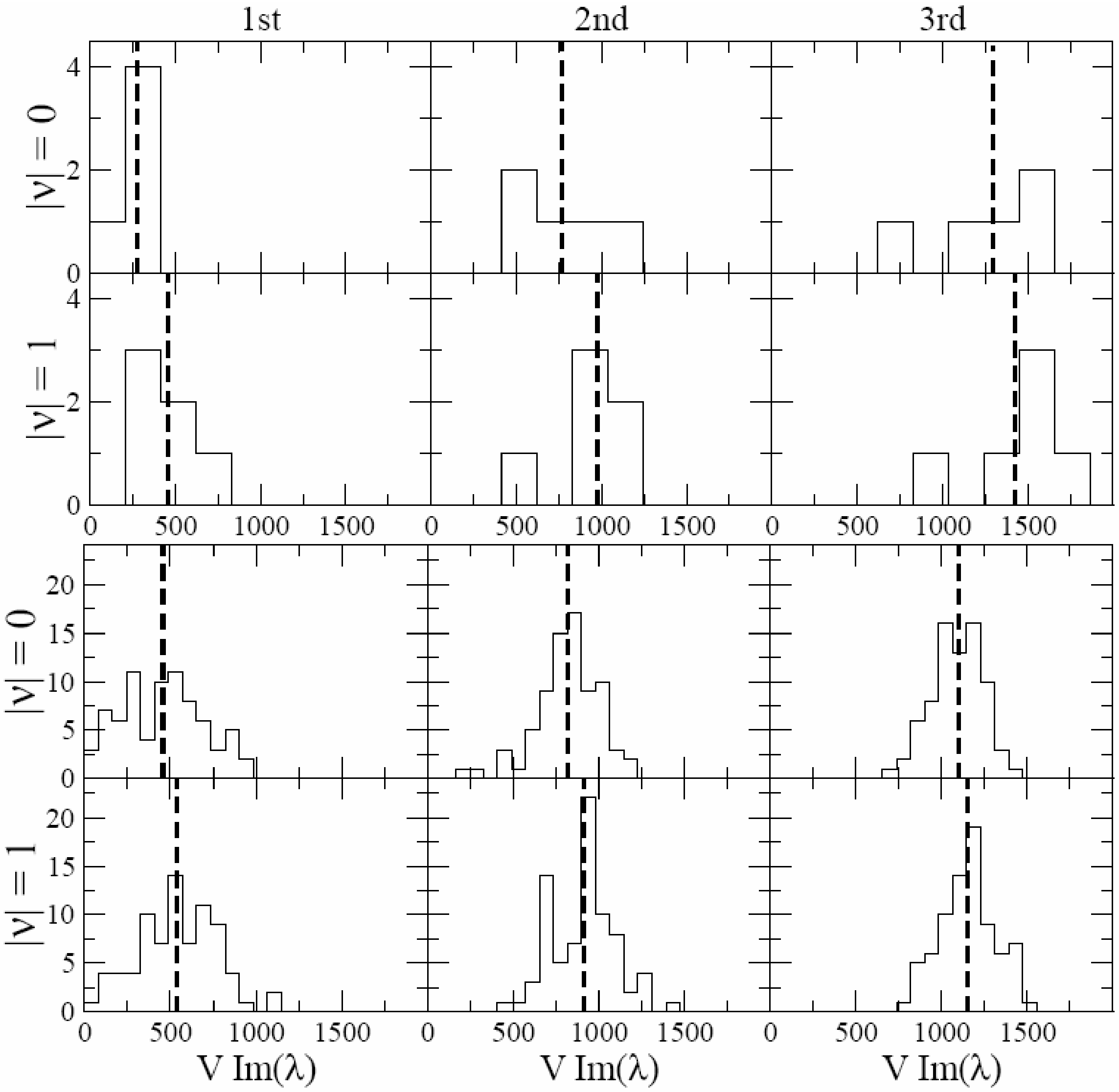


Figure 1: Iso-surfaces at time slices close to the maximum of the scalar density. Rows 1 and 2: typical case of a single lump; row 1: ρ_0 ($I = 15.6$), row 2: ρ_5 ($I = 12.1$). Rows 3 and 4: several lumps in one eigenvector; row 3: ρ_0 ($I = 3.2$), row 4: ρ_5 ($I = 2.4$).

Smallest non-real eigenvalues: Volume scaling

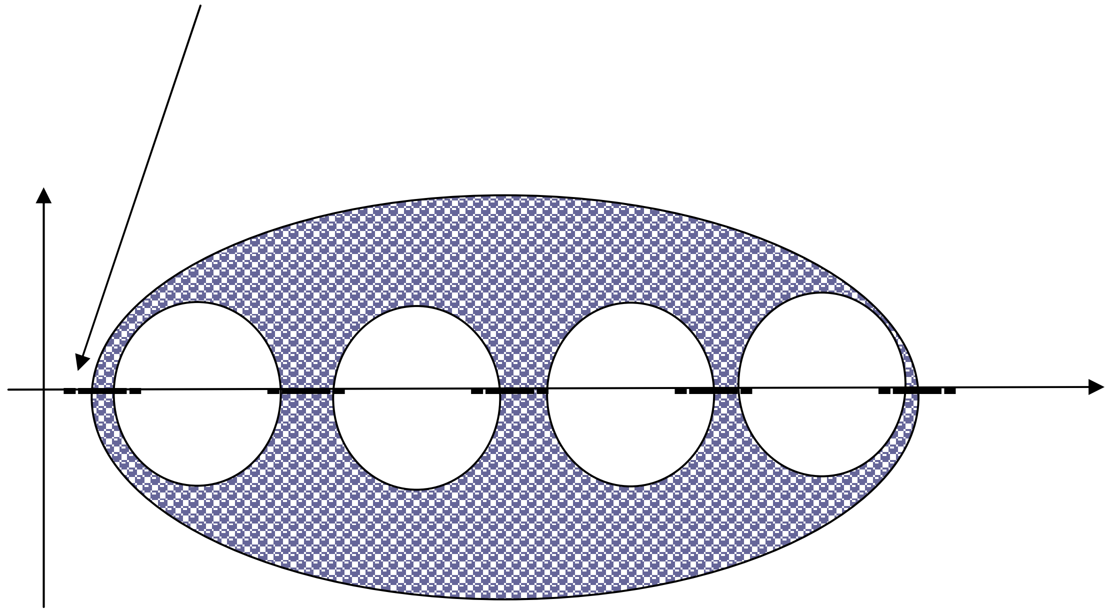
12^3x24

8^3x16



The issue of the fluctuating real eigenvalues

- The simple Wilson Dirac operator breaks the chiral symmetry badly:
  - Duplication of fermions
  - No chiral zero modes
  - Scattered eigenvalues, spurious small eigenmodes

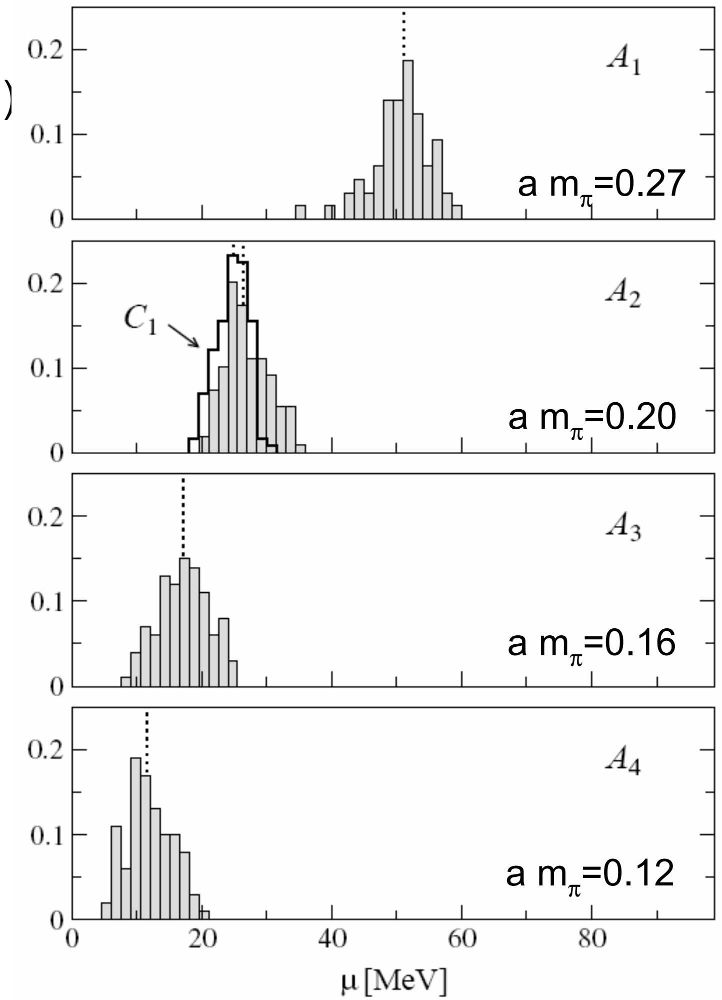


Smallest eigenvalue distribution

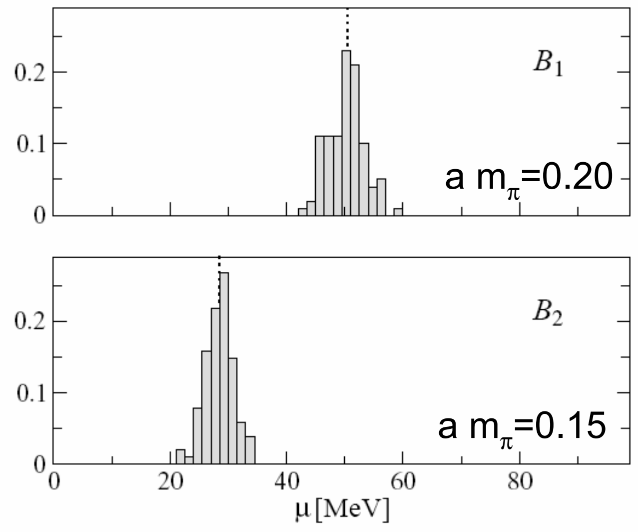
DelDebbio et al.
JHEP 0602(2006)

Study of the spectral gap of the hermitian Dirac operator for dynamical Wilson fermions

24^3x32

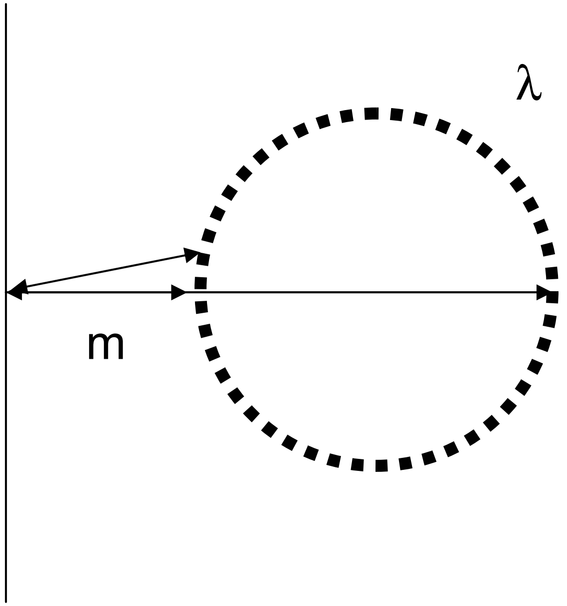


32^3x64

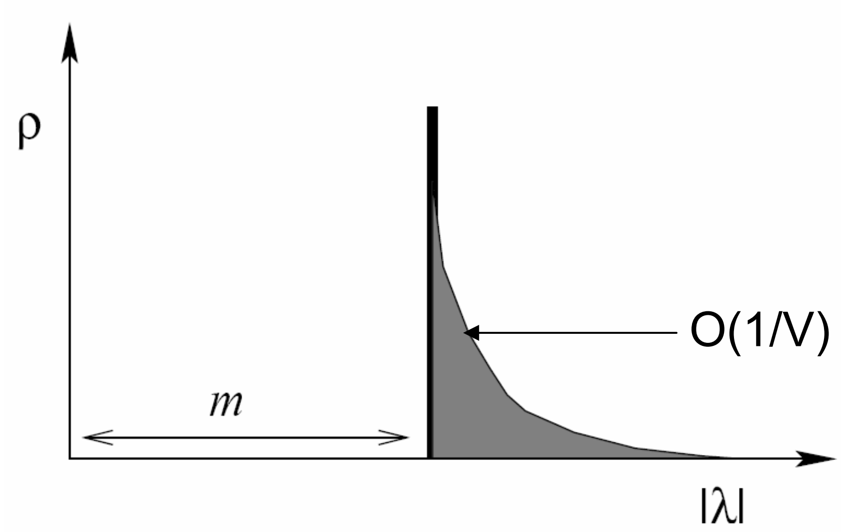


- AWI-mass ~ median of distribution
- Distribution becomes narrower with lattice size

Smallest eigenmode for hermitian overlap operator

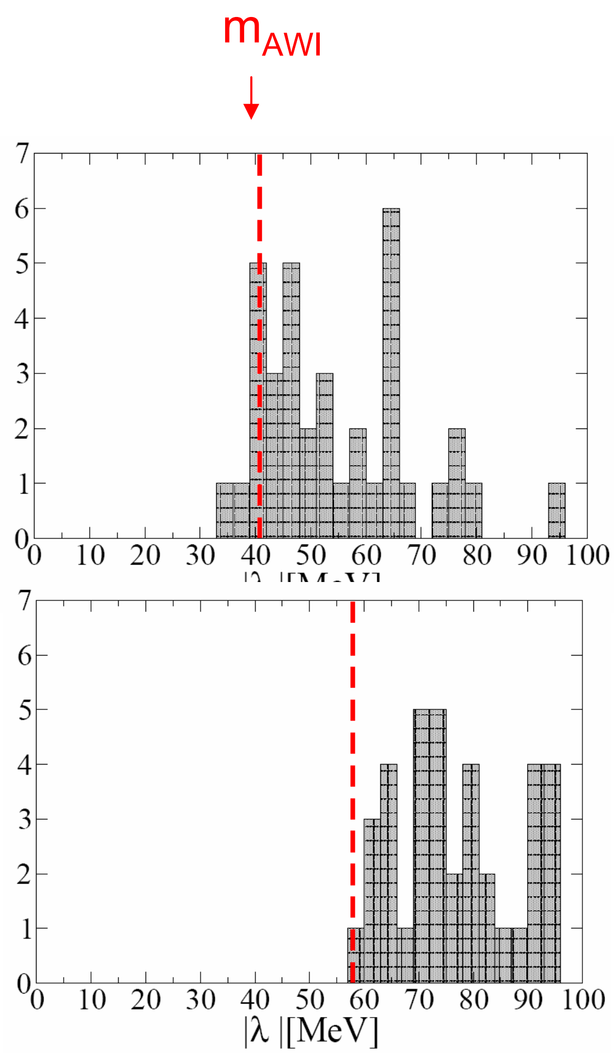


- Can be obtained from |λ| of the normal overlap operator
- Exact real modes=topological charge
- Complex modes: RMT distribution for volume - dependence



Smallest eigenmode for D_{CI}

12^3x24
a m=0.02
a m_π=0.29



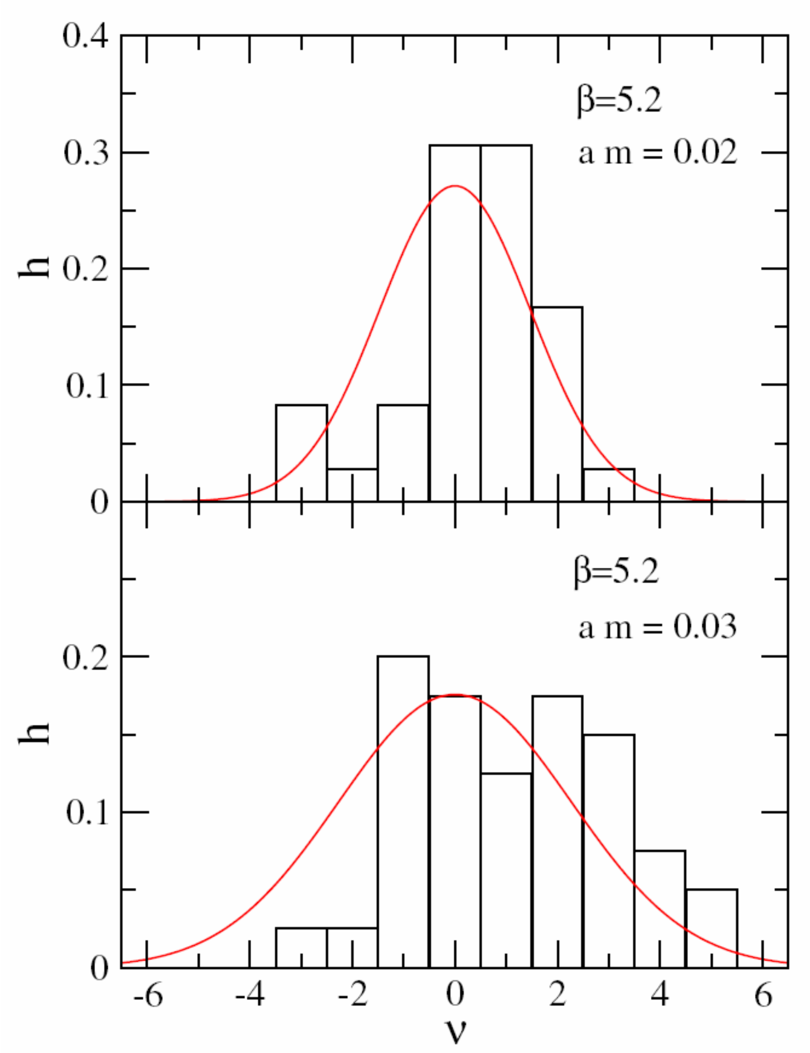
12^3x24
a m=0.03
a m_π=0.38

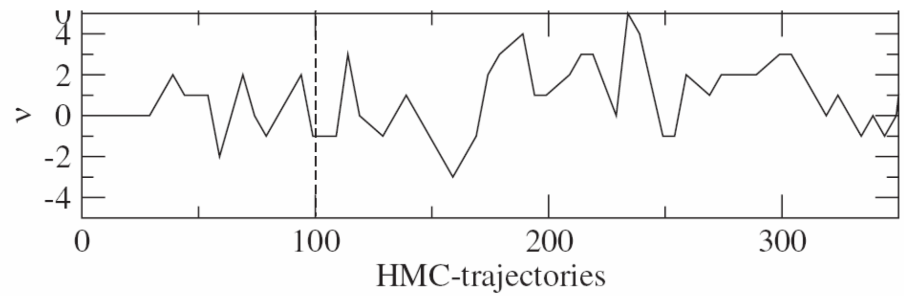
$$\frac{\langle \partial_t A_4(\vec{p}=\vec{0},t)P(0)\rangle}{\langle P(\vec{p}=\vec{0},t)P(0)\rangle} \equiv 2m_{\mathrm{AWI}}$$

AWI-mass is closer to l.h. edge of the distribution

1. Action, parameters and updating
2. Monitoring equilibration and quality
3. **Some results**

Topology and tunneling





(Only every 5th config. was measured)

Topological susceptibility

$$\chi_{\text{top}} = \langle \nu^2 \rangle / V$$

$$\chi_{\text{top}}^{\text{quenched}} = \frac{f_\pi^2 m_{\eta'}^2}{2N_f}$$

Quenched simulations:
~ (190 MeV)^4

$$\chi_{\text{top}}^{\text{dyn}} = -\frac{m\Sigma}{N_f} + \mathcal{O}(m^2)$$

Our (dyn.) result
a m=0.03: (166(8) MeV)^4
a m=0.02: (146(8) MeV)^4

However: Possibly very long fluctuations … large systematic error

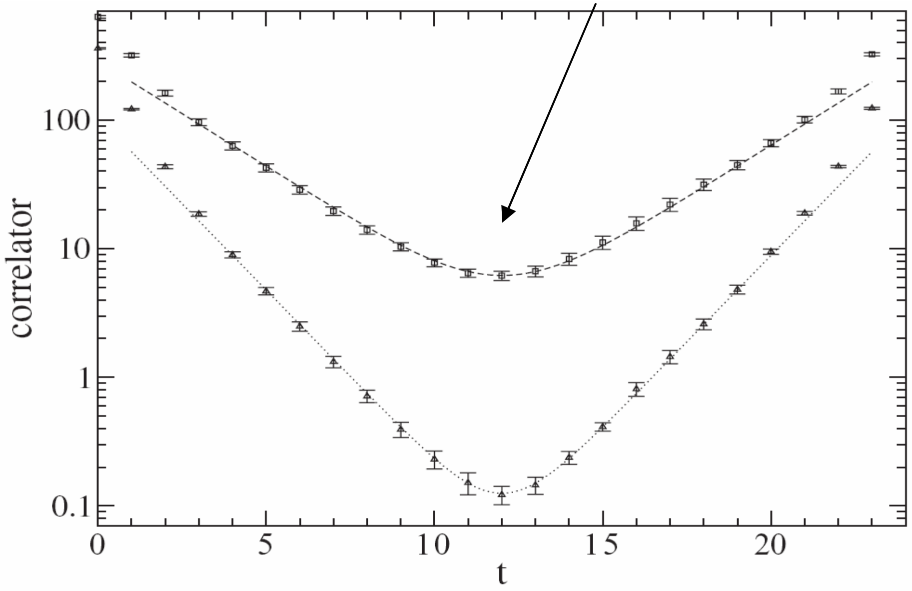
Meson masses

$$e^{-Mt} + e^{-M(N-t)}$$

$$P = \overline{d}\gamma_5 u$$

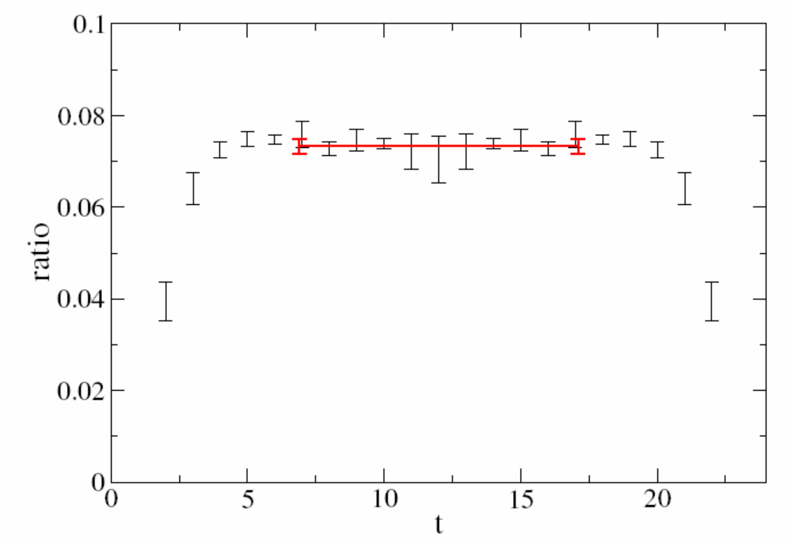
$$A_4 = \overline{d}\gamma_5\gamma_4 u$$

$$V_k = \overline{d}\gamma_k u$$



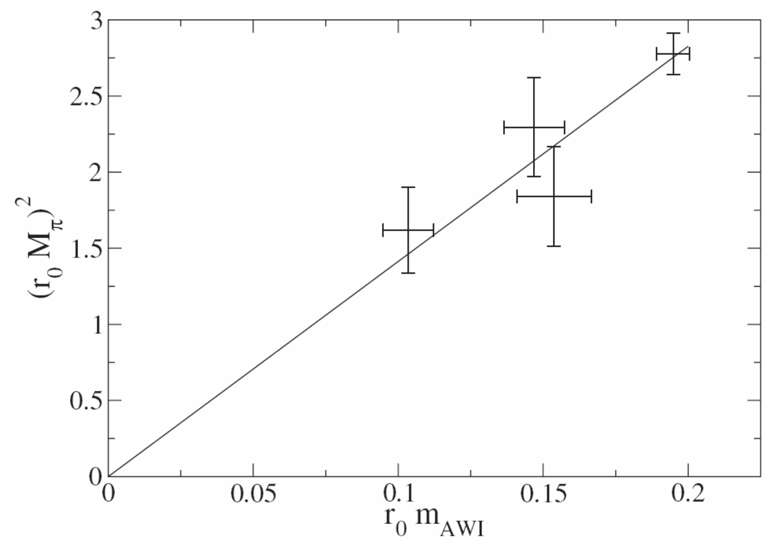
#	aM_π	aM_ρ	M_π/M_ρ	M_π [MeV]	M_ρ [MeV]
a	0.292(10)	0.535(35)	0.55(5)	501(44)	918(109)
b	0.378(8)	0.619(30)	0.61(4)	597(42)	977(96)
c	0.326(18)	0.502(21)	0.65(6)	534(48)	823(62)
d	0.431(8)	0.626(18)	0.69(3)	657(16)	954(33)

AWI-mass



$$\frac{\langle \partial_t A_4(\vec{p} = \vec{0}, t) P(0) \rangle}{\langle P(\vec{p} = \vec{0}, t) P(0) \rangle} \equiv 2 m_{\mathrm{AWI}}$$

12^3x24
β=5.2
a m=0.03



am_{AWI}	$r_0^2 M_\pi^2$	$r_0 m_{\mathrm{AWI}}$
0.025(1)	1.62(28)	0.103(9)
0.037(1)	2.29(33)	0.147(10)
0.037(2)	1.84(33)	0.154(13)
0.050(1)	2.78(14)	0.195(6)

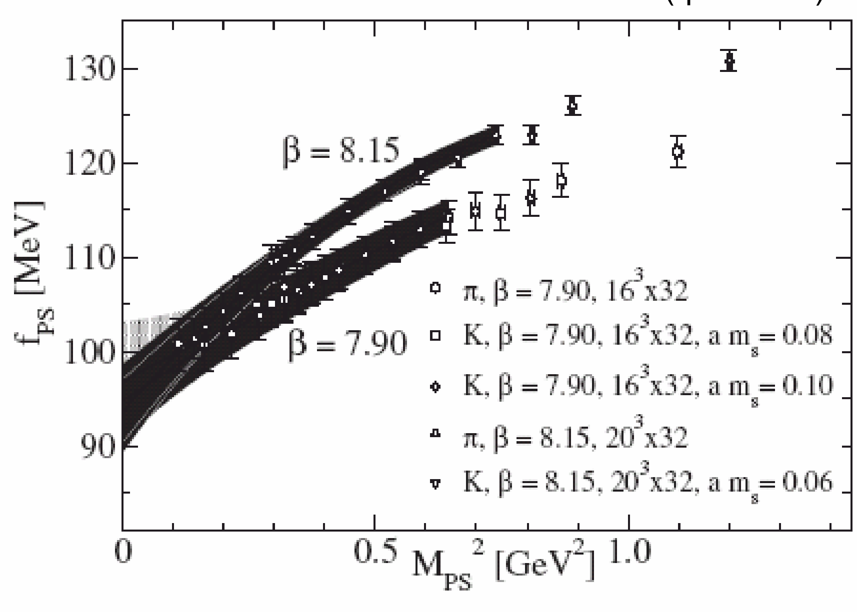
Pion decay constant

quenched

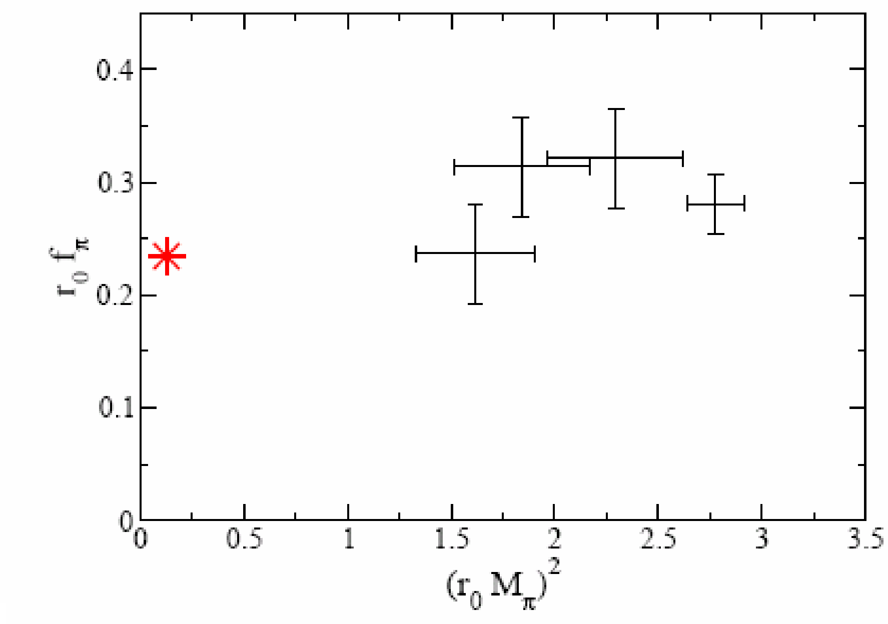
Gattringer, Huber, CBL (2005)

this work (2 dyn. quarks)

(quenched)



(dynamical)



Outlook

- Improved updating
- 16^3x32 for a=0.15 fm (i.e., 2.4 fm size)
- Pion mass down to 300 MeV