Workshop Summary and assorted remarks

Andreas Kronfeld

$$[\det_4 M]^{1/4} \doteq \det_1(\not{D} + m)?$$

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Why me?

- JWN: For crying out loud, Andreas, the INT staff need the title of your talk!!!
- ASK: Uh, I didn't send in a title, John, because I don't have anything to say.
- JWN: Great! In that case, you can be the summary speaker.

The Stakes

- Highest: LQCD aims to help experiments find new phenomena in *B* decay.
- Lower: $[\det_4 M]^{1/4}$ just has to be as rigorous as the alternatives:
  - Partially quenched chiral PT
  - Non-perturbative violations of power counting in Symanzik LEL

What to Summarize?

- You heard the talks; I'm not wiser than you.
- It still helps to have a crisp synopsis of what (I think) we've learned.
- I'll include some remarks on material not presented here.
- Feel free to shout at me.

What I tell others

- The validity of $[\det_4 M]^{1/4}$ has not been proven valid.
- It hasn't been proven invalid.
- The lattice community has to sort it out.
- The lattice community will sort it out.

Why are we here?

- LQCD with $[\det_4 M]^{1/4}$ reproduces several "gold-plated" decay constants, masses, and mass splittings at the 2% level.
- LQCD with $[\det_4 M]^{1/4}$ successfully predicted a handful of gold-plated quantities at the 5-10% level.

- Some people can't believe it, because the validity of $[\det_4 M]^{1/4}$ hasn't been proven rigorously. (What in QFT has been?)
- Some people won't believe it, for rather unscientific reasons. Alas.
- The validity of LQCD with $[\det_4 M]^{1/4}$ must be resolved.

The Criticisms

- $[\det_4 M]^{1/4}$ is (obviously) non-local.
- $[\det_4 M]^{1/4}$ (obviously) violates unitarity.
- Staggered quarks mess up thresholds.
- Topology.
- $[\det_4 M]^{1/4}$ ambiguous when $\mu \sim m$.
- Staggered quarks fail for $m < 0$.

Taste Basis

$$[\det_4 M]^{n_f/4} = \det_4 \begin{pmatrix} \begin{matrix} D+m & 0 \\ 0 & D+m \end{matrix} & aN \\ aN & \begin{matrix} D+m & 0 \\ 0 & D+m \end{matrix} \end{pmatrix}^{n_f/4}$$

- The reason we discuss $[\det_4 M]^{1/4}$, instead of $[\det_{16} M]^{1/16}$, is that aN doesn't vanish.
- Susskind, Kawamoto & Smit, Sharatchandra, Thun & Weisz showed how to reduce 16/4.

Obviously Non-Local?

- Some argue that $[\det_4 M]^{1/4}$ is non-local, just because $M^{1/4}$ is [Bunk *et al.*].

- Counter-argument:

$$[\det_4 M]^{n_f/4} = \det_4 \begin{pmatrix} D+m & 0 & \multicolumn{2}{c}{\multirow{2}{*}{aN}} \\ 0 & D+m & & \\ \multicolumn{2}{c}{\multirow{2}{*}{aN}} & D+m & 0 \\ & & 0 & D+m \end{pmatrix}^{n_f/4}$$

$$= \det_1 (D+m)^{n_f} \exp\left\{ \tfrac{n_f}{4} \operatorname{Tr} \ln \left[1 + a\mathcal{N}(D+m)^{-1} \right] \right\}$$

Golterman $\mu = 0$

- The second factor (MG called it T) is non-local at $a \neq 0$:
  - generates splittings between π_ξ
- No reason that it is non-local as $a \to 0$.
- No reason that $n_f \neq 4$ is worse.

Shamir

- Unfortunately, this kind of argument rests on expanding in fermion loops.
- YS showed a RG strategy, that could isolate the problem. Let $D_n = \mathcal{D}_n \times 1 + \Delta_n$:
  - 1) $|a_c \Delta_n| < C 2^{-n}$
  - 2) $\log[\det D_0 / \det D_n]$ is local

- The two requirements are rigorously satisfied in the free theory.

- They are plausible (*i.e.*, most believe) at every order in perturbation theory (though a staggered Reisz theorem would be nice).

- Nonperturbatively, all bets off (as for Wilson even with Reisz).

- Reduces ($a \to 0$) locality of $[\det_4 M]^{1/4}$ to existence of $\det_4 M$.

Obviously not Unitary?

- Some have said (but wisely not written down) that the sea and valence quarks are treated differently.
- Same action, but look carefully: taste projection vs. PQstagχPT.
- There are also some irrelevant and false arguments.

Das ist alles falsch!

- In a long discussion with a noted skeptic of $[\det_4 M]^{1/4}$, it was argued that misplaced thresholds of $\pi\pi$ scattering would violate unitarity (see Dürr's slides for drawing).
- They do make the extraction of the scattering length subtle.
- Not a problem of the 1/4 root per se.

- Noted skeptic also claimed that in two-loop perturbative QCD, the quark-quark thresholds are misplaced.
- As far as I know, the exact, discrete taste symmetry ensures that all taste receive the same mass renormalization.
- If so, I do not see how the claim could be true. Skeptic misspoke, or I misheard.

DeTar

- Nevertheless, for $a \neq 0$ there are clearly violations of unitarity.
- The lowest lying contributors to flavor-singlet taste-singlet correlators are undesired two-particle states.
- They can be understood using staggered chiral perturbation theory.

- The stagχPT analysis is partially quenched (see below).

- StagχPT shows how the unphysical states cancel each other in the continuum limit, leaving an unphysical piece of order a^2.

- Same argument presumable applies to glueballs: they are (incorrectly) unstable at $a \neq 0$, but widths vanish as a^2.

Follana

- For unimproved actions on coarse lattices, staggered quarks did not conform to expectations concerning topology.
- Dramatic improvement seen with improved actions and at finer lattice spacing.
- Quartets and zero modes emerge in concert.

- A new (at least to me) result concerned the comparison with random matrix theory

- Must focus on volumes so that all pions are in the ε-regime.

- Then there is good agreement, although ... [Damgaard].

A. Hasenfratz

- In Schwinger model, overlap and staggered determinants are roughly similar, for quark masses no too small.
- Many sore points in staggered, seem to surround approach to continuum limit. Do differences in 2d and 4d power counting make these studies less useful?

Golterman $\mu \neq 0$

- $[\det_4 M]^{1/4}$ is unacceptable when $\mu \sim m$.
- Some quartets surround origin and, hence, phase assignment unclear.
- So, for $\mu \neq 0$ don't use $[\det_4 M]^{1/4}$.
- Seems irrelevant for $\mu = 0$ though.

Bernard

- CB showed us how to formalize $[\det_4 M]^{1/4}$ in stagχPT using the replica trick.

- StagχPT is a loop expansion, so at any order the dependence on the number of replicas n_r is polynomial.

- Set $n_r = 1/4$.

- The construction is most transparent for 4 flavors, because the 1-flavor 4-taste theory is the same as the 4-flavor 1-taste theory.
- Solid argument to introduce splittings among the 4 flavors as mass insertions.
- Plausible argument to raise 4th flavor's mass so high that it decouples.

Partially Quenched

- This way of understanding stagχPT relies intimately on partially quenched χPT being sound.
- Normally [cf. Weinberg], the argument for an effective theory is that it is the most general parametrization of particle interactions, consistent with analyticity and unitarity.

- But partially quenched means that the number of sea and valence quarks differ.
- Not unitary.
- Do the standard arguments still go through? How should they be modified?
- NB: everyone will use PQχPT, even with other quarks, but it is less essential.

Dürr

- Valid for valence?
  - Yes, universal continuum limit.
- Valid for sea?
  - Yes, UV-filtered e-values match overlaps'.
- Valid in concert?
  - Take continuum limit first, then chiral.

Summary

- From Golterman and Shamir, I do not see a *new* nonlocality arising from the 1/4 root.
- It would be better to bring staggered fermions to the same footing as Wilson in pQCD [someone: generalize Reisz].
- Fear of nonperturbative violations of pQCD power counting seem misplaced.

- From DeTar, it is clear that violations of unitarity arise. They are very plausibly of order a^2.

- As one becomes more ambitious (going beyond gold-plated quantities), stagχPT becomes more and more important.

- As shown by Bernard, stagχPT requires PQχPT reasoning.

- A conceptual issue is to reexamine the assumptions of PQχPT [CB, PD, SRS?]
- A practical issue, would be to trace (at N^{n}LO) the uncertainty in gold-plated quantities (*i.e.*, well below thresholds) from, say, the a_0 contribution to the pion self-energy.

What I will tell others

- The validity of $[\det_4 M]^{1/4}$ has not been proven valid.
- It hasn't been proven invalid.
- The lattice community has to sort it out.
- The lattice community will sort it out.
- More evidence favors it than disfavors it.

And to critics

(reading this on the web)

- If you have a well-founded argument against staggered quarks, write it up. Your reluctance to do so weakens your position.