Calculating the kaon B parameter using mixed action lattice QCD

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Outline

I. Brief review of Lattice QCD + Flavor Physics

II. Different types of lattice fermions (staggered and domain wall), and Pros + Cons of each

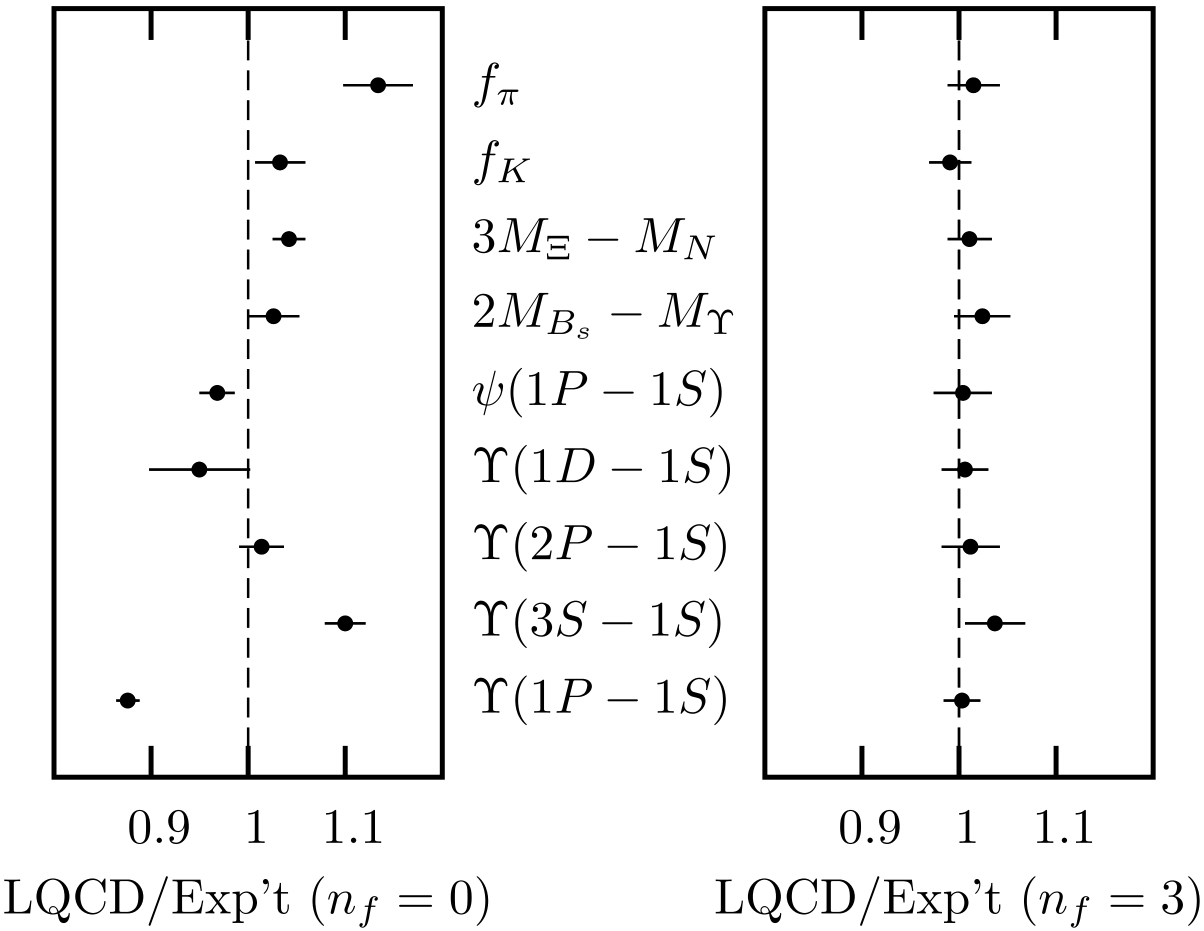
III. Mixed Actions and Chiral Effective Theory

IV. Our Proposal for B_K

Lattice QCD

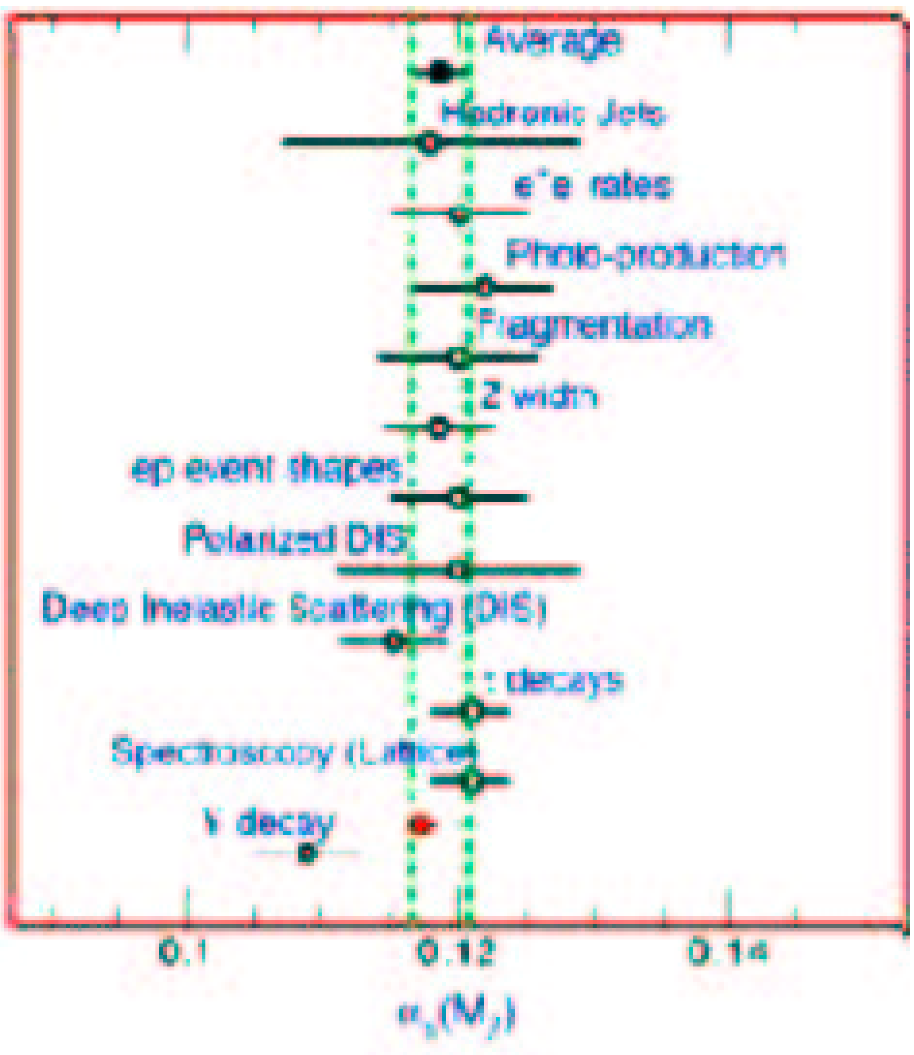
- Allows non-perturbative calculations from first principles
- Can be simulated on a computer using Monte Carlo methods
- Simulations require a finite-sized grid with lattice spacing a and size L
- Even with today's computers this is still a difficult task!

Successes of Lattice QCD



- Hadron spectroscopy – masses and decay constants
- *Good agreement for simple quantities!*

Further success: α_s



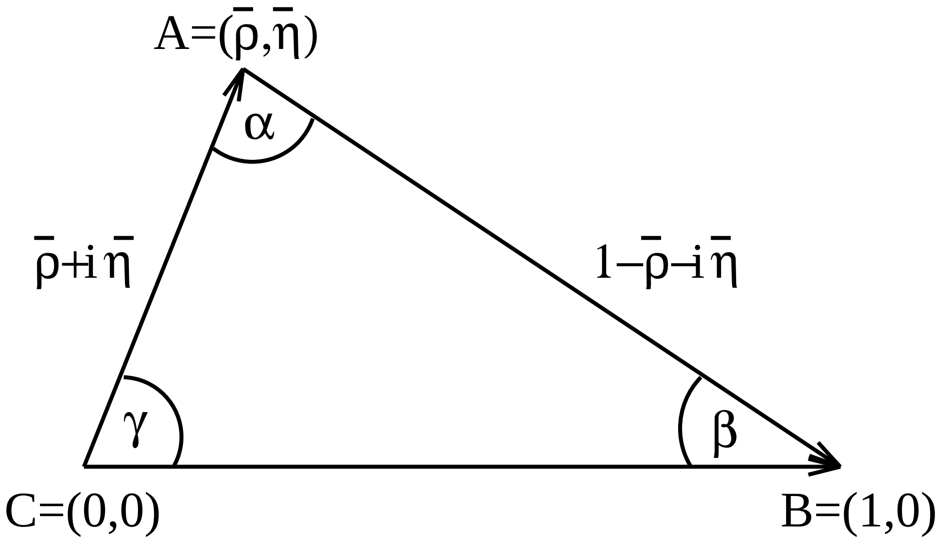
- The strong coupling constant (latest lattice result in RED)

What next? Flavor physics!

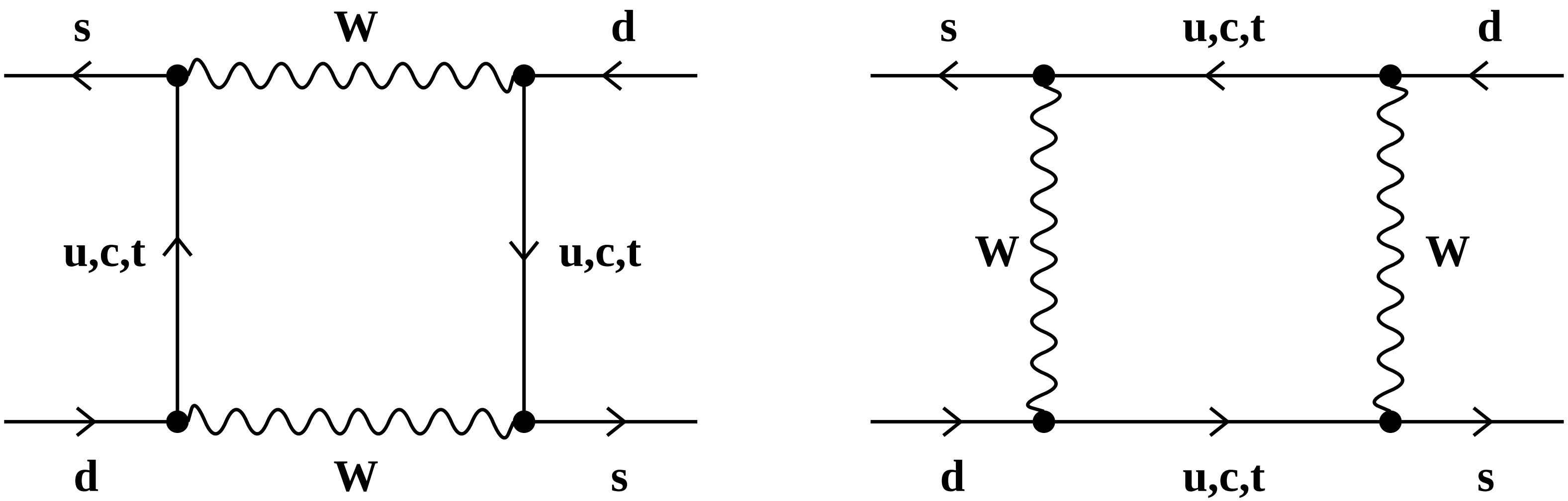
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \quad (1)$$

Unitarity implies:

$$V_{ub}V_{ub}^* + V_{cb}V_{cb}^* + V_{tb}V_{tb}^* = 0$$

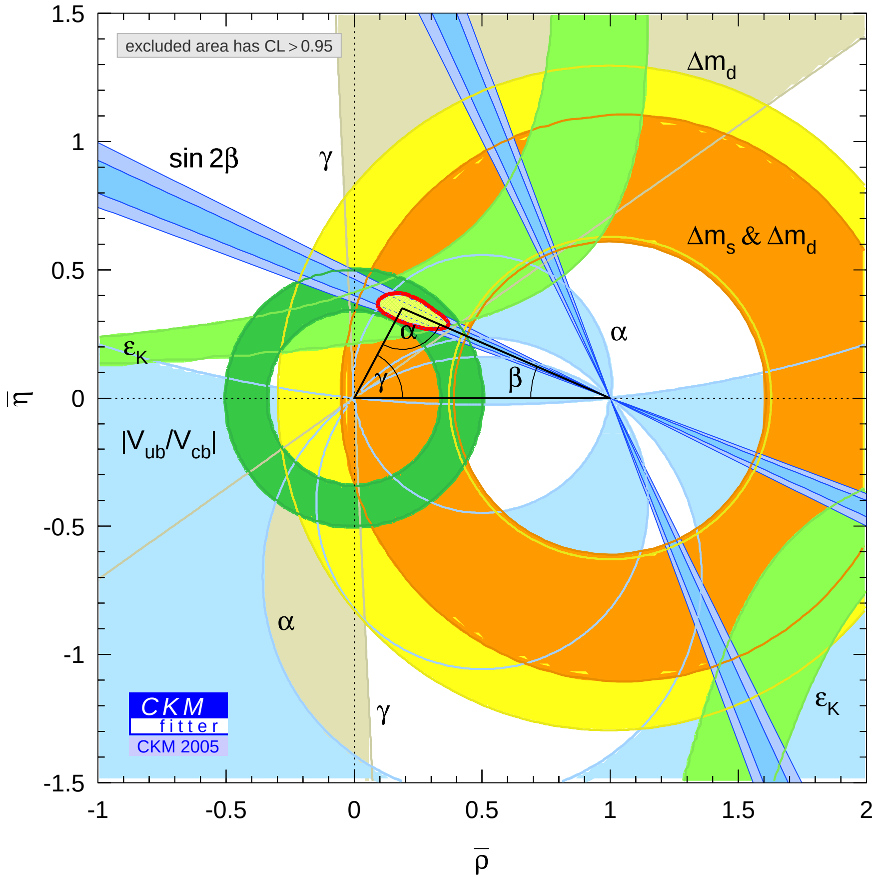


Kaon B parameter



$$B_K = \frac{\langle \overline{K^0} | [\overline{s_a} \gamma_\mu (1 - \gamma_5) d_a][\overline{s_b} \gamma_\mu (1 - \gamma_5) d_b] | K^0 \rangle}{\frac{8}{3} m_K^2 f_K^2}$$

Constraining the Unitarity Triangle



$$|\epsilon_K| = C_\epsilon B_K A^2 \overline{\eta}\{-\eta_1 S_0(x_c)(1-\lambda^2/2) + \eta_3 S_0(x_c, x_t) + \eta_2 S_0(x_t) A^2 \lambda^2 (1-\overline{\rho})\}$$

Current numerical status of B_K

- Benchmark calculation used in unitarity triangle fits [JLQCD]:

$$B_K^{NDR}(2\text{GeV}) = 0.628(42) \leftarrow \text{quenched!} \tag{2}$$

- Dynamical domain-wall fermions [RBC]:

$$B_K^{NDR}(2\text{GeV}) = 0.563(15)(21) \leftarrow \text{unknown extrapolation error} \tag{3}$$

- Dynamical staggered fermions [HPQCD,UKQCD]:

$$B_K^{NDR}(2\text{GeV}) = 0.618(18)(19)(130) \leftarrow \text{large mixing error} \tag{4}$$

Some discretizations of fermions:

- Staggered: A remnant of chiral symmetry is preserved, but additional flavors, called“tastes” are introduced. These vanish in the continuum limit, but must be accounted for in typical simulations.

- Domain wall: Good chiral properties due to an added dimension, but more expensive.

The Staggered Lattice Action

- The staggered action (in Euclidean space) appears simple:

$$S_F = (a^4) \sum_x \left(\frac{1}{a} \alpha_\mu(x) \left[\overline{\chi}(x)\chi(x + a\widehat{\mu}) - \overline{\chi}(x + a\widehat{\mu})\chi(x) \right] + m\overline{\chi}(x)\chi(x) \right)$$

  - χ and $\overline{\chi}$ are single fermionic (anticommuting) variables that live on the sites of the lattice, labeled by x
  - $\widehat{\mu}$ is a unit vector along the μth lattice direction
- $\Rightarrow$ Staggered action really the naive quark action of four degenerate quark species on a lattice of spacing $2a$
- Because the spinor components are spread out (staggered) over a hypercube – *staggered fermions*

Pros and Cons of Staggered Fermions

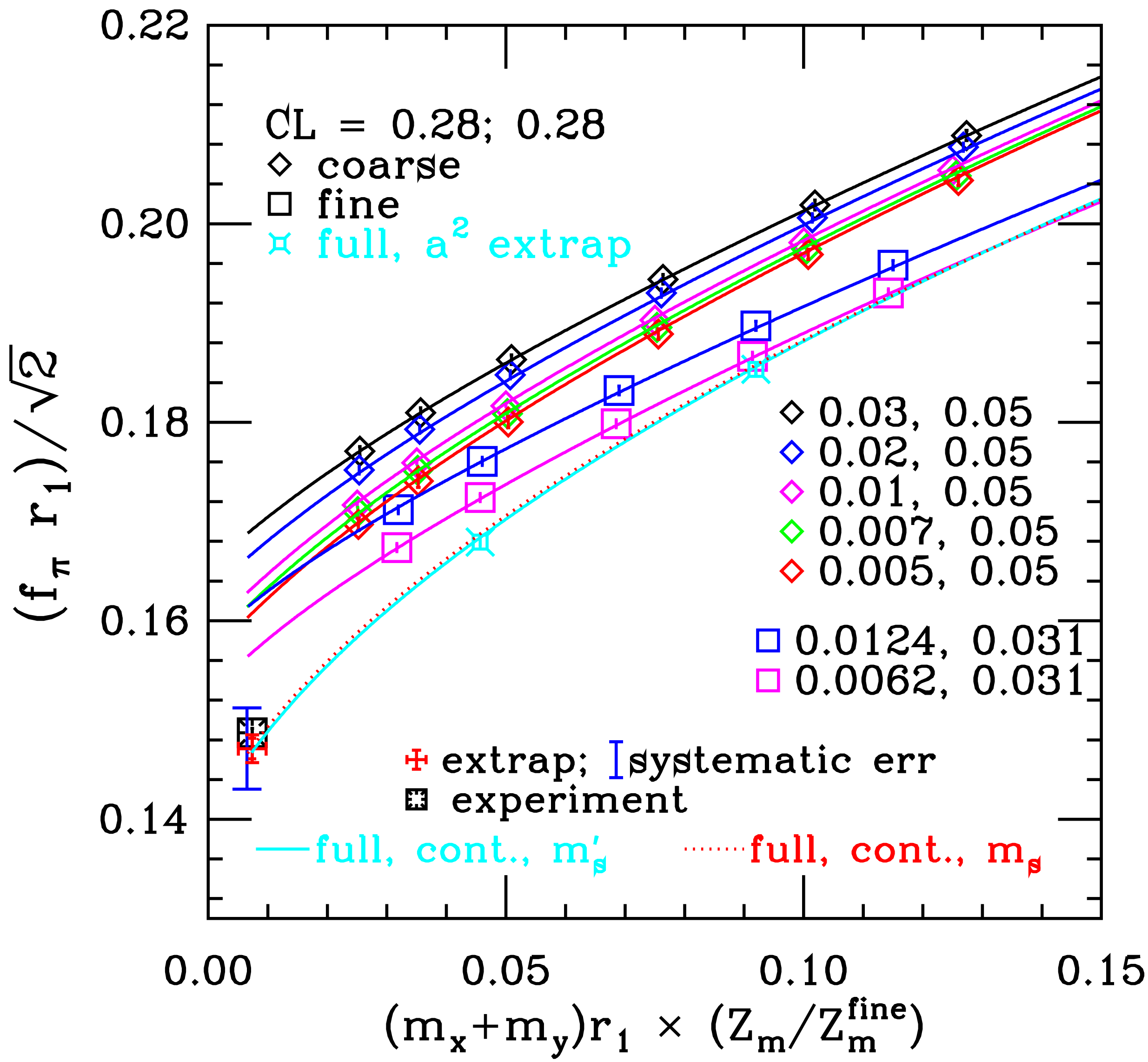
Pros:

- They are CHEAP!
- There is a remnant of chiral symmetry

Cons:

- Taste breaking
- Nonperturbative renormalization is very complicated
- The “4th root trick” (?)

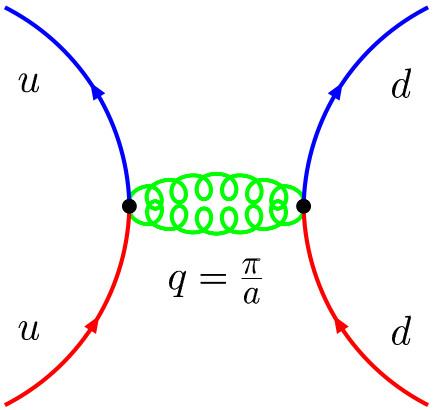
Chiral Extrapolation



Taste Symmetry Breaking

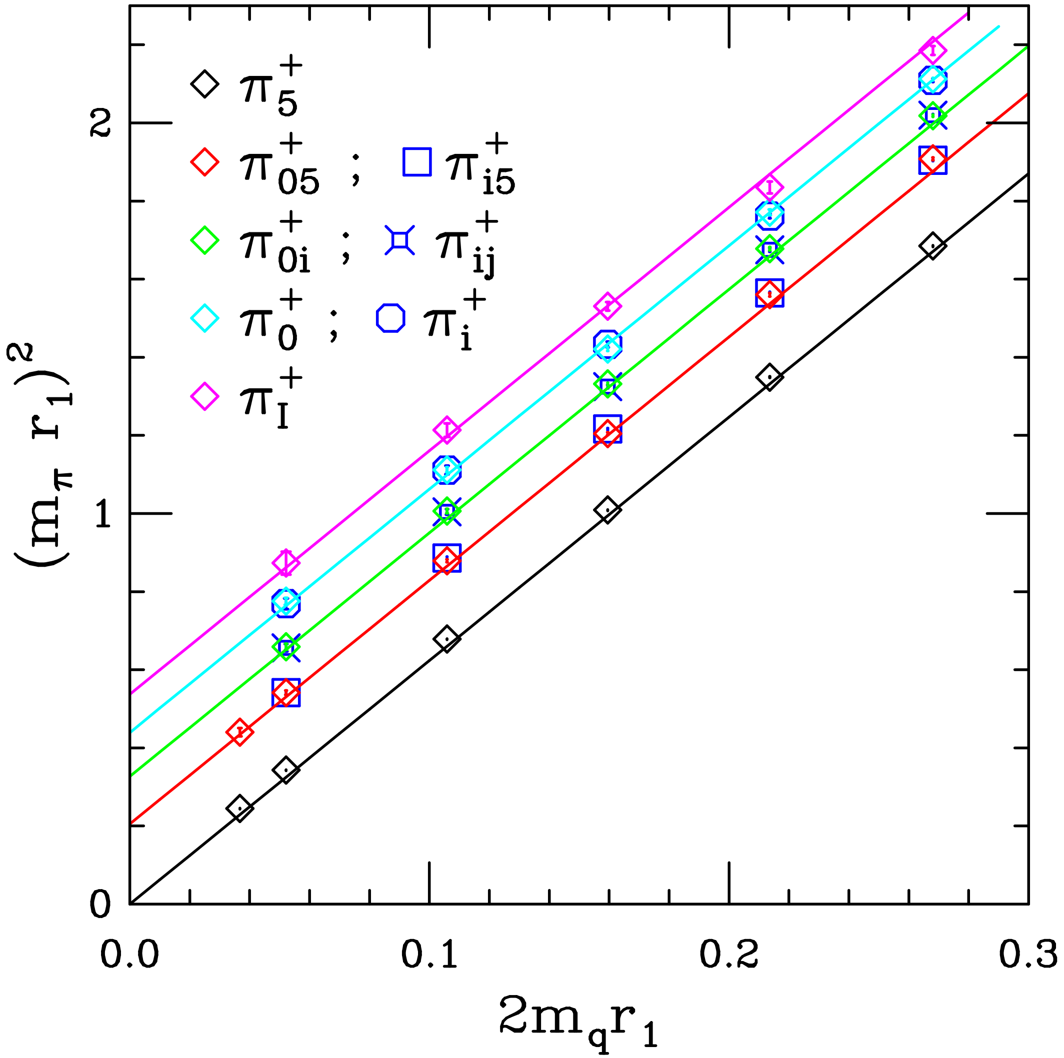
- Staggered quarks come in 4 tastes ⇒ staggered mesons come in *16 tastes*
- Labeled by the *taste matrix* in the lattice operator: $\pi_T \equiv \overline{Q}_i(\gamma_5 \otimes \xi_T)Q_j$

1 Singlet – ξ_I	1 Goldstone – ξ_5	
4 Vector – ξ_μ	4 Axial – $\xi_{\mu 5}$	6 Tensor – $\xi_{\mu\nu}$



- On the lattice, quarks of one taste can turn into another by exchanging high-momentum gluons

Taste Splittings



The “Fourth Root Trick"

- In the continuum limit, the four staggered tastes become degenerate
- In principle, taste breaking can be removed by taking the continuum limit, but in practice one must take the fourth root at finite lattice spacing. It is possible that the continuum limit of this theory is not QCD
- This is an open theoretical issue: It has not been proven correct, but it has not been proven wrong
- Staggered simulation agree well with experiment, at least for the simplest quantities!
- I will assume the validity of the 4th root trick in the rest of this talk

B_K on the Lattice

- Lattice version of B_K operator *mixes* with other lattice operators:

$$\mathcal{O}_K^{staggered,cont} = \mathcal{O}_K^{staggered,lat} + \frac{\alpha}{4\pi}[\text{taste } P \text{ ops.}] + \underbrace{\frac{\alpha}{4\pi}[\text{other taste ops.}] + \alpha^2[\text{all taste ops.}] + a^2[\text{all taste ops.}] + \ldots}_{\text{neglected in lattice simulations}}$$

- 1-loop matching coefficients between $\mathcal{O}_K^{staggered}$ and four-fermion lattice operators are known and are numbers of order $\alpha/4\pi$
- Lattice simulations really calculate this matrix element:

$$\langle \overline{K^0_1}_P | \mathcal{O}_{1-loop}(\text{taste } P) | K^0_{2\,P} \rangle \equiv \mathcal{M}_{\text{lat}}$$

- $\Rightarrow$ The lattice matrix element is not what is desired:
- Extra tastes make nonperturbative renormalization difficult to apply
- Currect 2+1 staggered calculations implement lattice-to-continuum matching to 1-loop in α_s [Becher, Gamiz + Melnikov; Lee + Sharpe]
- *Matching is the dominant source of error* – 20% of B_K

Domain Wall Fermions

$$S_F = \sum_{x,y,s,s'} \overline{\psi}(\delta_{s,s'}\gamma_\mu D^\mu_{x,y} + \delta_{x,y}\gamma_\mu D^\mu_{s,s'})\psi \quad (5)$$

- First term couples points on the same 4D slice and is just a standard Wilson lattice Dirac operator
- Second couples points in the 5^{th} dimension:

$$\gamma_\mu D^\mu_{s,s'} = [P_R\delta_{s+1,s'} + P_L\delta_{s-1,s'} - \delta_{s,s'}] \quad (6)$$

Domain Wall Fermions

- Construct 4D light quark fields from left and right handed projections of the full 5D field on the boundary:

$$q(x) = P_L\psi(x,0) + P_R\psi(x, N_S - 1) \quad (7)$$

$$\overline{q}(x) = \overline{\psi}(x,0)P_L + \overline{\psi}(s, N_S - 1)P_R \quad (8)$$

- Lefthanded quarks bound to the $s = 0$ boundary. Righthanded quarks bound to the $s = N_s - 1$ boundary
- In the limit $L_s \to \infty$, no coupling between LH and RH components
- For finite L_s (the distance between the domain walls), the overlap between LH and RH fields generates an exponentially small residual quark mass: $am_{res} \propto exp(-\lambda L_s)$

Pros and Cons of DWF

Pros:

- near exact chiral symmetry
- Nonperturbative renormalization can be carried out numerically

Cons:

- Very expensive!
- Ensembles are not yet publicly available

Mixed Action Lattice QCD

Lattice calculations have two stages:
1. Generate a sample of field configurations weighted by $exp(-S_QCD)$ using importance sampling methods:

$$e^{-S_{QCD}} = \det(\gamma_\mu D^\mu + m) e^{-S_{glue}} \tag{9}$$

2. Measure the ensemble average of a given operator (such as a propagator) on these background configurations:

$$(\gamma_\mu D^\mu + m)^{-1} \tag{10}$$

A different choice of Dirac operator during these steps leads to a mixed action simulation.
If the action is the same but the sea and valence masses are different, this is partially quenched QCD.

Mixed Actions: The Best of Both Worlds

Staggered Sea:

- Light dynamical quarks, so small error from chiral extrapolation
- Publicly available MILC lattices

Domain Wall Valence:

- No taste breaking in valence sector
- Can account for operator mixing nonperturbatively

Review of ChPT

Operators are constructed out of the unitary chiral matrix field Σ,

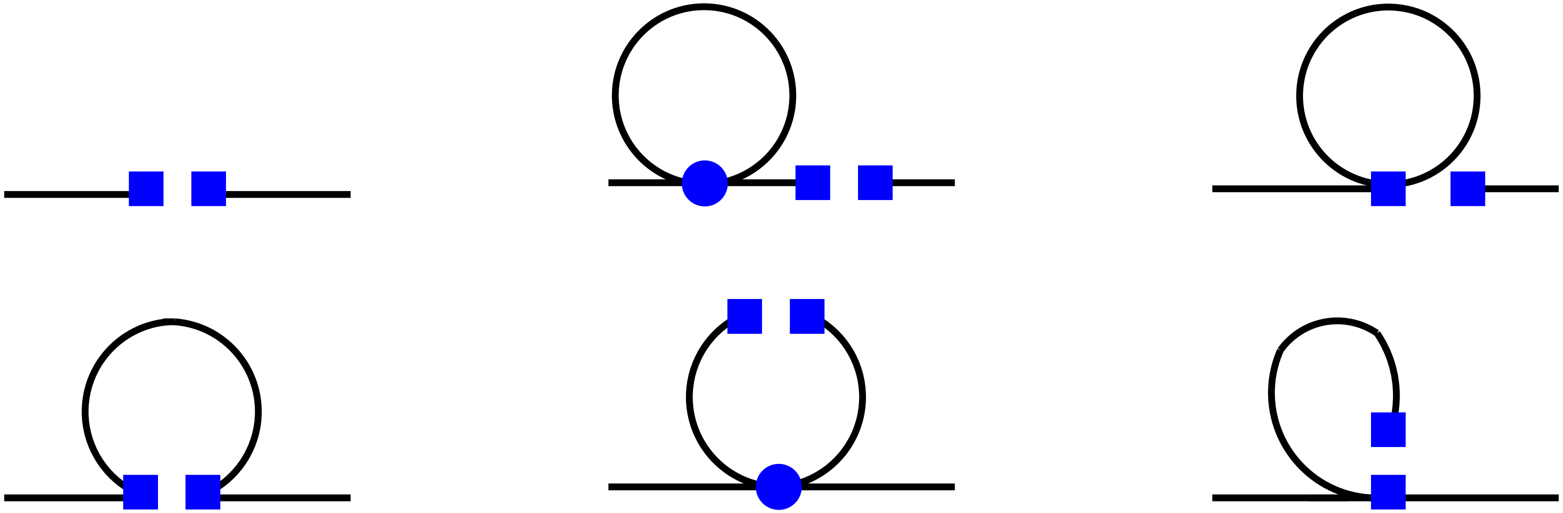
$$\Sigma = \exp\left[\frac{2i\phi^a\lambda^a}{f}\right], \tag{11}$$

where λ^a are proportional to the Gell-Mann matrices with $\text{tr}(\lambda_a\lambda_b) = \delta_{ab}$, ϕ^a are the real pseudoscalar-meson fields, and f is the meson decay constant in the chiral limit.

Expansion in quark masses and momentum

The theory is nonrenormalizable! At each order in ChPT new terms appear which must be obtained nonperturbatively, or from experiment.

B_K at NLO in continuum ChPT



$$B_K = B_0 \left[1 - \frac{6}{(4\pi f)^2} m_K^2 \ln \frac{m_K^2}{\mu^2} \right] + C m_K^2$$

Continuum effective action for lattice theories

- Use method of Symanzik to construct continuum-level effective theory

$$S_{\text{lat}} = S_{\text{dim.4}} + aS_{\text{dim.5}} + a^2 S_{\text{dim.6}} \quad (12)$$

- The lattice spacing, a, is now an explicit parameter
- New higher dimension operators contain all lattice effects

Mixed Action ChPT

- Ghost quarks are introduced with $m_{ghost} = m_v{}^{alence}$, as in partially quenched ChPT, to cancel the valence contributions in the loops

- No terms linear in a because there are no dimension 5 operators compatible with all lattice symmetries

- Three types of dimension 6 operators

  1. Contain only sea quarks - “usual” staggered ChPT
  2. Contain both valence and sea - this is new
  3. Contain only valence quarks - nothing new beyond the continuum

The Chiral Effective Mixed Action

$$\mathcal{L}_{\text{MA}\chi\text{PT}} = \frac{f^2}{8}\text{Tr}(\partial_\mu \Sigma \partial_\mu \Sigma^\dagger) - \frac{1}{4}\mu f^2 \text{Tr}(\mathcal{M}\Sigma + \mathcal{M}\Sigma^\dagger) + a^2\mathcal{U}_S + a^2\mathcal{U}_V$$

$a^2\mathcal{U}_S$ from mixed valence and sea 4-fermion operators
$a^2\mathcal{U}_V$ from sea quark only 4-fermion operators

Tree level valence-valence pions proportional to quark masses like in the continuum:

$$m_{xy}^2 = \mu(m_x + m_y) \tag{13}$$

Tree level sea-sea pions are:

$$m_{SS'}^2 = \mu(m_S + m_{S'}) + a^2\Delta_t \tag{14}$$

Mixed valence-sea pions

Mixed valence-sea pions at tree level are given by:

$$m_{SV}^2 = \mu(m_S + m_V) + a^2\Delta_{Mix} \tag{15}$$

This new parameter $a^2\Delta_{Mix}$ has not been measured yet. It can be obtained by looking at the mixed quark spectrum.

Fortunately, it does not appear in B_K at NLO!

B_K in Mixed Action ChPT

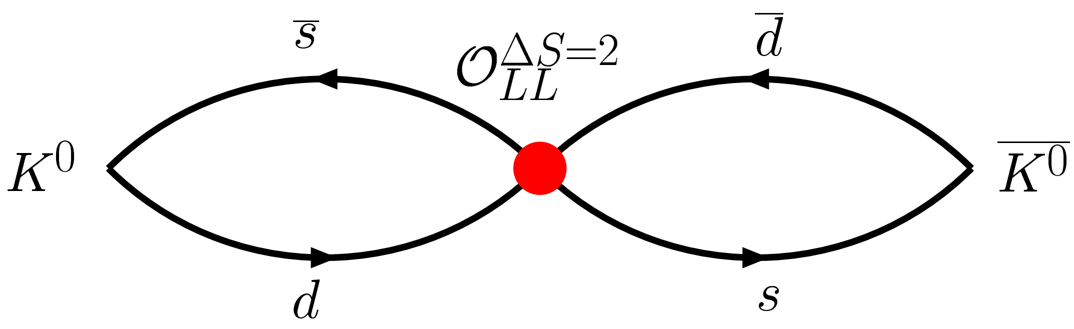
$$\left(\frac{B_K}{B_0}\right)^{\text{PQ},2+1} = 1 + \frac{1}{16\pi^2 f^2 m_{xy}^2}\left[I_{conn} + I_{disc}^{2+1} + c_1 a^2 m_{xy}^2 + c_2 m_{xy}^4 + c_3(m_X^2 - m_Y^2) + c_4 m_{xy}^2(2m_D^2 + m_S^2)\right], \tag{16}$$

$$I_{conn} = 2m_{xy}^4 \tilde{l}(m_{xy}^2) - l(m_X^2)(m_X^2 + m_{xy}^2) - l(m_Y^2)(m_Y^2 + m_{xy}^2) \tag{17}$$

$$I_{disc}^{2+1} = \frac{1}{3}(m_X^2 - m_Y^2)^2 \frac{\partial}{\partial m_X^2}\frac{\partial}{\partial m_Y^2}\left\{\sum_j l(m_j^2)\left(m_{xy}^2 + m_j^2\right) R_j^{[3,2]}(\{M_{XY,I}^{[3]}\};\{\mu_I^{[2]}\})\right\} \tag{18}$$

Proposal

- MILC 2+1 lattices with staggered (Asqtad) lattices are publicly available for a variety of sea quark masses and lattice spacings
- Chroma lattice QCD software package contains optimized code for making domain-wall propagators
- We will be writing code to make the B_K 3-pt function using pre-existing libraries in lattice QCD software QDP++



- We will be sharing propagators with LHP and NPLQCD collaborations to save computing time

Conclusions

- B_K is an important parameter for constraining physics beyond the Standard Model. It must be known to 5% to have impact
- We believe that multiple methods should be used in order to gain confidence in the predictions
- Mixed Action simulations combine the best of both worlds of staggered and domain wall lattice fermions
- We have calculated B_K to NLO in mixed action ChPT and are working on the numerical side